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# Antennas

## Volume 1

# General Principles

E Roubine and J C Bolomey

Translated by  
Meg Sanders



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**HEMISPHERE PUBLISHING CORPORATION**

A subsidiary of Harper & Row, Publishers, Inc.

Washington    New York    London

English translation © 1987 North Oxford Academic  
Publishers Ltd

Original French language edition  
(Antennes: Introduction Générale) © 1986 Masson, Paris

Revised and updated 1987

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English edition first published 1987 by North Oxford  
Academic Publishers Ltd, a subsidiary of Kogan Page  
Ltd, 120 Pentonville Road, London N1 9JN

Published in the U.S.A. by  
Hemisphere Publishing Corporation

**Library of Congress Cataloging-in-Publication Data**

Antennes. English.

Antennas.

Translation of: Antennes.

Includes bibliographies and indexes.

Contents: v. 1. General principles / E. Roubine and  
J. C. Bolomey—v. 2. Applications / S. Drabowitch and  
C. Ancona.

I. Antennas (Electronics) I. Roubine, E. (Elie)

II. Title.

TK7871.6.A513 1987 621.38'028'3 87-12111

ISBN 0-89116-278-X (v. 1)

ISBN 0-89116-279-8 (v. 2)

Set by Eta Services (Typesetters) Ltd, Beccles, Suffolk  
Printed and bound in Great Britain

**Antennas**  
**Volume 1**  
**General Principles**

# Foreword

This work was inspired by lectures given by its authors at the École Supérieure d'Électricité. It is not intended as a treatise on antennas. The mere size of this volume clearly rules out any such exhaustive study, which would fill an encyclopaedia. To pare down this enormous body of knowledge has been the greatest challenge to the authors, who have constantly felt the conflict between the necessary task of reducing and editing their lectures, and the desire to present modern and sometimes innovatory technical knowledge in as complete a way as possible.

The development of antennas, from the time of the first radio link established by Hertz in his well known experiment of 1887, shows the scale of the subject.

Marconi entered the field in 1901, with the first transatlantic reception, and this tremendous achievement has tended to overshadow those of the pioneers. But the success of his venture was based on Hertz's discharger and the antenna planned by Popov for the detection of storms over several kilometres, a Branly coherer attached to a lightning conductor (1896).

This early period was also marked by considerable theoretical activity. Hertz developed the theory of the dipole, Max Abraham that of the very thin prolate spheroid. Poincaré considered the cylinder, using integral equations. Even the results of Sommerfeld's work on terrestrial propagation, which formed a basis for the development of powerful long-wave links, had an influence on such large installations as Croix d'Hins or Sainte Assise (1921).

The distant ionospheric propagation of HF signals was discovered by amateurs shortly after the First World War. The period between the wars was marked by an increased number of arrays and rhombic antennas, while in parallel, radiodiffusion MF broadcasting was developed. This was the period of pylon antennas. Finally, the ideas of Hertz were readopted, with the first attempts at microwave direct links. The first parabolic reflector theory was developed by Darbord in 1931, and the notion of electromagnetic detection was already becoming accepted.

The use of radar in the Second World War led to an increased role for microwave antennas, the importance of which has been confirmed with the passing of time. In this context, electromagnetism proper once again took on the major role that the empiricism of the inter-war period, as exemplified by the use of the pylon antenna, had somewhat ignored. The return to electromagnetic orthodoxy also allowed the contradictions of the apparently natural model of the equivalent line to be overcome, and the observed impedances to be confirmed. This led to a turning point in the theory of common antennas: those developed along the lines of the cylindrical dipole (Hallen, 1938), the bicone

(Schelunkoff, 1941), and the prolate spheroid (Stratton and Chu, 1941) soon became famous. With hindsight, the role of the particularly difficult problem of input impedance can be seen to have been exaggerated, but the fact remains that research carried out in this context contributed to a clearer understanding of the functioning of antennas.

The benefits of electromagnetic theory in the area of microwaves have proved far more durable. It renewed the theory of diffraction, and the attention devoted to it in universities all over the world is significant of its basic importance.

The contemporary period of antennas is, above all, marked by the related needs of radar, radio astronomy and space technologies, with renewed analysis, which is frequently highly sophisticated (arrays, for example), using new and improved means of numerical calculation.

This rapid survey is intended not only to show how ambitious a project it would be to attempt an exhaustive treatise on the subject of antennas, but also to explain the need for economy in this work.

The two volumes that make up this work are each divided into two parts:

- Volume 1    Part 1: Theoretical introduction (Roubine)  
              Part 2: General properties of antennas (Bolomey)
- Volume 2    Part 1: Large antennas (Drabowitch)  
              Part 2: Small antennas and plasma-embedded antennas  
                          (Ancona)

Part 1 is a summary of the results of classical electromagnetism on which the theory of antennas is based. The intention is not to reconsider this body of knowledge, which has given rise to many remarkable discoveries, but simply to illustrate the contribution of modern analysis in the simplification of calculation and the generalization of their results. A certain degree of abstraction is accepted by today's students.

Less is generally known on the subject of optics, which was unfortunately not included in most study courses until quite recently. For this reason, the basic elements are summarized in Part 1, which ends with some of the concepts of electromagnetic diffraction and a study of various of its techniques.

Part 2 considers problems of fields and radiated power, gain, impedance and coupling, etc., common to all antennas the dimensions of which are measured in wavelengths. The content of this section is standard, but fundamental, and has been expressed from a modern viewpoint. A considerable amount of space has been devoted to the problem of reception and to numerical methods. These have had a significant effect on the subject, and justify the reduction in the consideration of theoretical developments in impedance to the minimum. The major concern has been to produce a text that can be used by engineers, without requiring an in-depth study of Part 1. The presentation of experimental techniques clearly underlies this preoccupation.

The second volume is deliberately devoted to applications, and is roughly divided between large and small antennas, based on the themes most representative of the 'state of the art', and which provide plenty of scope for

instruction. The authors, practised engineers who have contributed to the development of their particular fields, are also lecturers, hence their interest in an integrated presentation that will be of practical use to future workers.

In Vol. 2, Part 1 basically concerns large (in terms of wavelength) antennas, such as the many ground-based antennas. This is a large subject in itself but, rather than compiling a catalogue of antennas, the author has concentrated on these methods developed in the most important recent studies and which are based on real models. A desire for greater coherence has led him to concentrate on revolving structures with rotational symmetry in which the concept of a hybrid mode allows primary sources, corrugated horns, diffraction at the focus and at infinity in reflectors of revolution to be treated with similar formulae.

The same preoccupations are evident in Part 2, which concerns a subject of no small importance, despite its title and the lack of attention devoted to it in education, but in which the antennas under consideration amply show its significance. Scientific advance in problems in this area is frankly difficult, yet technical developments, particularly in space links, are making the way forward clearer all the time. The study of antennas in an ionized environment is a typical example, and one with which the author is very familiar, having contributed as a physicist.

It is to be hoped that, despite the reservations expressed above, this work forms a coherent unit. Nonetheless, the authors were given considerable latitude, which may have resulted in an occasional lack of consistency in notation or some repetition, despite attempts at coordination.

My pleasure in presenting the contributions of my co-authors is due, in part, to the facts that all three have at some time been my students and—although the one is not necessarily due to the other—that all are respected experts in their field.

Finally, I would like to express my gratitude to the École Supérieure d'Électricité, Thomson-CSF and the Société Technique d'Application et de Recherche Electronique, whose material aid greatly assisted in the publishing of this work.

**E Roubine**

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Part 1

# **Theoretical introduction**



# Electromagnetic field of antennas

## A: Purpose of Part 1

The first part of this work is intended as a summary of the elements of theoretical electromagnetism necessary to the study of the physical bases of antenna function. Despite the fact that these elements are to be found in many other texts, it was judged useful to present the most important results here, even if only to clarify the notation used. References are provided for derivations of a purely technical mathematical nature, which have not been included here.

It was felt that there could be considerable interest in presenting such a classic subject from a more modern viewpoint than is customary. The reader will notice the economy of method provided by the theory of distribution: almost immediate derivation of boundary conditions and of Huygens' theorem, direct expression of radiated fields without the use of potentials, simple handling of convolutions, etc.

Furthermore, the use of the complex combination  $E + jH$  of the electric and magnetic fields, suggested by their correlative properties, was a deliberate choice. The idea is an old one (Silberstein, 1907), but seems to have been used only tentatively by other authors. The reason for this is, doubtless, the ambiguity that can be created by the complex representation of the fields themselves, with sinusoidal time dependence. The use of bicomplex algebra with four units allows the problem to be overcome. The compactness of expression which results is striking (Appendix 1).

At very high frequency traditional geometric optics is an approximation which is very useful in problems of radiation and diffraction. Certain basic results have been summarized here, for two reasons. The first is that optics has unfortunately been absent from most university electrical engineering courses for some years, and many students know little or nothing about it. The second reason is that it has given rise to an ingenious extension which in recent years has become standard for the solution of diffraction problems (the geometric theory of diffraction).

Diffraction is naturally involved in the study of antennas (reflectors, apertures, and reception antennas). This is why theory is discussed at the end of Part 1.

## B: Maxwell's equations—physical viewpoint

### 1: Maxwell's equations

The electrical properties of any medium are determined by charges, currents and an electromagnetic (e.m.) field. If the charges possess, at each point  $x = (x_1, x_2, x_3)$  and at each instant  $t$  the densities  $\rho(x, t)$  (in C/m<sup>3</sup>) and  $\vec{J}(x, t)$  (in A/m<sup>2</sup>), these densities are related, as in fluid mechanics, by the conservation or continuity equation:

$$\operatorname{div} \vec{J} + \partial_t \rho = 0 \quad (1)$$

The e.m. field is made up of four vectorial fields:

an electrical field and an electric induction:  $\vec{E}(x, t)$  (in V/m) and  $\vec{D}(x, t)$  (in C/m);

a magnetic field and a magnetic induction:  $\vec{H}(x, t)$  (in A/m) and  $\vec{B}(x, t)$  (in T [teslas]).

Maxwell's equations are the system of partial differential equations that describe the local interactions:

$$\operatorname{curl} \vec{E} + \partial_t \vec{B} = 0 \quad (\text{induction law}) \quad (2)$$

$$\operatorname{curl} \vec{H} - \partial_t \vec{D} = \vec{J} \quad (\text{Ampere's law}) \quad (3)$$

$$\operatorname{div} \vec{D} = \rho \quad (\text{Gauss's electric law}) \quad (4)$$

$$\operatorname{div} \vec{B} = 0 \quad (\text{Gauss's magnetic law}) \quad (5)$$

Gauss's two laws are immediate consequences of (1), (2) and (3). For the present, only those areas in which the fields are twice continuously differentiable relative to the coordinates and time will be considered.

### 2: Simple (or perfect) media

These are characterized by two constants  $\varepsilon$  and  $\mu$  such that the following relationships exist:

$$\vec{D} = \varepsilon \vec{E} \quad (6)$$

$$\vec{B} = \mu \vec{H} \quad (7)$$

The media are linear, isotropic and homogeneous.  $\varepsilon$  is the absolute permittivity (or absolute dielectric constant) and is expressed in F/m.  $\mu$  is the absolute permeability, expressed in H/m.

The simplest example (and the most important in this context) is that of the vacuum:  $\varepsilon = \varepsilon_0$ ,  $\mu = \mu_0$ , where the constants  $\varepsilon_0$  and  $\mu_0$  are fixed by the system of units. In the so-called coordinated systems:

$$\varepsilon_0 \mu_0 c^2 = 1 \quad (8)$$

where  $c$  has the dimensions of velocity and is numerically equal to that of light *in vacuo*. The approximation  $c = 3 \times 10^8$  m/s is quite sufficient here. So in the

rationalized MKSA system:

$$\begin{aligned}\epsilon_0 &= \frac{1}{36\pi} 10^{-9} \text{ F/m} \\ \mu_0 &= 4\pi 10^{-7} \text{ H/m}\end{aligned}\tag{9}$$

With a more precise value ( $c = 2.9979 \dots \times 10^8$ ) one could, for example, adjust  $\mu_0$  versus  $\epsilon_0$ .

Simple media are defined by a relative permittivity  $\epsilon_r$  and a relative permeability  $\mu_r$ , such that:

$$\begin{aligned}\epsilon &= \epsilon_0 \epsilon_r \\ \mu &= \mu_0 \mu_r\end{aligned}\tag{10}$$

The relative permeability and permittivity, both dimensionless, are given in tables.

### 3: Propagation of the electromagnetic field

Apart from the charges and currents, in simple media we have:

$$\begin{aligned}\text{curl } \vec{\mathcal{E}} + \mu \partial_t \vec{\mathcal{H}} &= 0 \\ \text{curl } \vec{\mathcal{H}} - \epsilon \partial_t \vec{\mathcal{E}} &= 0 \\ \text{div } \vec{\mathcal{E}} &= 0 \\ \text{div } \vec{\mathcal{H}} &= 0\end{aligned}\tag{11}$$

By introducing the operator  $\Delta = \text{grad div} - \text{curl curl}$ :

$$\begin{aligned}\Delta \vec{\mathcal{E}} - \epsilon \mu \partial_t^2 \vec{\mathcal{E}} &= 0 \\ \Delta \vec{\mathcal{H}} - \epsilon \mu \partial_t^2 \vec{\mathcal{H}} &= 0\end{aligned}\tag{12}$$

The fields  $\vec{\mathcal{E}}$  and  $\vec{\mathcal{H}}$  obey the vectorial wave equation (Dalembert's equation). They propagate by waves with velocity:

$$v = \frac{1}{\sqrt{\epsilon \mu}} = \frac{c}{\sqrt{\epsilon_r \mu_r}}\tag{13}$$

In particular, they propagate in a vacuum with velocity  $c$ .

This first, major consequence of Maxwell's equations is the origin of Maxwell's electromagnetic theory of light (the electric vector being the light vector).

It was Hertz who, 20 years later (1887), confirmed the propagation of the e.m. field experimentally, and was thereafter considered to be the 'inventor' of radio. His experiments constituted the first transmissions of the type later to be called Hertzian. Hertz's discharger and resonator were the first antennas.

Note that that relationship (13) defines the refractive index of the medium:

$$n = \frac{c}{v} = \sqrt{\epsilon_r \mu_r}\tag{14}$$

This permits design with lenses of normal ( $\mu_r = 1$ ) or artificial dielectric.



## 4: Conductive media

### (a) Conductivity

Conductive media obey Ohm's law:

$$\vec{J} = \sigma \vec{E} \quad (15)$$

Conductivity  $\sigma$  is expressed in  $\Omega^{-1}/\text{m}$  or  $\text{S}/\text{m}$  (mhos/m). This is the reciprocal of resistivity (expressed in  $\Omega \text{ m}$ ). In pure metals (except for mercury) it is of the order of  $10^7$  (Ag:  $6.1 \times 10^7$ ; Cu:  $5.8 \times 10^7$ ).

It is easy to verify that the relaxation relationship applies for internal charges:

$$\rho(x, t) = \rho(x, 0)e^{-t/\tau}, \quad \tau = \frac{\epsilon}{\sigma} \quad (16)$$

where the relaxation time  $\tau$  is extremely low in good conductors. The charges are carried to the surface almost instantaneously.

### (b) Perfect conductors

The above suggests the idealized model of infinite conductivity. This simplifies the calculations for metallic conductors to a considerable degree, and the results constitute an approximation which is largely adequate for most applications.

### (c) Applied current

It is sometimes useful, in the study of antennas, to distinguish between 'ohmic' current (15) and 'external' current  $\vec{J}_a$  considered as a source applied to the antenna. In regions where the two coexist, there will be a total current  $\sigma \vec{E} + \vec{J}_a$ , which will constitute the second term of (3).

## 5: Anisotropic media

These are used in Vol. 2, Part 2. The inductions are only related linearly to the corresponding fields. In a cartesian coordinate system (which could be assumed to be rectangular), the following relationships exist between components:

$$\mathcal{D}_i = \sum_j \epsilon_{ij} \mathcal{E}_j \quad (17)$$

$$\mathcal{B}_i = \sum_j \mu_{ij} \mathcal{H}_j \quad (18)$$

or in matrix form, for example for the first:

$$\begin{pmatrix} \mathcal{D}_1 \\ \mathcal{D}_2 \\ \mathcal{D}_3 \end{pmatrix} = \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{pmatrix} \begin{pmatrix} \mathcal{E}_1 \\ \mathcal{E}_2 \\ \mathcal{E}_3 \end{pmatrix} \quad (19)$$

In a homogeneous medium the coefficients are constants, but they depend on the coordinate system chosen ( $e_1, e_2, e_3$ ). An intrinsic significance is often given to