Jean-Jacques Chattot

Computational Aerodynamics and Fluid Dynamics

Scientific Computation

An Introduction

Springer

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With 77 Figures

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To my wife Adrienne
and son Eric

Preface

The field of computational fluid dynamics (CFD) has matured since the author was first introduced to electronic computation in the mid-sixties. The progress of numerical methods has paralleled that of computer technology and software. Simulations are used routinely in all branches of engineering as a very powerful means for understanding complex systems and, ultimately, improve their design for better efficiency.

Today's engineers must be capable of using the large simulation codes available in industry, and apply them to their specific problem by implementing new boundary conditions or modifying existing ones.

The objective of this book is to give the reader the basis for understanding the way numerical schemes achieve accurate and stable simulations of physical phenomena, governed by equations that are related, yet simpler, than the equations they need to solve. The model problems presented here are linear, in most cases, and represent the propagation of waves in a medium, the diffusion of heat in a slab, and the equilibrium of a membrane under distributed loads. Yet, regardless of the origin of the problem, the partial differential equations (PDE's) reflect the physical phenomena to be modeled and can be classified as being of hyperbolic, parabolic or elliptic type. The numerical treatment depends on the equation type that can represent several physical situations as diverse as heat conduction and viscous fluid flow. Nonlinear model problems are also presented and solved, such as the transonic small disturbance equation and the equations of gas dynamics. The model problems are given a full treatment, from the exact analytical solution, the analysis of the scheme's consistency and accuracy, the study of stability, to the detailed implementation of the scheme and of the boundary and/or initial conditions. It is the author's hope that this will entice the reader to write his/her own programs, and by doing so, learn more about CFD than a book can teach.

Davis, March 2002 *Jean-Jacques Chattot*

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Table of Contents

1. Introduction

1.1 Motivation

The material in this book is based on lecture notes on computational fluid dynamics (CFD) that the author has developed over the past twenty years in France, at Centre National d'Etudes Supérieures de Mécanique and at the Université de Paris-Sud, and in the US at the University of California, Davis.

It is intended for senior undergraduate and first year graduate students who will be developing or using codes in the simulation of fluid flows or other physical phenomena governed by partial differential equations (PDEs).

It is the belief of the author, that a numerical method is not fully understood until it has been coded by the user and applied in simulation; each model and scheme in this book is presented with this goal in mind.

1.2 Content

The book is self contained and kept at a simple enough level that the reader will not need further references in order to understand the material.

The approach is based on the finite difference method (FD), which is widely employed as a method of discretization on cartesian mesh systems, in the physical domain, or in the computational domain after coordinate transformation. The extension to the finite volume method on arbitrary mesh sytems, including unstructured meshes, although feasible with a similar approach, would require all analyses to be performed numerically, instead of analytically in closed form, as is the case here.

The book is organized in chapters that build up each on material covered in the previous chapters, particularly Chaps. 2, 3 and 4 and Chaps. 8–11. Chapters 5, 6 and 7 can be read in any order.

The basics of the finite difference method are presented in Chap. 2. The tools that will be used throughout the book are introduced: the Taylor expansion and the complex mode analysis, which requires some complex algebra. They are the tools for the accuracy and stability analyses.

Chapter 3 is devoted to ordinary differential equations (ODEs) and their integration. ODEs represent an important particular case of partial differential equations (PDEs), when the number of independent variables reduces to one, either due to the nature of the physical problem (e.g. a time-dependent problem reaches a steady-state), or because the solution is expanded in terms of polynomials with unknown coefficients (Fourier series, etc...) in all but one independent variable.

Chapter 4 is a simple, but general discussion of PDEs, their type and classification, the notion of characteristic surfaces, compatibility relations and the jump conditions associated with conservation laws. For further understanding of this complex topic, a reference is recommended to the reader. This chapter is pivotal in making the connection between the physical phenomena of wave propagation, diffusion and equilibrium, and their mathematical counterparts, via the existence or non-existence of characteristics and their interpretation.

Chapters 5, 6 and 7 concern the linear model equations of hyperbolic, parabolic and elliptic type, respectively. Classical schemes are briefly reviewed and discussed in terms of accuracy and stability. Practical aspects of the implementation of selected schemes are presented; these should help the reader develop his/her own programs for the proposed methods. These models offer the opportunity to touch upon the subject of the solution of large linear algebraic systems of equations when implicit schemes and iterative techniques are discussed. The Thomas algorithm for tridiagonal matrices is described in detail.

Chapter 8 is devoted to the convection-diffusion equation, a model for the Navier-Stokes equations.

Chapter 9 presents the method of Murman and Cole, which, in many respects, is a precursor to the advanced CFD schemes of today. The author holds it in particular affection as he was asked as first assignment to implement it to simulate transonic flows past profiles and bodies of revolution at ONERA, France, in the early seventies.

Chapter 10, "treatment of nonlinearities" is a short complement in techniques of linearization, in an attempt to understand how linear stability analysis can still be of use in the context of nonlinear problems.

Chapter 11 represents the application of the previous material to a system, namely, the equations of gas dynamics, to study its type, its jumps and its solutions. An extension of the Murman-Cole scheme is introduced and results for a shock tube problem, the converging-diverging nozzle flow and the start-up of a supersonic wind tunnel, are given. This chapter contains advanced material such as eigenvalues and eigenfunctions. It can be skipped at the undergraduate level.

In each chapter, short problems are proposed to the reader, to illustrate a point or complete a proof that is only sketched. These problems should not require more than one page of derivations and calculations to complete. In Appendix A, more elaborate problems are proposed, taken from final exams in class. These should be completed in a two-hours time frame. The solutions to these problems are found in Appendix B.

Acknowledgement

The author is indebted to his first "Maîtres", Maurice Holt, Professor Emeritus, University of California Berkeley, USA, and Roger Peyret, Professor Emeritus, Universite de Nice, France, for their enthusiasm for CFD and their mentorship, that had such profound impact on his career.

2. Basics of the Finite-Difference Method

2.1 Representation of a Function by Discrete Values

Let $u = f(x)$ be a continuous and differentiable function to a sufficient order in the interval $[a, b]$. A discretization of $[a, b]$ with constant step h is introduced:

$$x_i = a + (i-1)h, \quad x_1 = a, \quad x_{ix} = b, \quad h = \frac{b-a}{ix-1},$$

ix is the total number of points of the discretization, h is the discretization step. Let $u_i = f(x_i)$ (see Fig. 2.1).

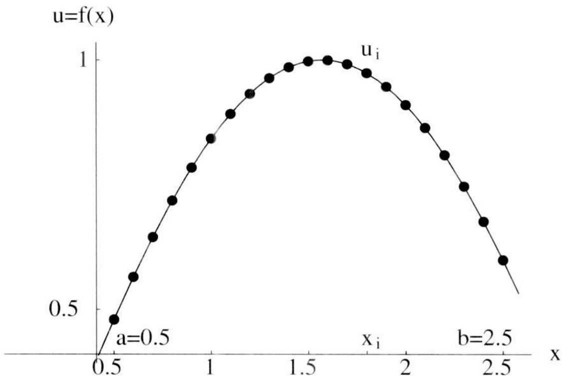

Fig. 2.1. Function $u = f(x)$

Remark: When $f(x)$ is given, the discrete values $u_i = f(x_i)$ are uniquely defined. However, if the discrete values $f(x_i)$ are given, it is not possible to find a unique $u = f(x)$ such that $u_i = f(x_i)$ without any further information about f (such as being a polynomial of some sort). Going from the continuous to the discrete carries with it a loss of information. In the finite difference method there is no hypothesis concerning the variation of the function between the points. This is in contrast to the finite-volume or finite-element methods where some assumption is made concerning the variation of the function between the points.

2.2 Representation of a First Derivative

Let $f(x)$ be a continuous function, differentiable as many times as needed in $[a,b]$. Then there exists a Taylor expansion about any point x_i:

$$u_{i+1} = f(x_{i+1}) = f(x_i+h) = f(x_i)+hf'(x_i)+\frac{h^2}{2!}f''(x_i)+\frac{h^3}{3!}f'''(x_i)+O(h^4).$$

$O(h^4)$ is the remainder and indicates that the unwritten terms are of fourth-order and higher. Since $f(x_i) = u_i$ we can write

$$\frac{u_{i+1}-u_i}{h} = f'(x_i) + \frac{h}{2!}f''(x_i) + \frac{h^2}{3!}f'''(x_i) + O(h^3). \tag{2.1}$$

As $h \to 0$, $(u_{i+1}-u_i)/h \to f'(x_i)$ thus the left-hand side of (2.1) is by definition a *finite-difference approximation* (FD) of the first derivative f' at point x_i. The leading term in the error is

$$\frac{h}{2!}f''(x_i) = O(h), \ as\ h \to 0.$$

It is said that the scheme is first-order accurate. It is also qualified as a *one-sided* or *advanced* or *forward* finite-difference scheme.

Replace h by $-h$ and get

$$u_{i-1} = f(x_{i-1}) = f(x_i-h) = f(x_i)-hf'(x_i)+\frac{h^2}{2!}f''(x_i)-\frac{h^3}{3!}f'''(x_i)+O(h^4)$$

and

$$\frac{u_i-u_{i-1}}{h} = f'(x_i) - \frac{h}{2!}f''(x_i) + \frac{h^2}{3!}f'''(x_i) + O(h^3). \tag{2.2}$$

Again, by definition, the left-hand side is a FD approximation of f' at x_i. It is *one-sided* (*retarded* or *backward*) and first-order accurate.

Define the average of the two previous schemes: ((2.1)+(2.2))/2

$$\frac{u_{i+1}-u_{i-1}}{2h} = f'(x_i) + \frac{h^2}{3!}f'''(x_i) + O(h^4). \tag{2.3}$$

This is a *centered* FD approximation for f' at x_i; it is second-order accurate. Note that the remainder is of fourth-order. This is because we have taken into account the fact that the odd-order derivatives vanish in the combination.

Non-centered schemes are not necessarily first-order accurate as the previous results may suggest. The following non-centered scheme is second-order accurate:

$$\frac{-u_{i+2}+4u_{i+1}-3u_i}{2h} = f'(x_i) - \frac{h^2}{3}f'''(x_i) + O(h^3).$$