ENGINEERING MECHANICS

VOLUME 2

DYNAMICS

SECOND EDITION

J.L. MERIAM AND L.G. KRAIGE

COMBINED EDITION

SI VERSION

ENGINEERING MECHANICS

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DYNAMICS

SECOND EDITION

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FOREWORD

The innovations and contributions of Dr. James L. Meriam to the field of engineering mechanics cannot be overstated. He has undoubtedly had as much influence on instruction in mechanics during the last thirty years as any one individual. His first books on mechanics in 1951 literally reconstructed undergraduate mechanics and became the definitive textbooks for the next decade. His texts were logically organized, easy to read, directed to the average engineering undergraduate, and were packed with exciting examples of real-life engineering problems superbly illustrated. These books became the model for other engineering mechanics texts in the 1950s and 1960s.

Dr. Meriam began his work in mechanical engineering at Yale University where he earned his B.E., M. Eng., and Ph.D. degrees. He had early industrial experience with Pratt and Whitney Aircraft and the General Electric Company, which stimulated his first contributions to mechanics in mathematical and experimental stress analysis. In the Second World War he served in the U.S. Coast Guard.

Dr. Meriam was a member of the faculty of the University of California, Berkeley, for twenty-one years where he served as Professor of Engineering Mechanics, Assistant Dean of Graduate Studies, and Chairman of the Division of Mechanics and Design. In 1963 he became Dean of Engineering at Duke University where he devoted his full energies to the development of its School of Engineering. In 1972 Professor Meriam followed his desire to return to full-time teaching and served as Professor of Mechanical Engineering at California Polytechnic State University, and more recently as a visiting professor at the University of California, Santa Barbara. Professor Meriam has always placed great emphasis on teaching, and this trait has been recognized by his students wherever he has taught. At Berkeley in 1963 he was the first recipient of the Outstanding Faculty Award of Tau Beta Pi, given primarily for excellence in teaching, and in 1978 received the Distinguished Educator Award for Outstanding Service to Engineering Mechanics Education from the American Society for Engineering Education.

Professor Meriam was the first author to show clearly how the method of virtual work in statics can be employed to solve a class of problems largely neglected by previous authors. In dynamics, plane motion became understandable, and in his later editions, three-dimensional kinematics and kinetics received the same treatment. He is credited with original developments in the theory of variable-mass dynamics, which are contained in his *Dynamics, 2nd Edition*. Professor Meriam has also been a leader in promoting the use of SI units, and his *SI Versions* of *Statics* and *Dynamics* published in 1975 were the first mechanics textbooks in SI units in this country.

Welcomed as a coauthor of this second edition of ENGINEERING MECHANICS is Dr. L. Glenn Kraige whose interests and qualifications in mechanics place him in a position of singular excellence to make significant contributions to mechanics education. Dr. Kraige earned his B.S., M.S., and Ph.D. degrees at the University of Virginia, principally in aerospace engineering, and he currently serves as Associate Professor of Engineering Science and Mechanics at Virginia Polytechnic Institute and State University.

In addition to his recognized research and publications in the field of spacecraft dynamics, Professor Kraige has devoted his attention to the teaching of mechanics at both introductory and advanced levels. His teaching earned him the Outstanding Educator Award from his department in 1977, the Phillip J. Sporn Award for outstanding teaching of engineering subjects in 1978, and the William E. Wine Award for outstanding university-level teaching in 1984. In his teaching, Professor Kraige stresses the development of analytical capabilities and at the same time the strengthening of physical insight and engineering judgment. More recently, he has also become a leader in the development of motion simulation software for use on personal computers.

The second edition of ENGINEERING MECHANICS promises to exceed the high standards set by the previous edition. The new edition contains one of the most outstanding collections of instructive and interesting problems yet assembled. With increased emphasis on the analytical rigor and unity of mechanics, and with continued emphasis on challenging engineering applications, the new text will appeal to a wide audience of students, teachers, and engineers, and will further extend the authors' contributions to mechanics.

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PREFACE
To the Student

As you undertake the study of engineering mechanics, first statics and then dynamics, you will be building a foundation of analytical capability for the solution of a great variety of engineering problems. Modern engineering practice demands a high level of analytical capability, and you will find that your study of mechanics will help you immensely in developing this capacity.

In engineering mechanics we learn to construct and solve mathematical models which describe the effects of force and motion on a variety of structures and machines that are of concern to engineers. In applying our principles of mechanics we formulate these models by incorporating appropriate physical assumptions and mathematical approximations. Both in the formulation and solution of mechanics problems you will have frequent occasion to use your background in plane and solid geometry, scalar and vector algebra, trigonometry, analytic geometry, and calculus. Indeed, you are likely to discover new significance to these mathematical tools as you make them work for you in mechanics.

Your success in mechanics (and throughout engineering) will be highly contingent upon developing a well-disciplined method of attack from hypothesis to conclusion in which the applicable principles are applied rigorously. Years of experience in teaching and engineering disclose the importance of developing the ability to represent one's work in a clear, logical, and concise manner. Mechanics is an excellent place in which to develop these habits of logical thinking and effective communication.

ENGINEERING MECHANICS contains a large number of sample problems in which the solutions are presented in detail. Also included in these examples are helpful observations that mention common errors and pitfalls to be avoided. In addition, the book contains a large selection of simple, introductory problems and problems of intermediate difficulty to help you gain initial confidence and understanding of each new topic. Also included are many problems which illustrate significant and contemporary engineering situations to stimulate your interest and help you to develop an appreciation for the many applications of mechanics in engineering.

We are pleased to extend our encouragement to you as a student of mechanics. We hope this book will provide both help and stimulation as you develop your background in engineering.

J. L. Meriam
Santa Barbara, California
January 1987

L. Glenn Kraige
Blacksburg, Virginia
January 1987

PREFACE
To the Instructor

The primary purpose of the study of engineering mechanics is to develop the capacity to predict the effects of force and motion in the course of carrying out the creative design function of engineering. Successful prediction requires more than a mere knowledge of the physical and mathematical principles of mechanics. Prediction also requires the ability to visualize physical configurations in terms of real materials, actual constraints, and the practical limitations which govern the behavior of machines and structures. One of our primary objectives in teaching mechanics should be to help the student develop this ability to visualize, which is so vital to problem formulation. Indeed, the construction of a meaningful mathematical model is often a more important experience than its solution. Maximum progress is made when the principles and their limitations are learned together within the context of engineering application.

Courses in mechanics are often regarded by students as a difficult requirement and frequently as an uninteresting academic hurdle as well. The difficulty stems from the extent to which reasoning from fundamentals, as distinguished from rote learning, is required. The lack of interest that is frequently experienced is due primarily to the extent to which mechanics is presented as an academic discipline often lacking in engineering purpose and challenge. This attitude is traceable to the frequent tendency in the presentation of mechanics to use problems mainly as a vehicle to illustrate theory rather than to develop theory for the purpose of solving problems. When the first view is allowed to predominate, problems tend to become overly idealized and unrelated to engineering with the result that the exercise becomes dull, academic, and uninteresting. This approach deprives the student of much of the valuable experience in formulating problems and thus of discovering the need for and meaning of theory. The second view provides by far the stronger motive for learning theory and leads to a better balance between theory and application. The crucial role of interest and purpose in providing the strongest possible motive for learning cannot be overemphasized.

Furthermore, we should stress the view that, at best, theory can only approximate the real world of mechanics rather than the view that the real world approximates the theory. This difference in philosophy is indeed basic and distinguishes the *engineering* of mechanics from the *science* of mechanics.

During the past twenty years there has been a strong trend in engineering education to increase the extent and level of theory in the engineering-science courses. Nowhere has this trend been felt more than in mechanics courses. To the extent that students are prepared to handle the accelerated treatment, the trend is beneficial. There is evidence and justifiable concern, however, that a significant disparity has more recently appeared between coverage and comprehension. Among the contributing factors we note three trends. First, emphasis on the geometric and physical meanings of prerequisite mathematics appears to have diminished. Second, there has been a significant reduction and even elimination of instruction in graphics, which in the past served to enhance the visualization and representation of mechanics problems. Third, in advancing the mathematical level of our treatment of mechanics there has been a tendency to allow the notational manipulation of vector operations to mask or replace geometric visualization. Mechanics is inherently a subject which depends on geometric and physical perception, and we should increase our efforts to develop this ability.

One of our responsibilities as teachers of mechanics is to use the mathematics which is most appropriate for the problem at hand. The use of vector notation for one-dimensional problems is usually trivial; for two-dimensional problems it is often optional; but for three-dimensional problems it is quite essential. As we introduce vector operations in two-dimensional problems, it is especially important that their geometric meaning be emphasized. A vector equation is brought to life by a sketch of the corresponding vector polygon, which often discloses through its geometry the shortest solution. There are, of course, many mechanics problems where the complexity of variable interdependence is beyond the normal powers of visualization and physical perception, and reliance on analysis is essential. Nevertheless, our students become better engineers when their abilities to perceive, visualize, and represent are developed to the fullest.

As teachers of engineering mechanics we have the strongest obligation to the engineering profession to set reasonable standards of performance and to uphold them. In addition, we have a serious responsibility to encourage our students to think for themselves. Too much help with details that students should be reasonably able to handle from prerequisite subjects can be as bad as too little help and can easily condition them to becoming overly dependent on others rather than to exercise their own initiative and ability. Also, when mechanics is subdivided into an excessive number of small compartments, each with detailed and repetitious instructions, students can have difficulty seeing the "forest for the trees" and, consequently, fail to perceive the unity of mechanics and the far-reaching applicability of its few basic principles and methods.

This second edition of ENGINEERING MECHANICS, as with the first edition, is written with the foregoing philosophy in mind. It is intended primarily for the first engineering course in mechanics, generally taught in the second year of study. The book omits a number of the more advanced topics contained in the senior author's more extensive treatments, *Statics* and *Dynamics, 2nd Edition* (1971) and *SI Version* (1975), and is designed especially to facilitate self-study. To this end a major feature of the book is the extensive treatment of sample problems, which are presented in a single-page format for more convenient study. In addition to presenting the solution in detail, each sample problem also contains comments and cautions keyed to salient points in the solution and printed in colored type. These comments alert students to common pitfalls and should provide a valuable aid to their self-study efforts.

A special feature of the second edition is the inclusion of selected problems that are especially appropriate for computer solution. These problems are separately identified and are placed at the end of the Problem Review articles that conclude most of the chapters. With the increasing availability of personal computers, attention to their role in mechanics education must be addressed. The authors wish to emphasize that the experience in formulating problems, where reason and judgment are developed, is vastly more important for the student than is the manipulative exercise in carrying out the solution. For this reason the time devoted to computer usage in mechanics should be carefully limited. On the other hand, the computer can be used to reveal results in special problems where conventional solutions are awkward or inadequate. The computer-oriented problems in the second edition are included to help guide the student into the type of problem for which machine solution is especially beneficial in revealing some critical conclusion not otherwise readily discernable. To conserve adequate time for problem formulation, it is suggested that the student be encouraged or required to develop and carry out an appropriate computer program for only a limited few of these special problems.

In the second edition increased attention has been given to identifying the more complete mathematical solutions to selected problems which illustrate the importance of matching the number of dependent variables with the number of available independent equations.

Volume 2, Dynamics, contains 114 sample problems and 1313 unsolved problems from which a wide choice of assignments can be made. Of these problems over 50 percent are totally new with the balance selected from the preceding editions. Each problem set begins with relatively simple, uncomplicated problems to help students gain confidence with the new topic. Many practical problems and examples of interesting engineering situations drawn from a wide range of applications are represented in the problem collection. Simple numerical values have been used throughout, however, so as not to complicate the solutions and divert attention from the principles. The problems are arranged generally in order of increasing difficulty, and the answers to most of them are given. The more difficult problems are identified by the symbol ▶ and may often be used to provide a comprehensive classroom experience when solved by the instructor. All numerical solutions have been carried out and checked with an electronic calculator without rounding intermediate values. Consequently, the final answers should be correct to within the number of significant figures cited. The authors are confident that the book is exceptionally free from error.

ENGINEERING MECHANICS is written in a style which is both concise and friendly. The major emphasis is focused on basic principles and methods rather than on a multitude of special cases. Strong effort has been made to show both the cohesiveness of the relatively few fundamental ideas and the great variety of problems which these few ideas will solve.

The logical division between particle dynamics and rigid-body dynamics, with each part treating the kinematics prior to the kinetics, has been followed in Chapters 2 and 3. This arrangement greatly facilitates a more thorough and rapid excursion in rigid-body dynamics with the prior benefit of a comprehensive introduction to particle dynamics.

Chapter 3 on particle kinetics focuses on the three basic methods, force-mass-acceleration, work-energy, and impulse-momentum. The special topics of impact, central-force motion, and relative motion are grouped together in Section D of Chapter 3 on special applications and serve as optional material to be assigned according to instructor preference and available time. With this arrangement the attention of the student is focused more strongly on the three basic approaches to kinetics which are developed in Sections A, B, and C of the chapter.

Chapter 4 on systems of particles is an extension of the principles of motion for a single particle and develops the general relationships which are so basic to modern comprehension of dynamics. The chapter also includes the topics of steady mass flow and variable mass, which may be considered as optional material depending on the time available.

In Chapter 5 on the kinematics of rigid bodies in plane motion, emphasis is placed jointly on geometrical visualization and analysis and on rigorous mathematical solution of the vector equations of relative velocity and relative acceleration. This dual approach serves the purpose of reinforcing the meaning of vector mathematics.

Chapter 6 on the kinetics of rigid bodies has been redesigned to place greater emphasis on the basic equations which govern all categories of plane motion. Strong dependence is placed on the identification of knowns and unknowns and on the necessary and sufficient motion equations which guarantee solution. Special emphasis is also placed on forming the direct equivalence between the actual applied forces and couples and their $m\bar{a}$ and $\bar{I}\alpha$ resultants. In this way the versatility of the moment principle is emphasized, and the student is encouraged to think directly in terms of resultant dynamics effects.

Chapter 7, which may be treated as optional, provides a basic introduction to three-dimensional dynamics which is sufficient to solve many of the more common space-motion problems. For students who later pursue more advanced work in dynamics, Chapter 7 will provide a solid foundation. Gyroscopic motion with steady precession is treated in two ways. The first approach makes use of the analogy between the relation of force and linear-momentum vectors and the relation of moment and angular-momentum vectors. With this treatment the student can understand the gyroscopic phenomenon of steady precession and handle most of the engineering problems on gyros without a detailed study of three-dimensional dynamics. The second approach makes use of the more general momentum equations for three-dimensional rotation where all components of momentum are accounted for.

Chapter 8 is devoted to the topic of vibrations, which has been expanded to a separate chapter. This increased emphasis will be especially useful for engineering students whose only exposure to vibrations is acquired in the basic dynamics course.

Moments and products of inertia of mass are included in Appendix B, and Appendix C contains a summary review of selected topics of elementary mathematics that the student should be prepared to use in mechanics.

Special recognition is due Dr. A. L. Hale of the Bell Telephone Laboratories for his continuing contribution in the form of invaluable suggestions and accurate checking of the manuscript. Dr. Hale has rendered similar service to all previous versions of the senior author's books on mechanics, and his input has been a great asset. In addition, appreciation is expressed to Professor J. M. Henderson of the University of California, Davis, for helpful comments and suggestions of selected problems. Contribution by the staff of John Wiley & Sons, especially editor Bill Stenquist and illustrator John Balbalis, during the planning and production of the book reflects a high degree of professional competence and is duly recognized. The contribution of Virginia Polytechnic Institute and State University through the generous support of Dr. Paul Torgersen, Dean of the College of Engineering, and Dr. Daniel Frederick, Head of the Department of Engineering Science and Mechanics, is especially acknowledged. Finally, we wish to acknowledge the patience and forebearance of our wives, Julia and Dale, during the many hours required to prepare this manuscript.

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Santa Barbara, California
January 1987

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January 1987

CONTENTS

INTRODUCTION TO DYNAMICS 1

1/1 *HISTORY AND MODERN APPLICATIONS*

Dynamics is that branch of mechanics which deals with the motion of bodies under the action of forces. The study of dynamics in engineering usually follows the study of statics, which deals with the action of forces on bodies at rest. Dynamics has two distinct parts—*kinematics*, which is the study of motion without reference to the forces which cause motion, and *kinetics*, which relates the action of forces on bodies to their resulting motions. The student of engineering will find that a thorough comprehension of dynamics will provide one of his most useful and powerful tools for analysis in engineering.

Historically, dynamics is a relatively recent subject compared with statics. The beginning of a rational understanding of dynamics is credited to Galileo (1564–1642), who made careful observations concerning bodies in free fall, motion on an inclined plane, and motion of the pendulum. He was largely responsible for bringing a scientific approach to the investigation of physical problems. Galileo was continually under severe criticism for refusing to accept the established beliefs of his day, such as the philosophies of Aristotle which held, for example, that heavy bodies fall more rapidly than light bodies. The lack of accurate means for the measurement of time was a severe handicap to Galileo, and further significant development in dynamics awaited the invention of the pendulum clock by Huygens in 1657. Newton (1642–1727), guided by Galileo's work, was able to make an accurate formulation of the laws of motion and, hence, to place dynamics on a sound basis. Newton's famous work was published in the first edition of his *Principia*,* which is generally recognized as one of the greatest of all recorded contributions to knowledge. In addition to stating the laws governing the motion of a particle, Newton

* The original formulations of Sir Isaac Newton may be found in the translation of his *Principia* (1687), revised by F. Cajori, University of California Press, 1934.