Control Systems Engineering

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Preface

This text is an introduction to control systems engineering. Such a course is typically taken by undergraduates in mechanical, electrical, chemical, and aerospace engineering. It is assumed that the student has a background in calculus and college physics (mechanics, thermodynamics, and electrical circuits). Any other required material in physics and mathematics (e.g., differential equations, transforms, and matrices) is developed in the text and its appendices.

Teaching, and therefore writing a text in the control systems discipline present a great challenge for several reasons. The control engineer can be called on to develop mathematical models for a variety of applications involving hydraulic, pneumatic, thermal, mechanical, and electrical systems and therefore must be familiar with the modeling techniques appropriate to each area. Here, the challenge is to develop the student's ability to create a model that is sufficiently detailed to capture the dominant dynamic characteristics of the system yet simple enough to be useful for design purposes. Technological developments in the control field have been rapid, as evidenced by the increasing use of digital computers as controllers. Treatment of digital control necessitates covering sampled-data systems but without neglecting the fundamentals of continuous-time system analysis. In addition to controller hardware developments, the field has seen substantial improvement in computational techniques. Formulations using the state space approach provide a good basis for developing general algorithms that are well suited for computer-aided design purposes. On the other hand, the methods of "classical" control theory are still widely used and their utility has been significantly enhanced by modern computational techniques involving computer simulation and computer graphics. Today's engineer must be familiar with the classical methods and the new computational improvements.

We have attempted to satisfy these needs by introducing the required concepts and methods in a balanced and gradual way. First, a unified energy-based approach to modeling is presented. Examples from different fields are given to show the student how to develop relatively low-order models that are adequate for many applications. Next, there is a presentation of analysis techniques for such models. Emphasis is given to physical interpretations of the response patterns exhibited by these models and to the relationship between the form and complexity of the model and the resulting analytical requirements. Digital-simulation techniques are introduced to allow the student to deal with nonlinearities and other complexities that occur in many applications.

The basic principles of feedback control systems are then introduced. Commonly used controller hardware is introduced, and the analysis techniques developed earlier are applied to the design of such systems. Presentation of feedback control theory as merely a mathematical exercise is avoided, and the physical interpretations and practical considerations of the designs are emphasized. An in-depth coverage of a variety of design methods is included. The text concludes with a detailed study of digital control, including analysis of sampled-data systems and discrete-time models and the implementation of digital controllers.

Readers familiar with *Modeling, Analysis, and Control of Dynamic Systems*† will notice some additions and deletions as well as a substantial rearrangement of the material. This was done to better accommodate a particular control systems course sequence that is used at many schools. Most of the differences in existing sequences at various schools can be categorized by the relative emphasis given the following topics:

1. Modeling
2. Prediction and analysis of system response
3. Computer simulation methods
4. "Classical" control system design, emphasizing transfer functions and graphical methods, such as the root locus and frequency response plots
5. Digital control
6. "Modern" analysis and control techniques using matrix methods.

At the University of Rhode Island, the first five topics are covered in a two-semester sequence. The last topic, modern control methods, is covered at the graduate level and therefore not included in this text. This is one of the major differences from the previous text. The other differences are the addition of a new, separate chapter on computer simulation and the consolidation of the material on modeling, analytical techniques, and digital control into distinct chapters. This arrangement better reflects the course sequence offered at many schools.

It is my view that an undergraduate control systems sequence should have the following objectives:

1. To introduce the terminology and the basic principles of modeling, analysis, and feedback control of dynamic systems. A general understanding of these topics is important even for graduates who will not be actively involved in designing control systems, because such systems are encountered in many applications and their characteristics should be understood. This introduction should also prepare students for continuing their education in the controls area, either with self-study by reading the literature or attending short courses or graduate school.
2. To develop in students an ability to design control systems for applications likely to be encountered by a control systems specialist. This objective is important for those with possible career objectives in the controls area.

There is much disagreement in the academic community over how the topic of modern control theory fits into the preceding objectives. Having no delusions about settling this argument, nevertheless, we summarize the views of the classical school of thought on this subject, because it is for this group that the present text has been developed. Modern control theory refers to a collection of techniques developed over

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†W. J. Palm, III, *Modeling, Analysis, and Control of Dynamic Systems*, John Wiley, New York, 1983.

the last 30 years, which deals with matrix methods, eigen analysis, and applications of such optimization methods as the calculus of variations. In the years since those methods were developed, there have been few significant implementations, and most of them have been in a single applications area, the aerospace industry. In addition, these applications are very complex and therefore not easily covered at the undergraduate level. It is interesting to note that the availability of low-cost, powerful microprocessor controllers has not led to a substantial increase in the applications of modern control theory Such methods are useful for theoretical developments in control research and such related areas as signal processing. For this reason, these methods should be treated at the graduate level.

Modern control theory was developed primarily to deal with system models of high order. But most control systems applications encountered by a B.S. engineering graduate require controlling a physical system that can be represented by a differential equation of first or second order. With integral control action, the order is increased by one, so that the resulting system model is usually of third order or less. Thus, most applications can be handled with relatively low-order models, and the methods of classical control theory have proved to be well suited for these applications. It is unfortunate that the term *classical* suggests out-of-date, because this is not the case. In fact, classical design methods have been greatly enhanced by the availability and low cost of digital computers for systems analysis and simulation. The graphical tools of classical design can now be more easily used with computer graphics and the effects of nonlinearities and model approximations evaluated by computer simulation. The use of low-cost computers as digital controllers means that some of the limitations imposed on classical designs are eliminated and more flexibility now exists in the choice of the control algorithm. This text has been written in light of these new developments.

This is not to say that the control systems community has learned very little in the last 30 years. The state space and matrix *notation* introduced by modern control theory, as opposed to its *design* methods, has become widespread in the literature because of its ability to represent algorithms concisely. For this reason, many computer-aided design packages use this notation, and therefore it is helpful for control engineers to be familiar with it. Chapter 5, which deals with computer methods, contains an introduction to this notation.

Chapter 1 establishes the viewpoint of dynamic systems analysis and control and its associated modeling requirements. Terminology and an overview of analytical techniques are presented. Zero-order system examples provide a simple introduction to system analysis, because they require only algebraic models. Linearization, system diagrams, and the properties of feedback are introduced in this way.

The general structure of dynamic modeling based on energy principles and integral causality is developed in Chapter 2. Basic principles for modeling simple mechanical, electrical, fluid, and thermal systems are treated. Examples are given that introduce commonly used hardware.

The analysis of system response begins in Chapter 3. First- and second-order models are used as a vehicle for introducing free- and forced-linear system response to the common inputs: the step, ramp, and sine functions. Simple substitution methods are used to obtain the response. This approach, although less "automatic", serves to strengthen students' understanding of the basic response patterns of linear systems. Stability, dominant-root approximations, performance specifications, and linearization of dynamic models are also introduced.

The Laplace transform method for predicting system response is introduced in Chapter 4. This also provides a basis for introducing transfer functions and block diagram algebra. Such related concepts as impedance, frequency response plots, and the initial and final value theorems are presented. Treatment of impulse response, the effects of numerator dynamics, and signal flow graphs conclude the chapter.

Chapter 5 discusses computer simulation methods. The Euler, predictor-corrector, and Runge–Kutta methods are first developed for first-order models and then extended to include higher order cases. The utility of the state variable form is demonstrated, and methods for obtaining this form from the transfer function are given. Practical considerations for simulation are treated next, with a discussion of how linear or linearized analysis can be used to guide the simulation, and the use of step inputs with numerator dynamics. Finally, an introduction to matrix methods and their use in computer-aided design is given.

Chapter 6 presents the basics of feedback control. The common two-position and proportional-integral-derivative types of control laws are analyzed for first- and second-order systems. Typical hardware is described, and the implementation of the control laws with electronic, pneumatic, and hydraulic elements is developed.

A determined effort has been made throughout the text to justify and illustrate the analytical methods in terms of their practical applications. Book length prevents treating design problems in the fine detail needed in practice, and some design considerations, such as economics and reliability, are difficult to cover within the scope of this study. Nevertheless, the discussion and examples should give students as much of a feel for the design process as is possible in an academic environment. To this end, a number of topics that have not been given sufficient emphasis in the past are consolidated into Chapter 7. Methods for selecting the controller gains, and for interpreting the effects of modeling approximations, are among the design issues discussed. All physical elements are power-limited, and design methods are given for taking the associated saturation nonlinearities into account. The classical design approaches of feed-forward, feedback, and cascade compensation are presented, followed by some newer methods, such as state variable feedback, decoupling control, and pseudo-derivative feedback. The latter topic shows that control systems design is an exciting field with much to be discovered.

Control systems design has always relied heavily on graphical methods. Chapter 8 develops the root locus method. Although computer programs are readily available for generating these plots, designers must be able to sketch them first, at least roughly, in order to use the programs efficiently and interpret the results. The material in Chapter 8 is organized in this spirit. The chapter concludes with some examples of how the root locus plot is used for control systems design.

Chapter 9 uses the root locus plot, and other graphical methods—the Bode plot and the Nyquist criterion—to obtain a deeper understanding of control and compensation methods. The important topics of lead and lag compensation are covered in detail.

Chapter 10 introduces digital systems, and develops the necessary background for the analysis of sampled-data systems and discrete-time models. Sampling, quantization, and coding effects in analog-to-digital conversion are discussed. The Laplace transform of a sampled-time function provides a natural way of developing the z transform. This transform is then used to obtain the forced response for common input functions. Sampled-data systems result when a digital-to-analog converter couples a digital device to a continuous-time system, such as in computer control of a mechanical load. The zero-order hold model of this coupling is used with the z transform to develop methods for analyzing this important class of systems. In Chapter 11 digital equivalents of the analog control laws developed in previous chapters are discussed, and some examples are given of control algorithms that take advantage of the unique features of digital control. The concept of digital filtering is introduced, and analyzed with time and frequency domain techniques. The chapter concludes with a detailed development of a computer control system for regulating the speed of a dc motor.

Seven appendices are included for reference information and review purposes to fill in gaps in the reader's mathematical preparation, and for further study of some specialized topics. Appendix A deals with useful analytical techniques, such as the Taylor and Fourier series, matrix analysis, and complex numbers. A self-contained development of the Laplace transform appears in Appendix B. Appendix C gives a useful FORTRAN program implementing the Runge–Kutta integration scheme. Reduction of complicated systems diagrams is facilitated with Mason's rule (Appendix D). Analog simulation is covered in Appendix E. Appendix F presents the Routh–Hurwitz and Jury stability criteria. Appendix G contains tables of physical data, units, and conversion factors.

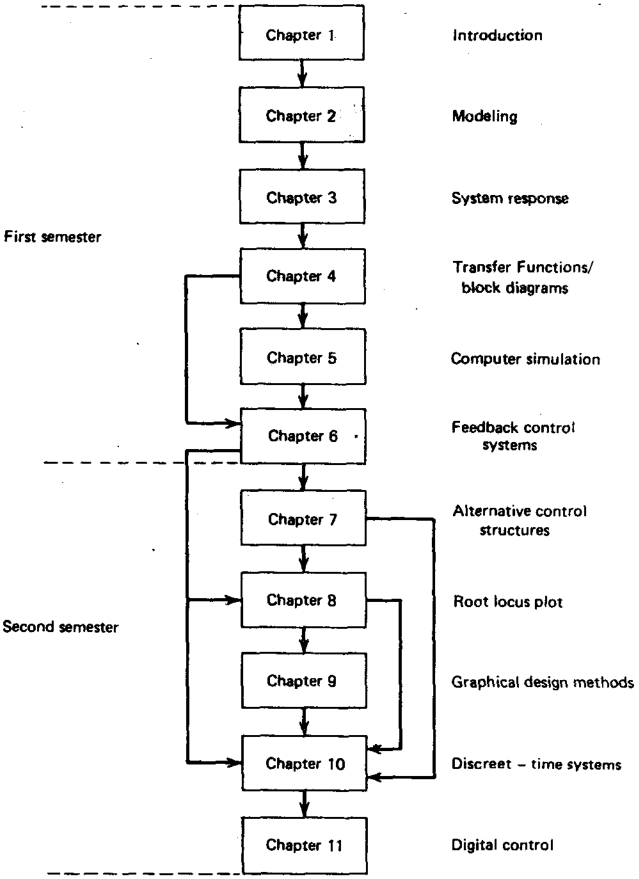
This text contains ample material for a two-semester course in control systems. The first course can introduce modeling, analysis, and control principles, while the second course is more specialized, dealing with control systems design. The text has been designed to be flexible, and can accommodate differences in courses because of differences in the students' mathematical preparation, previous training in modeling and system dynamics, and availability of computing resources for course work. The following chart shows how the various chapters can be covered, depending on the preceding factors.

For those wishing only a brief introduction or review of physical models, Sections 2.1–2.5 provide the necessary material, including the development of models of important mechanical and electrical control systems hardware. Because fluid and thermal systems are more difficult to model, Sections 2.6–2.9, which cover these systems, can be omitted if a briefer coverage of modeling is required.

Chapter 3 treats solution methods for differential equations and can be covered quickly if the students have had a prior course in this subject. The chapter should not be skipped, however, because it develops terminology and the students' understanding of how these methods are applied in engineering situations.

Chapter 4 introduces Laplace transform methods, to which some students might have had prior exposure in a mathematics course. However, transfer functions and block diagrams are rarely covered in such courses. Since these topics are essential in the succeeding chapters, its coverage should not be omitted.

Chapter 5 covers computer simulation methods. If computer facilities, especially those with xy-plotting capability, are available to the students, this chapter provides a



particularly instructive and motivating experience. However, if such facilities are not available, or if time is limited, this chapter can be omitted, because the following chapters do not depend on its results.

Chapter 6 is the basic chapter on feedback control systems and therefore is one of the core chapters that cannot be omitted. Beyond Chapter 6, there are several possible combinations if time is not available to cover all of the remaining chapters. These are shown in the chart shown earlier.

At the University of Rhode Island, second-semester junior students have already had a course in engineering analysis but no modeling course prior to beginning the control systems sequence. The analysis course covers ordinary differential equations, Laplace transforms, and matrices. Therefore, we can treat some of the material in Chapters 3 and 4 by way of a review. The required junior course thus covers the first six chapters; a senior professional elective covers Chapters 7–11. No formal laboratory accompanies the courses. If one is available, the text contains enough information on hardware to serve as a reference for the laboratory sessions.

With the exception of Appendix C, no specific computer or calculator language has been used to present the concepts, and no assumptions have been made concerning the availability of such equipment to the student. However, if it is available, there are many opportunities to use it throughout the text. Programmable calculators can be put to good use in response calculations for evaluating complicated functions, simulating nonlinear systems, and root finding (a good example is programming the solution of a cubic polynomial, since third-order systems are so common). Digital computers are also convenient for the applications mentioned earlier, especially with high-order systems. In addition, they can be used for matrix operations and generating frequency response and root locus plots. Sources for such programs are given at appropriate points in the text.

I am especially grateful to Bill Stenquist of John Wiley & Sons for suggesting this project and for his enthusiasm and support. Thanks are also due to Professors Thomas J. Kim, Frank M. White, and Charles D. Nash, Jr. of the University of Rhode Island, for useful discussions and encouragement. The suggestions of several anonymous reviewers were helpful. Marilyn Buckingham's editing of the manuscript was very thorough, and is much appreciated. My wife, Mary Louise, and our children. Aileen, Billy, and Andy, deserve my special gratitude for their interest and for providing an environment conducive to writing. Finally, I wish to dedicate this work to my parents, for their lifelong support.

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CHAPTER ONE
Introduction

Modeling, analysis, and control of dynamic systems have interested engineers for a long time. Within recent years, the subject has increased in importance for three reasons. Before the invention of the digital computer, calculations required for meaningful applications of the subject were often too time consuming and error prone to be seriously considered. Thus, gross simplifications were made, and only the simplest models of transient behavior were used, if at all. Now, of course, the widespread availability of computers, as well as pocket calculators, allows us to consider more detailed models and more complex algorithms for analysis and design.

Second, with this increased computational power, engineers have correspondingly increased the performance specifications required of their designs to make better use of limited materials and energy, for example, or to improve safety. This leads to the need for more detailed models, especially with regard to predicting transient behavior.

Finally, using computers as system elements for measurement and control now allows more complex algorithms to be employed for data analysis and decision making. For example, intelligent instruments with microprocessors can now calibrate themselves. This increased capability requires a better understanding of dynamic systems so that the full potential of these devices can be realized.

1.1 SYSTEMS

The term *system* has become widely used today, and as a result, its original meaning has been somewhat diluted. *A system is a combination of elements intended to act together to accomplish an objective.* For example, an electrical resistor is an element for impeding the flow of current, and it is usually not considered to be a system in the sense of our definition. However, when it is used in a network with other resistors, capacitors, inductors, etc., it becomes part of a system. Similarly, a car's engine is a system whose elements are the carburetor, the ignition, the crankshaft, etc. On a higher level, the car itself can be regarded as a system with the engine as an element. Since nothing in nature can be completely isolated from everything else, we see that our selection of the "boundaries" of the system depends on the purpose and the limitations of our study. This in part accounts for the widespread use of the term *system*, since almost everything can be considered a system at some level.

The Systems Approach

Automotive engineers interested in analyzing the car's overall performance would not have the need nor the time to study in detail the design of the gear train. They most likely would need to know only its gear ratio. Given this information, they would then consider the gear train as a "black box." This term is used to convey the fact that the details of the gear train are not important to the study (or at least constitute a luxury