MODELLING LARGE SYSTEMS

P C Roberts

Limits to growth revisited

blished by Taylor and Francis Ltd in association with
the Operational Research Society

Modelling Large Systems

Peter C. Roberts

ORASA text No. 4

Taylor & Francis · London · 1978

First published 1978 by Taylor & Francis Ltd,
10–14 Macklin Street, London WC2B 5NF

ISBN 0 85066 170 6

Printed and bound in the United Kingdom by
Taylor & Francis (Printers) Ltd, Rankine Road,
Basingstoke, Hampshire RG24 0PR.

Distributed in the United States and its Territories
by Halsted Press (a division of John Wiley & Sons Inc.)
605 Third Avenue, New York, N.Y. 10016.

Modelling Large Systems

Introduction

There are few phrases that give rise to such emotion in the world of the traditional academic or scientific adviser than does 'world modelling'. Born of contentious argument, fostered by naive and simplistic propaganda, the subject lives on in a world of exaggeration and partisanship. It is a debate that has forced major issues into public awareness when more traditional analysts would have suppressed them entirely, but on the whole neither side has emerged from the debate with much credit.

One of the revealing features of the debate has been the way in which so many of the contenders have revealed their ignorance of the fundamental principles of systems modelling—an ignorance which seems shocking until one tries to find a reasoned statement of those principles. There is a serious gap in the literature at this point and this book is an attempt to fill part of that gap. It is written by the Head of the Systems Analysis Research Unit of the Department of the Environment in the UK, which is one of the world's leading research teams in this area. The Unit is also unique in that it has approached its work without a political axe to grind, and started with a careful review of what had already been done before developing its own models.

The text is introductory—and should perhaps be made compulsory reading for anybody wishing to express an opinion on a world model, let alone build one. Its foundation is that of traditional science, and shows how scientific ideas develop naturally into work of this kind. But it also shows the constraints and conditions necessary for the development of a satisfactory model. It started out to be an explanatory text, but has become much more.

ROLFE TOMLINSON
General Editor

Preface

This is a book about world models though less than half of it deals directly with past or potential world models. The reason for this lies in the fact that world modelling brings into focus all of those deep seated problems of models concerned with credibility, verification, forecasting and 'who changes what' in real situations. Discussions about world modelling tend to generate much heat. I have attempted to shed some light in these pages, by examining the nuts and bolts of the modelling process.

I make no apology for extending the field of inquiry to the frontier with philosophy. All models, not only world models, force one to probe the philosophical underpinning. The natural sciences are replete with models of all sorts and the illustrative material for the early chapters draws heavily on these. I have argued that operational researchers should not forget their origins in the more traditional sciences.

My thanks are due to past and present members of the Systems Analysis Research Unit, who contributed to that peculiarly astringent style of critical discussion which forces the rejection of ill-founded assertions.

1977 PETER ROBERTS

Acknowledgements

My thanks for permission to reproduce illustrations are due to:

Blackwell Scientific Publications for Figure 2.1 from *Theoretical Ecology*, May 1976.

Wright Allen Press Inc for Figures 2.2, 2.3, 2.4, 2.5 and 2.6 from Meadows, D. L., 1970, Dynamics of Commodity Production and Cycles; for Figure 5.4 from Forrester, J. A., 1971, *World Dynamics*.

John C. Fisher for Figure 4.1 from Fisher, J. C. & Pry, R. H., June 1970, *A Simple Substitution Model of Technological Change*, Report GE 70-C-25.

F. Meyer for Figures 5.1, 5.2 and 5.3 from Meyer, F. and Vallee, J., 1975, *The Dynamics of Long Term Growth*, Technological Forecasting and Social Change 7 (285-300).

Contents

1. Science and systems

Throughout recorded history writers have sought the marks that distinguish human kind from beasts of the field. For the religious it is a simple distinction between those with and those without immortal souls. In times noted by a stronger taste for the empirical the distinction was 'man the toolmaker'. More appealing is 'man the artist', and for a touch of whimsy, *homo ludens*—'man the games player'. No doubt there are others and they collectively illustrate a higher level, an all embracing distinction, *homo generalis*—'man the generalizer'; the categorizer, botanist, propounder of natural law, seeker for similarity amidst the differences, and unity within diversity. In one way or another we all enjoy the process of reductionism, whereby the complexity of nature is explained by one general principle. A frisson of pleasure and awe runs through us at the idea that all the wealth of diversity in the physical world is created out of a handful of elements, or that the primates have topologically identical bone structures, or that all plane right-angled triangles are governed by the Pythagorean rule (including all the plane right-angled triangles that could ever be drawn).

As usual, snobbery and elitism abound. Rutherford, in his usual pithy style, wrote on the lower orders of the scientific world: 'There is physics, and there is chemistry (which is an inferior form of physics)—and there is stamp collecting'. You may not like what he says, but you know what he means. If Rutherford could claim a place in the scientific aristocracy for physics because of the generality of its statements, we should award honourable mention to the system scientists. Their generalizations are of function, of

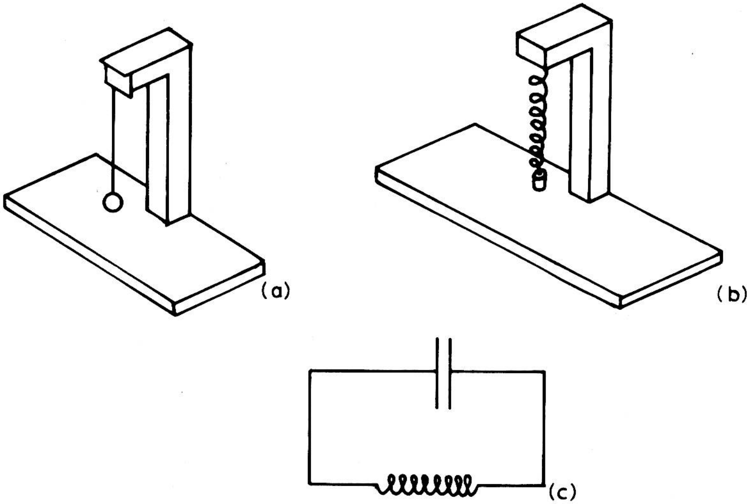

FIGURE 1.1 Three forms of oscillatory system: pendulum, vertical spring with suspended mass, series circuit of inductor and capacitor.

modes of behaviour, of dynamic properties, but their interest is in the structure, not the stuff. Consider the systems of Figure 1.1:

(*a*) Pendulum.
(*b*) Vertical spring with suspended mass.
(*c*) Series circuit of inductor and capacitor.

The materials are very different, the parameters that determine behaviour are specific to each example, but at a more abstract level these systems belong to the same class. Ignore damping and a single differential equation can be used to define the motion (whether of pendulum bob, suspended mass or current in the circuit).

Thus,

$$\frac{d^2x}{dt^2} = -kx \tag{1.1}$$

where x is a measure of the displacement from equilibrium such that:

For the pendulum, x = angle of separation from the vertical (for small angles, say $< 5°$).

For the spring, x = vertical distance of the mass from the rest position.

For the circuit, x = potential difference between the plates of the capacitor; t = time; k = a positive constant unique to each system, but always with the dimensions of the inverse square of time.

It can be shown that the differential equation integrates to yield a relation of the form:

$$x = a \sin \omega (t + \varphi) \tag{1.2}$$

where $\omega = 1/\sqrt{k}$ and a, φ are integration constants. This is the equation of an oscillator—a sine wave: repetitive cycles in which the sequence of x values recurs again and again every $2\pi/\omega$ units of time.

Though simple in character, this equation and its solution exemplifies that feature of systems which provokes interest. The component parts—the bob, string, spring, mass, capacitor, inductor—are none of them intrinsically oscillatory; but arranged in suitable relationships, the combinations of components can possess oscillatory properties. For larger systems, those with many component parts, this idea is brought out by contrasting the micro behaviour (that of the components) with the macro behaviour (that of the total system), Thus, we may contrast the chaotic dance of the gas molecules inside a balloon (the micro aspect) with the apparently steady and uniform outward force per unit area keeping the balloon inflated (the macro aspect). At the biological level, the outward manifestations of muscular contraction derive from changes to the concentrations of ions present in the cells. Hence the concept of reducing the elaborate behaviour of biological entities to the chemistry of their component molecules, and thence to the physics rules governing which molecules are formed from given elements.

A more evocative term has been coined for the systems that we call life forms: 'emergent properties'. Those qualities we associate with life, like self-replication, are regarded as emerging inevitably from large but dull amino acid molecules. It has to be said that the virus or even the amoeba emerging in this way does not challenge credibility, but that it takes an ardent reductionist to believe, say, in Bach's violin concerto in A minor 'emerging' from a mass of protoplasm.

The newcomer could be forgiven for thinking that systems science, systems engineering, systems analysis and cybernetics were brand-new shiny products of the second half of the twentieth century, since it is only in that period that the terms have come into common use. The truth is, however, that important ideas in systems thinking have a long pedigree, and some of the most striking and successful results of the systems approach belong to classical science. Indeed, so heavily dependent are many modern applications on classic models that it is worthwhile tracing their precedents, if only to appreciate that a few profound insights by the early giants can spawn generations of model builders in emulation.

Of all the classical examples which could be used, the story of the development of models for understanding the solar system is the most striking, in the sense it conveys of successive penetration to ever simpler components or principles from which the variety of outward behaviour emerges: a reduction further and further back to a most satisfying and elegant synthesis. In its last, or perhaps one should say 'current' form, the application of the system rules is called 'celestial mechanics'. This has a prosaic flavour compared with the 'music of the spheres', but it would not be inapposite to describe the end result as a 'divine simplicity'.

Celestial mechanics

The evolution in understanding of planetary motion has been of interest to historians of politics, religion and scientific thought.[1] Those planets visible to the naked eye (Mercury, Venus, Mars, Jupiter and Saturn) have apparent motions which are perplexing. For most of the time they move against the background of fixed stars in stately regular courses, but from time to time each one slows and then makes a short, retrograde step followed by a further reversal and return to forward motion. The first quantified explanation for this effect of which we are aware is due to Ptolemy in about A.D. 100. The ideas of reductionism and simple underlying principles are there, based on an idea from Aristotle who had declared that perfect motion is circular motion. So the Ptolemaic system is an ingenious mix of epicycles and deferents, circles upon circles which, given a geocentric system yields a rough approximation to the planets' motions including the retrograde phases. The heliocentric system (offered speculatively by Anaxogoras) was revived by Copernicus with the advantage of explaining retrograde motion much more elegantly, but Kepler took the next crucial step in abandoning the circles for ellipses. The Kepler laws of planetary motion give very good agreement with observation. Up to this point we see descriptions of the system with higher accuracy and using fewer parameters, but low in explanatory power (a mere five planets). Newton postulated a breath-taking generalization that not only did the elliptical orbits follow from a rule of attractive force between sun and planet varying inversely with the square of the intervening distance, but that there was a root cause of this, namely that every particle attracts every other particle according to the product of the particle masses, and inversely as the square of the particle separation. This exquisitely simple proposition has tremendous explanatory power: the motions of planets, comets, moons; tidal variation; oblateness of the earth spheroid, precession of the equinoxes and variation of pendulum swing with altitude–latitude. The remarkable success which attended this essay, to reduce a great diversity of celestial and terrestial phenomena to a universal underlying law, has encouraged three centuries of scientists and a quarter of a century of operational researchers and systems analysts to do likewise.

Urban mechanics 1

The number of objects larger than, say, a mile in diameter, that occupy the space that we call the solar system is large. The main features of the motions of these objects are puzzling, and the finer features baffling, without the guidance of the gravitational principle. But if such a unification as the Newtonian law can be demonstrated for one large system, there is reason to hope that similar feats could be achieved for others. One of the best known recent system applications following the same basic concept has been to the traffic flows in urban areas.

The traffic flows in a modern city are on a scale which certainly qualify for description as a 'large system'. In orders of magnitude there are 10^6 people, 10^4 links and nodes and 10^9 trips made in a large city each year. The capital investment in roads, vehicles, track, tunnels, bridges, etc. is a significant proportion of total public investment; the running costs in fuel, lighting, maintenance, policing and hospital care are at a similar level, and the time outlay of travellers is perhaps a third of their disposable hours (time not used in working, sleeping and eating). A set of transportation modellers has, therefore, come into being with the mission of understanding and explaining the properties of the system. This is not just an academic exercise: at any rate, the transportation modellers in government service have the goal of estimating the cost effectiveness of changes to the transport system.

People are not particles, and the 'forces' that drive them hither and thither are not gravitational in nature; nevertheless, gravity provides a useful analogy on which to build a transportation model. Suppose the ground area of the city is divided into small 'zones'. It is not unreasonable to assume that the number of trips made between zone i and zone j would be inversely related to the difficulty of getting from i to j. Although distance_{ij} may be a rough measure of difficulty, a preferred variable is cost_{ij}, and because people value time as well as money, cost of getting from zone i to zone j should take account of both the monetary expenditure and the perceived value of travel time (and conceivably, the cost of dis-discomfort). Nothing can yet be said about the 'deterrence function', the manner in which the number of trips_{ij} depends upon the generalized cost_{ij}—certainly we have no right to assume a square law dependence. Nevertheless, we can see that this law has the right pattern of behaviour, providing steadily increasing deterrence as cost increases.

It is easy to see the other constituents of the transportation model as being analogous to mass. The floor area of the buildings in the zone has been proffered as one possible analogue to mass. A more ingenious way of securing the analogue is to argue that the number of trips from source i to destination j is proportional to the total number of trips with source i to all other zones. Similarly, it can be hypothesized that the number of trips with source i and destination j is proportional to the total number of trips with destination j starting from all other zones, including i. These innocent, and even apparently tautologous assumptions, have far reaching implications. Expressed in mathematical form we have:

Let

T_{ij} = number of trips source i, destination j,
c_{ij} = generalized cost of trip between i and j,
$\sum T_{ij} = O_i$ total number of trips source i to all destinations,
$\sum_i T_{ij} = D_j$ total number of trips destination j starting from all origins,

and let $f(c_{ij})$ be a deterrence function.
Then

$$T_{ij} = A_i B_j \frac{O_i D_j}{f(c_{ij})} \tag{1.3}$$

Compare this and the equation for the gravitational attraction F between two point masses m_i and m_j separated by a distance d:

$$F_{ij} = \frac{G\, m_i m_j}{d^2{}_{ij}} \tag{1.4}$$

In this expression, one constant only is needed for all i, j—the gravitational constant G. In the transportation expression, on the other hand, many constants A_i and B_j are required specific to the contexts of i and j respectively. The reason for this modification, rather than using a single universal constant, can be understood by imagining an increase of O_i to $O_i \times 10$ and of D_j to $D_j \times 10$. Clearly the resulting T_{ij} would not be as large as $\times$ 100 compared with the original T_{ij}. Such a manner of T_{ij} increase would mean that total trips among all zones was dependent on the boundaries chosen for the zones, and this choice is arbitrary.

Before developing the transportation model further, it will be helpful to consider another piece of invention from classical science.

Gas Mechanics

A gas consists of a large number of molecules moving about and rebounding from each other and from the walls of the containing vessel. If it is assumed that these motions are random, that there is no preferred direction of motion for any particular molecule at any specified time this provides a sufficient specification to define a model from which general deductions can be made using probability methods. In Appendix 1 the expressions for the distribution of molecular density and of molecular energy are derived. It appears that the most likely distribution of molecules is that in which equal volumes within the vessel contain equal numbers of molecules, the density tends to be uniform throughout the vessel. This is an unsurprising result but if the same technique is applied to the *energies* of the molecules, an unexpected result appears. In fact the most likely distribution is one in which the numbers of molecules in equi-width energy bands, of successively higher mean energy, declines geometrically. A plot of numbers of molecules against energy of those molecules is negative exponential in form. This result is far from obvious and quite elaborate experimental means has been needed to verify that, indeed, the energy distribution of real gases conforms to this rule.

Urban mechanics 2

This technique of modelling a system by deducing the most probable amongst all its possible states has prompted A. G. Wilson[2] to devise a similar approach for transportation. In Appendix 2 it is shown that a treatment of trip making, strictly parallel with that used to model gas molecules, can be used to identify the form of the deterrence function. The two constraints applying in the gas system are: