CLASSICAL DYNAMICS

A CONTEMPORARY APPROACH

JORGE V. JOSÉ • EUGENE J. SALETAN

经典动力学现代方法

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PREFACE

Among the first courses taken by graduate students in physics in North America is Classical Mechanics. This book is a contemporary text for such a course, containing material traditionally found in the classical textbooks written through the early 1970s as well as recent developments that have transformed classical mechanics to a subject of significant contemporary research. It is an attempt to merge the traditional and the modern in one coherent presentation.

When we started writing the book we planned merely to update the classical book by Saletan and Cromer (1971) (SC) by adding more modern topics, mostly by emphasizing differential geometric and nonlinear dynamical methods. But that book was written when the frontier was largely quantum field theory, and the frontier has changed and is now moving in many different directions. Moreover, classical mechanics occupies a different position in contemporary physics than it did when SC was written. Thus this book is not merely an update of SC. Every page has been written anew and the book now includes many new topics that were not even in existence when SC was written. (Nevertheless, traces of SC remain and are evident in the frequent references to it.)

From the late seventeenth century well into the nineteenth, classical mechanics was one of the main driving forces in the development of physics, interacting strongly with developments in mathematics, both by borrowing and lending. The topics developed by its main protagonists, Newton, Lagrange, Euler, Hamilton, and Jacobi among others, form the basis of the traditional material.

In the first few decades following World War II, the graduate Classical Mechanics course, although still recognized as fundamental, was barely considered important in its own right to the education of a physicist: it was thought of mostly as a peg on which to hang quantum physics, field theory, and many-body theory, areas in which the budding physicist was expected to be working. Textbooks, including SC, concentrated on problems, mostly linear, both old and new, whose solutions could be obtained by reduction to quadrature, even though, as is now apparent, such systems form an exceptional subset of all classical dynamical systems.

In those same decades the subject itself was undergoing a rebirth and expanding, again in strong interaction with developments in mathematics. There has been an explosion in the study of nonlinear classical dynamical systems, centering in part around the discovery of novel phenomena such as chaos. (In its new incarnation the subject is often also called Dynamical Systems, particularly in its mathematical manifestations.)

What made substantive new advances possible in a subject as old as classical mechanics are two complementary developments. The first consists of qualitative but powerful geometric ideas through which the general nature of nonlinear systems can be studied (including global, rather than local analysis). The second, building upon the first, is the modern computer, which allows quantitative analysis of nonlinear systems that had not been amenable to full study by traditional analytic methods.

Unfortunately the new developments seldom found their way into Classical Mechanics courses and textbooks. There was one set of books for the traditional topics and another for the modern ones, and when we tried to teach a course that includes both the old and the new, we had to jump from one set to the other (and often to use original papers and reviews). In this book we attempt to bridge the gap: our main purpose is not only to bring the new developments to the fore, but to interweave them with more traditional topics, all under one umbrella.

That is the reason it became necessary to do more than simply update SC. In trying to mesh the modern developments with traditional subjects like the Lagrangian and Hamiltonian formulations and Hamilton–Jacobi theory we found that we needed to write an entirely new book and to add strong emphasis on nonlinear dynamics. As a result the book differs significantly not only from SC, but also from other classical textbooks such as Goldstein's.

The language of modern differential geometry is now used extensively in the literature, both physical and mathematical, in the same way that vector and matrix notation is used in place of writing out equations for each component and even of indicial notation. We therefore introduce geometric ideas early in the book and use them throughout, principally in the chapters on Hamiltonian dynamics, chaos, and Hamiltonian field theory.

Although we often present the results of computer calculations, we do not actually deal with programming as such. Nowadays that is usually treated in separate Computational Physics courses.

Because of the strong interaction between classical mechanics and mathematics, any modern book on classical mechanics must emphasize mathematics. In this book we do not shy away from that necessity. We try not to get too formal, however, in explaining the mathematics. For a rigorous treatment the reader will have to consult the mathematical literature, much of which we cite.

We have tried to start most chapters with the traditional subjects presented in a conventional way. The material then becomes more mathematically sophisticated and quantitative. Detailed applications are included both in the body of the text and as Worked Examples, whose purpose is to demonstrate to the student how the core material can be used in attacking problems.

The problems at the end of each chapter are meant to be an integral part of the course. They vary from simple extensions and mathematical exercises to more elaborate applications and include some material deliberately left for the student to discover.

An extensive bibliography is provided to further avenues of inquiry for the motivated student as well as to give credit to the proper authors of most of the ideas and developments in the book. (We have tried to be inclusive but cannot claim to be exhaustive; we apologize for works that we have failed to include and would be happy to add others that may be suggested for a possible later edition.)

Topics that are out of the mainstream of the presentation or that seem to us overly technical or represent descriptions of further developments (many with references to the literature) are set in smaller type and are bounded by vertical rules. Worked Examples are also set in smaller type and have a shaded background.

The book is undoubtedly too inclusive to be covered in a one-semester course, but it can be covered in a full year. It does not have to be studied from start to finish, and an instructor should be able to find several different fully coherent syllabus paths that include the basics of various aspects of the subject with choices from the large number of applications and extensions. We present two suggested paths at the end of this preface.

Chapter 1 is a brief review and some expansion of Newton's laws that the student is expected to bring from the undergraduate mechanics courses. It is in this chapter that velocity phase space is first introduced.

Chapter 2 and 3 are devoted to the Lagrangian formulation. In them geometric ideas are first introduced and the tangent manifold is described.

Chapter 4 covers scattering and linear oscillators. Chaos is first encountered in the context of scattering. Wave motion is introduced in the context of chains of coupled oscillators, to be used again later in connection with classical field theory.

Chapter 5 and 6 are devoted to the Hamiltonian formulation. They discuss symplectic geometry, completely integrable systems, the Hamilton–Jacobi method, perturbation theory, adiabatic invariance, and the theory of canonical transformations.

Chapter 7 is devoted to the important topic of nonlinearity. It treats nonlinear dynamical systems and maps, both continuous and discrete, as well as chaos in Hamiltonian systems and the essence of the KAM theorem.

Rigid-body motion is discussed in Chapter 8.

Chapter 9 is devoted to continuum dynamics (i.e., to classical field theory). It deals with wave equations, both linear and nonlinear, relativistic fields, and fluid dynamics. The nonlinear fields include sine–Gordon, nonlinear Klein–Gordon, as well as the Burgers and Korteweg–de Vries equations.

In the years that we have been writing this book, we have been aided by the direct help and comments of many people. In particular we want to thank Graham Farmelo for the many detailed suggestions he made concerning both substance and style. Jean Bellisard was very kind in explaining to us his version and understanding of the famous KAM theorem, which forms the basis of Section 7.5.4. Robert Dorfman made many useful suggestions after he and John Maddocks had used a preliminary version of the book in their Classical Mechanics course at the University of Maryland in 1994–5. Alan Cromer, Theo Ruijgrok, and Jeff Sokoloff also helped by reading and commenting on parts of the book. Colleagues from the Theoretical Physics Institute of Naples helped us understand the geometry that lies at the basis of much of this book. Special thanks should also go to Martin Schwarz for clarifying many subtle mathematical points to Eduardo Piña, and to Alain Chenciner. We are particularly grateful to Professor D. Schlüter for the very many corrections that he offered. We also want to thank the many anonymous referees for their constructive criticisms and suggestions, many of which we have tried to incorporate. We should also thank the many students who have used early versions of the book. The questions they raised and their suggestions have been particularly helpful. In addition, innumerable discussions with our colleagues, both present and past, have contributed to the project.

Last, but not least, JVJ wants to thank the Physics Institute of the National University of Mexico and the Theoretical Physics Institute of the University of Utrecht for their kind hospitality while part of the book was being written. The continuous support by the National Science Foundation and the Office of Naval Research has also been important in the completion of this project.

The software used in writing this book was Scientific Workplace. Most of the figures were produced using Corel Draw.

TWO PATHS THROUGH THE BOOK

In conclusion we present two suggested paths through the book for students with different undergraduate backgrounds. Both of these paths are for one-semester courses. We leave to the individual instructors the choice of material for students with really strong undergraduate backgrounds, for a second-semester graduate course, and for a one-year graduate course, all of which would treat in more detail the more advanced topics.

Path 1. For the "traditional" graduate course.

Comment: This is a suggested path through the book that comes as close as possible to the traditional course. On this path the geometry and nonlinear dynamics are minimized, though not excluded. The instructor might need to add material to this path. Other suggestions can be culled from path two.

Chapter 1. Quick review.

Chapter 2. Sections 2.1, 2.2, 2.3.1, 2.3.2, and 2.4.1.

Chapter 3. Sections 3.1.1, 3.2.1 (first and third subsections), and 3.3.1.

Chapter 4. Sections 4.1.1, 4.1.2, 4.2.1, 4.2.3, and 4.2.4.

Chapter 5. Sections 5.1.1, 5.1.3, 5.3.1, and 5.3.3.

Chapter 6. Sections 6.1.1, 6.1.2, 6.2.1, 6.2.2 (first subsection), 6.3.1, 6.3.2 (first three subsections), and 6.4.1.

Chapter 7. Sections 7.1.1, 7.1.2, 7.4.2, and 7.5.1.

Chapter 8. Sections 8.1, 8.2.1 (first two subsections), 8.3.1., and 8.3.3 (first three subsections).

Path 2. For students who have had a good undergraduate course, but one without Hamiltonian dynamics.

Comment: A lot depends on the students's background. Therefore some sections are labeled IN, for "If New." If, in addition, the students' background includes Hamiltonian dynamics, much of the first few chapters can be skimmed and the emphasis placed on later material. At the end of this path we indicate some sections that might be added for optional enrichment or substituted for skipped material.

Chapter 1. Quick review.

Chapter 2. Sections 2.1.3, 2.2.2–2.2.4, and 2.4.

Chapter 3. Sections 3.1.1 (IN), 3.2, 3.3.1, 3.3.2 (IN), and 3.4.1.

Chapter 4. Sections 4.1.1 (IN), 4.1.2, 4.1.3, 4.2.1 (IN), 4.2.2, 4.2.3, and 4.2.4 (IN).

Chapter 5. Sections 5.1.1, 5.1.3, 5.2, 5.3.1, 5.3.3, 5.3.4 (first two subsections), and 5.4.1 (first two subsections).

Chapter 6. Sections 6.1.1, 6.1.2, 6.2.1, 6.2.2 (first and fourth subsections), 6.3.1, 6.3.2 (first four subsections), 6.4.1, and 6.4.4.

Chapter 7. Sections 7.1.1, 7.1.2, 7.2, 7.4, and 7.5.1–7.5.3.

Chapter 8. Sections 8.1, 8.2.1, 8.3.1, and 8.3.3.

Chapter 9. Section 9.1.

Suggested material for optional enrichment:

Chapter 2. Section 2.3.3.

Chapter 3. Section 3.1.2.

Chapter 4. Section 4.1.4.

Chapter 5. Sections 5.1.2, 5.3.4 (third subsection), and 5.4.1 (third and fourth subsections).

Chapter 6. Sections 6.2.3, 6.3.2 (fifth and sixth subsections), 6.4.2, and 6.4.3.

Chapter 7. Sections 7.1.3, 7.3, 7.5.4, and the appendix.

Chapter 8. Sections 8.2.2 and 8.2.3.

Chapter 9. Section 9.2.1.

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