

CLASSICAL DYNAMICS

A CONTEMPORARY APPROACH

JORGE V. JOSÉ • EUGENE J. SALETAN

经典动力学现代方法



CAMBRIDGE

世界图书出版公司

CLASSICAL DYNAMICS:

A CONTEMPORARY APPROACH

JORGE V. JOSÉ

and

EUGENE J. SALETAN

CAMBRIDGE
UNIVERSITY PRESS

世界图书出版公司

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS

The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York, NY 10011-4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa
<http://www.cambridge.org>

© Cambridge University Press 1998

This book is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 1998
Reprinted 2000, 2002

Typeset in Times Roman 10.75/14 pt. and Futura in \LaTeX [TB]

A catalog record for this book is available from the British Library

Library of Congress Cataloging in Publication Data

José, Jorge V. (Jorge Valenzuela) 1949–

Classical dynamics : a contemporary approach / Jorge V. José,

Eugene J. Saletan

p. cm.

1. Mechanics, Analytic. I. Saletan, Eugene J. (Eugene Jerome),

1924– . II. Title.

QA805.J73 1998

531'.11'01515 – dc21

97-43733

CIP

ISBN 0 521 63176 9 hardback

ISBN 0 521 63636 1 paperback

This reprint edition is published with the permission of the Syndicate of the Press of the University of Cambridge, Cambridge, England.

THIS EDITION IS LICENSED FOR DISTRIBUTION AND SALE IN THE PEOPLE'S
REPUBLIC OF CHINA ONLY, EXCLUDING TAIWAN, HONG KONG AND MACAO
AND MAY NOT BE DISTRIBUTED AND SOLD ELSEWHERE.

PREFACE

Among the first courses taken by graduate students in physics in North America is Classical Mechanics. This book is a contemporary text for such a course, containing material traditionally found in the classical textbooks written through the early 1970s as well as recent developments that have transformed classical mechanics to a subject of significant contemporary research. It is an attempt to merge the traditional and the modern in one coherent presentation.

When we started writing the book we planned merely to update the classical book by Saletan and Cromer (1971) (SC) by adding more modern topics, mostly by emphasizing differential geometric and nonlinear dynamical methods. But that book was written when the frontier was largely quantum field theory, and the frontier has changed and is now moving in many different directions. Moreover, classical mechanics occupies a different position in contemporary physics than it did when SC was written. Thus this book is not merely an update of SC. Every page has been written anew and the book now includes many new topics that were not even in existence when SC was written. (Nevertheless, traces of SC remain and are evident in the frequent references to it.)

From the late seventeenth century well into the nineteenth, classical mechanics was one of the main driving forces in the development of physics, interacting strongly with developments in mathematics, both by borrowing and lending. The topics developed by its main protagonists, Newton, Lagrange, Euler, Hamilton, and Jacobi among others, form the basis of the traditional material.

In the first few decades following World War II, the graduate Classical Mechanics course, although still recognized as fundamental, was barely considered important in its own right to the education of a physicist: it was thought of mostly as a peg on which to hang quantum physics, field theory, and many-body theory, areas in which the budding physicist was expected to be working. Textbooks, including SC, concentrated on problems, mostly linear, both old and new, whose solutions could be obtained by reduction to quadrature, even though, as is now apparent, such systems form an exceptional subset of all classical dynamical systems.

In those same decades the subject itself was undergoing a rebirth and expanding, again in strong interaction with developments in mathematics. There has been an explosion in

the study of nonlinear classical dynamical systems, centering in part around the discovery of novel phenomena such as chaos. (In its new incarnation the subject is often also called Dynamical Systems, particularly in its mathematical manifestations.)

What made substantive new advances possible in a subject as old as classical mechanics are two complementary developments. The first consists of qualitative but powerful geometric ideas through which the general nature of nonlinear systems can be studied (including global, rather than local analysis). The second, building upon the first, is the modern computer, which allows quantitative analysis of nonlinear systems that had not been amenable to full study by traditional analytic methods.

Unfortunately the new developments seldom found their way into Classical Mechanics courses and textbooks. There was one set of books for the traditional topics and another for the modern ones, and when we tried to teach a course that includes both the old and the new, we had to jump from one set to the other (and often to use original papers and reviews). In this book we attempt to bridge the gap: our main purpose is not only to bring the new developments to the fore, but to interweave them with more traditional topics, all under one umbrella.

- That is the reason it became necessary to do more than simply update SC. In trying to mesh the modern developments with traditional subjects like the Lagrangian and Hamiltonian formulations and Hamilton–Jacobi theory we found that we needed to write an entirely new book and to add strong emphasis on nonlinear dynamics. As a result the book differs significantly not only from SC, but also from other classical textbooks such as Goldstein's.

The language of modern differential geometry is now used extensively in the literature, both physical and mathematical, in the same way that vector and matrix notation is used in place of writing out equations for each component and even of indicial notation. We therefore introduce geometric ideas early in the book and use them throughout, principally in the chapters on Hamiltonian dynamics, chaos, and Hamiltonian field theory.

Although we often present the results of computer calculations, we do not actually deal with programming as such. Nowadays that is usually treated in separate Computational Physics courses.

Because of the strong interaction between classical mechanics and mathematics, any modern book on classical mechanics must emphasize mathematics. In this book we do not shy away from that necessity. We try not to get too formal, however, in explaining the mathematics. For a rigorous treatment the reader will have to consult the mathematical literature, much of which we cite.

We have tried to start most chapters with the traditional subjects presented in a conventional way. The material then becomes more mathematically sophisticated and quantitative. Detailed applications are included both in the body of the text and as Worked Examples, whose purpose is to demonstrate to the student how the core material can be used in attacking problems.

The problems at the end of each chapter are meant to be an integral part of the course. They vary from simple extensions and mathematical exercises to more elaborate applications and include some material deliberately left for the student to discover.

An extensive bibliography is provided to further avenues of inquiry for the motivated student as well as to give credit to the proper authors of most of the ideas and developments in the book. (We have tried to be inclusive but cannot claim to be exhaustive; we apologize for works that we have failed to include and would be happy to add others that may be suggested for a possible later edition.)

Topics that are out of the mainstream of the presentation or that seem to us overly technical or represent descriptions of further developments (many with references to the literature) are set in smaller type and are bounded by vertical rules. Worked Examples are also set in smaller type and have a shaded background.

The book is undoubtedly too inclusive to be covered in a one-semester course, but it can be covered in a full year. It does not have to be studied from start to finish, and an instructor should be able to find several different fully coherent syllabus paths that include the basics of various aspects of the subject with choices from the large number of applications and extensions. We present two suggested paths at the end of this preface.

Chapter 1 is a brief review and some expansion of Newton's laws that the student is expected to bring from the undergraduate mechanics courses. It is in this chapter that velocity phase space is first introduced.

Chapter 2 and 3 are devoted to the Lagrangian formulation. In them geometric ideas are first introduced and the tangent manifold is described.

Chapter 4 covers scattering and linear oscillators. Chaos is first encountered in the context of scattering. Wave motion is introduced in the context of chains of coupled oscillators, to be used again later in connection with classical field theory.

Chapter 5 and 6 are devoted to the Hamiltonian formulation. They discuss symplectic geometry, completely integrable systems, the Hamilton–Jacobi method, perturbation theory, adiabatic invariance, and the theory of canonical transformations.

Chapter 7 is devoted to the important topic of nonlinearity. It treats nonlinear dynamical systems and maps, both continuous and discrete, as well as chaos in Hamiltonian systems and the essence of the KAM theorem.

Rigid-body motion is discussed in Chapter 8.

Chapter 9 is devoted to continuum dynamics (i.e., to classical field theory). It deals with wave equations, both linear and nonlinear, relativistic fields, and fluid dynamics. The nonlinear fields include sine–Gordon, nonlinear Klein–Gordon, as well as the Burgers and Korteweg–de Vries equations.

In the years that we have been writing this book, we have been aided by the direct help and comments of many people. In particular we want to thank Graham Farnelo for the many detailed suggestions he made concerning both substance and style. Jean Bellisard was very kind in explaining to us his version and understanding of the famous KAM theorem, which forms the basis of Section 7.5.4. Robert Dorfman made many useful suggestions after he and John Maddocks had used a preliminary version of the book in their Classical Mechanics course at the University of Maryland in 1994–5. Alan Cromer, Theo Ruijgrok, and Jeff Sokoloff also helped by reading and commenting on parts of the book. Colleagues from the Theoretical Physics Institute of Naples helped us understand the geometry that lies at the basis of much of this book. Special thanks should also go

to Martin Schwarz for clarifying many subtle mathematical points to Eduardo Piña, and to Alain Chenciner. We are particularly grateful to Professor D. Schlüter for the very many corrections that he offered. We also want to thank the many anonymous referees for their constructive criticisms and suggestions, many of which we have tried to incorporate. We should also thank the many students who have used early versions of the book. The questions they raised and their suggestions have been particularly helpful. In addition, innumerable discussions with our colleagues, both present and past, have contributed to the project.

Last, but not least, JVJ wants to thank the Physics Institute of the National University of Mexico and the Theoretical Physics Institute of the University of Utrecht for their kind hospitality while part of the book was being written. The continuous support by the National Science Foundation and the Office of Naval Research has also been important in the completion of this project.

The software used in writing this book was Scientific Workplace. Most of the figures were produced using Corel Draw.

TWO PATHS THROUGH THE BOOK

In conclusion we present two suggested paths through the book for students with different undergraduate backgrounds. Both of these paths are for one-semester courses. We leave to the individual instructors the choice of material for students with really strong undergraduate backgrounds, for a second-semester graduate course, and for a one-year graduate course, all of which would treat in more detail the more advanced topics.

Path 1. For the “traditional” graduate course.

Comment: This is a suggested path through the book that comes as close as possible to the traditional course. On this path the geometry and nonlinear dynamics are minimized, though not excluded. The instructor might need to add material to this path. Other suggestions can be culled from path two.

Chapter 1. Quick review.

Chapter 2. Sections 2.1, 2.2, 2.3.1, 2.3.2, and 2.4.1.

Chapter 3. Sections 3.1.1, 3.2.1 (first and third subsections), and 3.3.1.

Chapter 4. Sections 4.1.1, 4.1.2, 4.2.1, 4.2.3, and 4.2.4.

Chapter 5. Sections 5.1.1, 5.1.3, 5.3.1, and 5.3.3.

Chapter 6. Sections 6.1.1, 6.1.2, 6.2.1, 6.2.2 (first subsection), 6.3.1, 6.3.2 (first three subsections), and 6.4.1.

Chapter 7. Sections 7.1.1, 7.1.2, 7.4.2, and 7.5.1.

Chapter 8. Sections 8.1, 8.2.1 (first two subsections), 8.3.1., and 8.3.3 (first three subsections).

Path 2. For students who have had a good undergraduate course, but one without Hamiltonian dynamics.

Comment: A lot depends on the students’s background. Therefore some sections are labeled IN, for “If New.” If, in addition, the students’ background includes Hamiltonian dynamics, much of the first few chapters can be skimmed and the

emphasis placed on later material. At the end of this path we indicate some sections that might be added for optional enrichment or substituted for skipped material.

Chapter 1. Quick review.

Chapter 2. Sections 2.1.3, 2.2.2–2.2.4, and 2.4.

Chapter 3. Sections 3.1.1 (IN), 3.2, 3.3.1, 3.3.2 (IN), and 3.4.1.

Chapter 4. Sections 4.1.1 (IN), 4.1.2, 4.1.3, 4.2.1 (IN), 4.2.2, 4.2.3, and 4.2.4 (IN).

Chapter 5. Sections 5.1.1, 5.1.3, 5.2, 5.3.1, 5.3.3, 5.3.4 (first two subsections), and 5.4.1 (first two subsections).

Chapter 6. Sections 6.1.1, 6.1.2, 6.2.1, 6.2.2 (first and fourth subsections), 6.3.1, 6.3.2 (first four subsections), 6.4.1, and 6.4.4.

Chapter 7. Sections 7.1.1, 7.1.2, 7.2, 7.4, and 7.5.1–7.5.3.

Chapter 8. Sections 8.1, 8.2.1, 8.3.1, and 8.3.3.

Chapter 9. Section 9.1.

Suggested material for optional enrichment:

Chapter 2. Section 2.3.3.

Chapter 3. Section 3.1.2.

Chapter 4. Section 4.1.4.

Chapter 5. Sections 5.1.2, 5.3.4 (third subsection), and 5.4.1 (third and fourth subsections).

Chapter 6. Sections 6.2.3, 6.3.2 (fifth and sixth subsections), 6.4.2, and 6.4.3.

Chapter 7. Sections 7.1.3, 7.3, 7.5.4, and the appendix.

Chapter 8. Sections 8.2.2 and 8.2.3.

Chapter 9. Section 9.2.1.

CONTENTS

<i>List of Worked Examples</i>	<i>page xix</i>
<i>Preface</i>	<i>xxi</i>
<i>Two Paths Through the Book</i>	<i>xxiv</i>
1 FUNDAMENTALS OF MECHANICS	1
1.1 Elementary Kinematics	1
1.1.1 Trajectories of Point Particles	1
1.1.2 Position, Velocity, and Acceleration	3
1.2 Principles of Dynamics	5
1.2.1 Newton's Laws	5
1.2.2 The Two Principles	6
Principle 1	7
Principle 2	7
Discussion	9
1.2.3 Consequences of Newton's Equations	10
Introduction	10
Force is a Vector	11
1.3 One-Particle Dynamical Variables	13
1.3.1 Momentum	14
1.3.2 Angular Momentum	14
1.3.3 Energy and Work	15
In Three Dimensions	15
Application to One-Dimensional Motion	18
1.4 Many-Particle Systems	22
1.4.1 Momentum and Center of Mass	22
Center of Mass	22
Momentum	24
Variable Mass	24
1.4.2 Energy	26
1.4.3 Angular Momentum	27

1.5	Examples	29
1.5.1	Velocity Phase Space and Phase Portraits	29
	The Cosine Potential	29
	The Kepler Problem	31
1.5.2	A System with Energy Loss	34
1.5.3	Noninertial Frames and the Equivalence Principle	38
	Equivalence Principle	38
	Rotating Frames	41
	Problems	42
2	LAGRANGIAN FORMULATION OF MECHANICS	48
2.1	Constraints and Configuration Manifolds	49
2.1.1	Constraints	49
	Constraint Equations	49
	Constraints and Work	50
2.1.2	Generalized Coordinates	54
2.1.3	Examples of Configuration Manifolds	57
	The Finite Line	57
	The Circle	57
	The Plane	57
	The Two-Sphere S^2	57
	The Double Pendulum	60
	Discussion	60
2.2	Lagrange's Equations	62
2.2.1	Derivation of Lagrange's Equations	62
2.2.2	Transformations of Lagrangians	67
	Equivalent Lagrangians	67
	Coordinate Independence	68
	Hessian Condition	69
2.2.3	Conservation of Energy	70
2.2.4	Charged Particle in an Electromagnetic Field	72
	The Lagrangian	72
	A Time-Dependent Coordinate Transformation	74
2.3	Central Force Motion	77
2.3.1	The General Central Force Problem	77
	Statement of the Problem; Reduced Mass	77
	Reduction to Two Freedoms	78
	The Equivalent One-Dimensional Problem	79
2.3.2	The Kepler Problem	84
2.3.3	Bertrand's Theorem	88
2.4	The Tangent Bundle TQ	92

2.4.1	Dynamics on TQ	92
	Velocities Do Not Lie in Q	92
	Tangent Spaces and the Tangent Bundle	93
	Lagrange's Equations and Trajectories on TQ	95
2.4.2	TQ as a Differential Manifold	97
	Differential Manifolds	97
	Tangent Spaces and Tangent Bundles	100
	Application to Lagrange's Equations	102
	Problems	103
3	TOPICS IN LAGRANGIAN DYNAMICS	108
3.1	The Variational Principle and Lagrange's Equations	108
3.1.1	Derivation	108
	The Action	108
	Hamilton's Principle	110
	Discussion	112
3.1.2	Inclusion of Constraints	114
3.2	Symmetry and Conservation	118
3.2.1	Cyclic Coordinates	118
	Invariant Submanifolds and Conservation of Momentum	118
	Transformations, Passive and Active	119
	Three Examples	123
3.2.2	Noether's Theorem	124
	Point Transformations	124
	The Theorem	125
3.3	Nonpotential Forces	128
3.3.1	Dissipative Forces in the Lagrangian Formalism	129
	Rewriting the EL Equations	129
	The Dissipative and Rayleigh Functions	129
3.3.2	The Damped Harmonic Oscillator	131
3.3.3	Comment on Time-Dependent Forces	134
3.4	A Digression on Geometry	134
3.4.1	Some Geometry	134
	Vector Fields	134
	One-Forms	135
	The Lie Derivative	136
3.4.2	The Euler-Lagrange Equations	138
3.4.3	Noether's Theorem	139
	One-Parameter Groups	139
	The Theorem	140
	Problems	143

4	SCATTERING AND LINEAR OSCILLATIONS	147
4.1	Scattering	147
4.1.1	Scattering by Central Forces	147
	General Considerations	147
	The Rutherford Cross Section	153
4.1.2	The Inverse Scattering Problem	154
	General Treatment	154
	Example: Coulomb Scattering	156
4.1.3	Chaotic Scattering, Cantor Sets, and Fractal Dimension	157
	Two Disks	158
	Three Disks, Cantor Sets	162
	Fractal Dimension and Lyapunov Exponent	166
	Some Further Results	169
4.1.4	Scattering of a Charge by a Magnetic Dipole	170
	The Störmer Problem	170
	The Equatorial Limit	171
	The General Case	174
4.2	Linear Oscillations	178
4.2.1	Linear Approximation: Small Vibrations	178
	Linearization	178
	Normal Modes	180
4.2.2	Commensurate and Incommensurate Frequencies	183
	The Invariant Torus T	183
	The Poincaré Map	185
4.2.3	A Chain of Coupled Oscillators	187
	General Solution	187
	The Finite Chain	189
4.2.4	Forced and Damped Oscillators	192
	Forced Undamped Oscillator	192
	Forced Damped Oscillator	193
	Problems	197
5	HAMILTONIAN FORMULATION OF MECHANICS	201
5.1	Hamilton's Canonical Equations	202
5.1.1	Local Considerations	202
	From the Lagrangian to the Hamiltonian	202
	A Brief Review of Special Relativity	207
	The Relativistic Kepler Problem	211
5.1.2	The Legendre Transform	212

5.1.3	Unified Coordinates on T^*Q and Poisson Brackets	215
	The ξ Notation	215
	Variational Derivation of Hamilton's Equations	217
	Poisson Brackets	218
	Poisson Brackets and Hamiltonian Dynamics	222
5.2	Symplectic Geometry	224
5.2.1	The Cotangent Manifold	224
5.2.2	Two-Forms	225
5.2.3	The Symplectic Form ω	226
5.3	Canonical Transformations	231
5.3.1	Local Considerations	231
	Reduction on T^*Q by Constants of the Motion	231
	Definition of Canonical Transformations	232
	Changes Induced by Canonical Transformations	234
	Two Examples	236
5.3.2	Intrinsic Approach	239
5.3.3	Generating Functions of Canonical Transformations	240
	Generating Functions	240
	The Generating Functions Gives the New Hamiltonian	242
	Generating Functions of Type	244
5.3.4	One-Parameter Groups of Canonical Transformations	248
	Infinitesimal Generators of One-Parameter Groups;	
	Hamiltonian Flows	249
	The Hamiltonian Noether Theorem	251
	Flows and Poisson Brackets	252
5.4	Two Theorems: Liouville and Darboux	253
5.4.1	Liouville's Volume Theorem	253
	Volume	253
	Integration on T^*Q ; The Liouville Theorem	257
	Poincaré Invariants	260
	Density of States	261
5.4.2	Darboux's Theorem	268
	The Theorem	269
	Reduction	270
	Problems	275
	Canonicity Implies PB Preservation	280

6 TOPICS IN HAMILTONIAN DYNAMICS	284
6.1 The Hamilton–Jacobi Method	284
6.1.1 The Hamilton–Jacobi Equation	285
Derivation	285
Properties of Solutions	286
Relation to the Action	288
6.1.2 Separation of Variables	290
The Method of Separation	291
Example: Charged Particle in a Magnetic Field	294
6.1.3 Geometry and the HJ Equation	301
6.1.4 The Analogy Between Optics and the HJ Method	303
6.2 Completely Integrable Systems	307
6.2.1 Action–Angle Variables	307
Invariant Tori	307
The ϕ^α and J_α	309
The Canonical Transformation to AA Variables	311
Example: A Particle on a Vertical Cylinder	314
6.2.2 Liouville’s Integrability Theorem	320
Complete Integrability	320
The Tori	321
The J_α	323
Example: the Neumann Problem	324
6.2.3 Motion on the Tori	328
Rational and Irrational Winding Lines	328
Fourier Series	331
6.3 Perturbation Theory	332
6.3.1 Example: The Quartic Oscillator; Secular Perturbation Theory	332
6.3.2 Hamiltonian Perturbation Theory	336
Perturbation via Canonical Transformations	337
Averaging	339
Canonical Perturbation Theory in One Freedom	340
Canonical Perturbation Theory in Many Freedoms	346
The Lie Transformation Method	351
Example: The Quartic Oscillator	357
6.4 Adiabatic Invariance	359
6.4.1 The Adiabatic Theorem	360
Oscillator with Time-Dependent Frequency	360
The Theorem	361
Remarks on $N > 1$	363
6.4.2 Higher Approximations	364

6.4.3	The Hannay Angle	365
6.4.4	Motion of a Charged Particle in a Magnetic Field	371
	The Action Integral	371
	Three Magnetic Adiabatic Invariants	374
	Problems	377
7	NONLINEAR DYNAMICS	382
7.1	Nonlinear Oscillators	383
7.1.1	A Model System	383
7.1.2	Driven Quartic Oscillator	386
	Damped Driven Quartic Oscillator; Harmonic Analysis	387
	Undamped Driven Quartic Oscillator	390
7.1.3	Example: The van der Pol Oscillator	391
7.2	Stability of Solutions	396
7.2.1	Stability of Autonomous Systems	397
	Definitions	397
	The Poincaré–Bendixon Theorem	399
	Linearization	400
7.2.2	Stability of Nonautonomous Systems	410
	The Poincaré Map	410
	Linearization of Discrete Maps	413
	Example: The Linearized Hénon Map	417
7.3	Parametric Oscillators	418
7.3.1	Floquet Theory	419
	The Floquet Operator \mathbf{R}	419
	Standard Basis	420
	Eigenvalues of \mathbf{R} and Stability	421
	Dependence on G	424
7.3.2	The Vertically Driven Pendulum	424
	The Mathieu Equation	424
	Stability of the Pendulum	426
	The Inverted Pendulum	427
	Damping	429
7.4	Discrete Maps; Chaos	431
7.4.1	The Logistic Map	431
	Definition	432
	Fixed Points	432
	Period Doubling	434
	Universality	442
	Further Remarks	444

7.4.2	The Circle Map	445
	The Damped Driven Pendulum	445
	The Standard Sine Circle Map	446
	Rotation Number and the Devil's Staircase	447
	Fixed Points of the Circle Map	450
7.5	Chaos in Hamiltonian Systems and the KAM Theorem	452
7.5.1	The Kicked Rotator	453
	The Dynamical System	453
	The Standard Map	454
	Poincaré Map of the Perturbed System	455
7.5.2	The Hénon Map	460
7.5.3	Chaos in Hamiltonian Systems	463
	Poincaré–Birkhoff Theorem	464
	The Twist Map	466
	Numbers and Properties of the Fixed Points	467
	The Homoclinic Tangle	468
	The Transition to Chaos	472
7.5.4	The KAM Theorem	474
	Background	474
	Two Conditions: Hessian and Diophantine	475
	The Theorem	477
	A Brief Description of the Proof of KAM	480
	Problems	483
	Number Theory	486
	The Unit Interval	486
	A Diophantine Condition	487
	The Circle and the Plane	488
	KAM and Continued Fractions	489
8	RIGID BODIES	492
8.1	Introduction	492
8.1.1	Rigidity and Kinematics	492
	Definition	492
	The Angular Velocity Vector ω	493
8.1.2	Kinetic Energy and Angular Momentum	495
	Kinetic Energy	495
	Angular Momentum	498
8.1.3	Dynamics	499
	Space and Body Systems	499
	Dynamical Equations	500
	Example: The Gyrocompass	503

Motion of the Angular Momentum \mathbf{J}	505
Fixed Points and Stability	506
The Poinot Construction	508
8.2 The Lagrangian and Hamiltonian Formulations	510
8.2.1 The Configuration Manifold \mathcal{Q}_R	510
Inertial, Space, and Body Systems	510
The Dimension of \mathcal{Q}_R	511
The Structure of \mathcal{Q}_R	512
8.2.2 The Lagrangian	514
Kinetic Energy	514
The Constraints	515
8.2.3 The Euler–Lagrange Equations	516
Derivation	516
The Angular Velocity Matrix Ω	518
8.2.4 The Hamiltonian Formalism	519
8.2.5 Equivalence to Euler's Equations	520
Antisymmetric Matrix–Vector Correspondence	520
The Torque	521
The Angular Velocity Pseudovector and	
Kinematics	522
Transformations of Velocities	523
Hamilton's Canonical Equations	524
8.2.6 Discussion	525
8.3 Euler Angles and Spinning Tops	526
8.3.1 Euler Angles	526
Definition	526
R in Terms of the Euler Angles	527
Angular Velocities	529
Discussion	531
8.3.2 Geometric Phase for a Rigid Body	533
8.3.3 Spinning Tops	535
The Lagrangian and Hamiltonian	536
The Motion of the Top	537
Nutation and Precession	539
Quadratic Potential; the Neumann Problem	542
8.4 Cayley–Klein Parameters	543
8.4.1 2×2 Matrix Representation of 3-Vectors and	
Rotations	543
3-Vectors	543
Rotations	544
8.4.2 The Pauli Matrices and CK Parameters	544
Definitions	544