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双语教材

微积分

（第六版）

Calculus (6th Edition)

詹姆斯·斯图尔特（James Stewart） 著

张乃岳 编译

黄志勇 审

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编译者前言

由詹姆斯·斯图尔特编写的《微积分》（*Calculus*）是一本在欧美高校中备受欢迎的微积分教材，本版是它的第六版，它采用了由浅入深的方式，向读者介绍了微积分的相关概念和分析解决问题的方法。

本书是在詹姆斯·斯图尔特编写的这本《微积分》（*Calculus*）的基础上删改而成的。我们在删改时充分考虑了中国高校教学和中国学生需求的特点与学校教学的课时要求，其中删改的内容主要包括：定积分的更多应用（further applications of integration），参数方程和极坐标（parametric equations and polar coordinates），向量空间和几何空间（vectors and the geometry of space），向量函数（vector functions），向量微积分（vector calculus）的大部分内容以及二阶微分方程（second-order differential equations）。此外，我们根据中国的实际情况，对原书中的一些应用性习题和实验性习题也进行了删减。

经过删减后，本书基本保留了原书的逻辑体系，最后成书包括十二章以及学习微积分所需的初等数学知识介绍。每一章的内容包含知识讲解、例题解析以及练习题三部分，书后附录中附有练习题的答案。此外，为了便于读者更好地理解数学中的一些英文关键术语的中文含义，我们在每一章的章末增加了关键术语的中英文对照表，读者在学习过程中可以参考。

本书可以作为中国高等院校微积分课程的双语教材和教师参考书，也可作为国际培训班中所有需要微积分教学的专业的数学教材，使用本书进行教学不仅可以使学生们掌握微积分的概念和计算技巧，也可以使学生提高自己的外语水平和能力。

编译者　张乃岳
北京大学

PREFACE

This *Metric International Version* differs from the regular version of *Calculus,* Sixth Edition, in several ways:

The units used in almost all of the examples and exercises have been changed from US Customary units to metric units. There are a small number of exceptions: In some engineering applications it may be useful for some engineers to be familiar with US units. And I wanted to retain a few exercises (for example, those involving baseball) where it would be inappropriate to use metric units.

I've changed the examples and exercises involving real-world data to be more international in nature, so that the vast majority of them now come from countries other than the United States. For example, there are now exercises and examples concerning Hong Kong postal rates; Canadian public debt; unemployment rates in Australia; hours of daylight in Ankara, Turkey; isothermals in China; percentage of the population in rural Argentina; populations of Malaysia, Indonesia, Mexico, and India; and power consumption in Ontario, among many others.

In addition to changing exercises so that the units are metric and the data have a more international flavor, a number of other exercises have been changed as well, the result being that about 10% of the exercises are different from those in the regular version.

PHILOSOPHY OF THE BOOK

The art of teaching, Mark Van Doren said, is the art of assisting discovery. I have tried to write a book that assists students in discovering calculus—both for its practical power and its surprising beauty. In this edition, as in the first five editions, I aim to convey to the student a sense of the utility of calculus and develop technical competence, but I also strive to give some appreciation for the intrinsic beauty of the subject. Newton undoubtedly experienced a sense of triumph when he made his great discoveries. I want students to share some of that excitement.

The emphasis is on understanding concepts. I think that nearly everybody agrees that this should be the primary goal of calculus instruction. In fact, the impetus for the current calculus reform movement came from the Tulane Conference in 1986, which formulated as their first recommendation:

Focus on conceptual understanding.

I have tried to implement this goal through the *Rule of Three*: "Topics should be presented geometrically, numerically, and algebraically." Visualization, numerical and graphical experimentation, and other approaches have changed how we teach conceptual reasoning in fundamental ways. More recently, the Rule of Three has been expanded to become the *Rule of Four* by emphasizing the verbal, or descriptive, point of view as well.

In writing the sixth edition my premise has been that it is possible to achieve conceptual understanding and still retain the best traditions of traditional calculus. The book contains elements of reform, but within the context of a traditional curriculum.

ALTERNATIVE VERSIONS

I have written several other calculus textbooks that might be preferable for some instructors. Most of them also come in single variable and multivariable versions.

- *Metric International Version Calculus, Early Transcendentals,* Sixth Edition, is similar to the present textbook except that the exponential, logarithmic, and inverse trigonometric functions are covered in the first semester.
- *International Student Edition Essential Calculus* is a much briefer book (800 pages), though it contains almost all of the topics in the present text. The relative brevity is achieved through briefer exposition of some topics and putting some features on the website.
- *Metric International Version Calculus: Concepts and Contexts,* Third Edition, emphasizes conceptual understanding even more strongly than this book. The coverage of topics is not encyclopedic and the material on transcendental functions and on parametric equations is woven throughout the book instead of being treated in separate chapters.
- *Calculus: Early Vectors* introduces vectors and vector functions in the first semester and integrates them throughout the book. It is suitable for students taking Engineering and Physics courses concurrently with calculus.

CONTENT

Diagnostic Tests The book begins with four diagnostic tests, in Basic Algebra, Analytic Geometry, Functions, and Trigonometry.

A Preview of Calculus This is an overview of the subject and includes a list of questions to motivate the study of calculus.

1 ▪ Functions and Models From the beginning, multiple representations of functions are stressed: verbal, numerical, visual, and algebraic. A discussion of mathematical models leads to a review of the standard functions, including exponential and logarithmic functions, from these four points of view.

2 ▪ Limits The material on limits is motivated by a prior discussion of the tangent and velocity problems. Limits are treated from descriptive, graphical, numerical, and algebraic points of view.

3 ▪ Derivatives The material on derivatives is covered in two sections in order to give students more time to get used to the idea of a derivative as a function. The examples and exercises explore the meanings of derivatives in various contexts. Higher derivatives are now introduced in Section 3.2.

4 ▪ Applications of Differentiation The basic facts concerning extreme values and shapes of curves are deduced from the Mean Value Theorem. Graphing with technology emphasizes the interaction between calculus and calculators and the analysis of families of curves.

5 ▪ Integrals The area problem and the distance problem serve to motivate the definite integral, with sigma notation introduced as needed. Emphasis is placed on explaining the meanings of integrals in various contexts and on estimating their values from graphs and tables.

6 ▪ Applications of Integration Here I present the applications of integration—area, volume, work, average value—that can reasonably be done without specialized techniques of integration. General methods are

emphasized. The goal is for students to be able to divide a quantity into small pieces, estimate with Riemann sums, and recognize the limit as an integral.

7 ▪ Inverse Functions This chapter discusses Inverse functions.

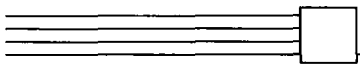
8 ▪ Techniques of Integration All the standard methods are covered but, of course, the real challenge is to be able to recognize which technique is best used in a given situation. Accordingly, in Section 8.5, I present a strategy for integration.

9 ▪ Differential Equations Modeling is the theme that unifies this introductory treatment of differential equations.

10 ▪ Infinite Sequences and Series The convergence tests have intuitive justifications as well as formal proofs. Numerical estimates of sums of series are based on which test was used to prove convergence.

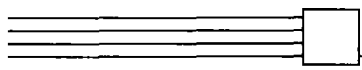
11 ▪ Partial Derivatives Functions of two or more variables are studied from verbal, numerical, visual, and algebraic points of view. In particular, I introduce partial derivatives by looking at a specific column in a table of values of the heat index (perceived air temperature) as a function of the actual temperature and the relative humidity. Directional derivatives are estimated from contour maps of temperature, pressure, and snowfall.

12 ▪ Multiple Integrals Contour maps and the Midpoint Rule are used to estimate the average snowfall and average temperature in given regions. Double and triple integrals are used to compute probabilities, surface areas, and (in projects) volumes of hyperspheres and volumes of intersections of three cylinders. Cylindrical and spherical coordinates are introduced in the context of evaluating triple integrals.



ANCILLARIES

Metric International Version Calculus, Sixth Edition, is supported by a complete set of ancillaries developed under my direction. Each piece has been designed to enhance student understanding and to facilitate creative instruction. The tables on pages xxi–xxii describe each of these ancillaries.



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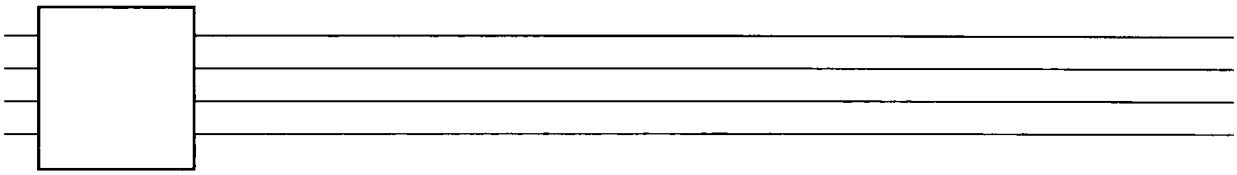
JAMES STEWART



TO THE STUDENT

Reading a calculus textbook is different from reading a newspaper or a novel, or even a physics book. Don't be discouraged if you have to read a passage more than once in order to understand it. You should have pencil and paper and calculator at hand to sketch a diagram or make a calculation.

Some students start by trying their homework problems and read the text only if they get stuck on an exercise. I suggest that a far better plan is to read and understand a section of the text before attempting the exercises. In particular, you should look at the definitions to see the exact meanings of the terms. And before you read each example, I suggest that you cover up the solution and try solving the problem yourself. You'll get a lot more from looking at the solution if you do so.

Part of the aim of this course is to train you to think logically. Learn to write the solutions of the exercises in a connected, step-by-step fashion with explanatory sentences—not just a string of disconnected equations or formulas.

The answers to the odd-numbered exercises appear at the back of the book, in Appendix I. Some exercises ask for a verbal explanation or interpretation or description. In such cases there is no single correct way of expressing the answer, so don't worry that you haven't found the definitive answer. In addition, there are often several different forms in which to express a numerical or algebraic answer, so if your answer differs from mine, don't immediately assume you're wrong. For example, if the answer given in the back of the book is $\sqrt{2} - 1$ and you obtain $1/(1 + \sqrt{2})$, then you're right and rationalizing the denominator will show that the answers are equivalent.

The icon indicates an exercise that definitely requires the use of either a graphing calculator or a computer with graphing software. But that doesn't mean that graphing devices can't be used to check your work on the other exercises as well. The symbol CAS is reserved for problems in which the full resources of a computer algebra system (like Derive, Maple, Mathematica, or the TI-89/92) are required.

You will also encounter the symbol ∅, which warns you against committing an error. I have placed this symbol in the margin in situations where I have observed that a large proportion of my students tend to make the same mistake.

Tools for Enriching Calculus, which is a companion to this text, is referred to by means of the symbol TEC and can be accessed from www.stewartcalculus.com. It directs you to modules in which you can explore aspects of calculus for which the computer is particularly useful. TEC also provides *Homework Hints* for representative exercises that are indicated by printing the exercise number in red: 15. These homework hints ask you questions that allow you to make progress toward a solution without actually giving you the answer. You need to pursue each hint in an active manner with pencil and paper to work out the details. If a particular hint doesn't enable you to solve the problem, you can click to reveal the next hint.

An optional CD-ROM that your instructor may have asked you to purchase is the *Interactive Video Skillbuilder,* which contains videos of instructors explaining two or three of the examples in every section of the text.

I recommend that you keep this book for reference purposes after you finish the course. Because you will likely forget some of the specific details of calculus, the book will serve as a useful reminder when you need to use calculus in subsequent courses. And, because this book contains more material than can be covered in any one course, it can also serve as a valuable resource for a working scientist or engineer.

Calculus is an exciting subject, justly considered to be one of the greatest achievements of the human intellect. I hope you will discover that it is not only useful but also intrinsically beautiful.

JAMES STEWART

DIAGNOSTIC TESTS

Success in calculus depends to a large extent on knowledge of the mathematics that precedes calculus: algebra, analytic geometry, functions, and trigonometry. The following tests are intended to diagnose weaknesses that you might have in these areas. After taking each test you can check your answers against the given answers and, if necessary, refresh your skills by referring to the review materials that are provided.

A DIAGNOSTIC TEST: ALGEBRA

1. Evaluate each expression without using a calculator.

(a) $(-3)^4$ (b) -3^4 (c) 3^{-4}

(d) $\dfrac{5^{23}}{5^{21}}$ (e) $\left(\dfrac{2}{3}\right)^{-2}$ (f) $16^{-3/4}$

2. Simplify each expression. Write your answer without negative exponents.

(a) $\sqrt{200} - \sqrt{32}$

(b) $(3a^3b^3)(4ab^2)^2$

(c) $\left(\dfrac{3x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2}$

3. Expand and simplify.

(a) $3(x + 6) + 4(2x - 5)$ (b) $(x + 3)(4x - 5)$

(c) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$ (d) $(2x + 3)^2$

(e) $(x + 2)^3$

4. Factor each expression.

(a) $4x^2 - 25$ (b) $2x^2 + 5x - 12$

(c) $x^3 - 3x^2 - 4x + 12$ (d) $x^4 + 27x$

(e) $3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$ (f) $x^3y - 4xy$

5. Simplify the rational expression.

(a) $\dfrac{x^2 + 3x + 2}{x^2 - x - 2}$ (b) $\dfrac{2x^2 - x - 1}{x^2 - 9} \cdot \dfrac{x + 3}{2x + 1}$

(c) $\dfrac{x^2}{x^2 - 4} - \dfrac{x + 1}{x + 2}$ (d) $\dfrac{\dfrac{y}{x} - \dfrac{x}{y}}{\dfrac{1}{y} - \dfrac{1}{x}}$

6. Rationalize the expression and simplify.

(a) $\dfrac{\sqrt{10}}{\sqrt{5}-2}$ (b) $\dfrac{\sqrt{4+h}-2}{h}$

7. Rewrite by completing the square.

(a) x^2+x+1 (b) $2x^2-12x+11$

8. Solve the equation. (Find only the real solutions.)

(a) $x+5=14-\frac{1}{2}x$ (b) $\dfrac{2x}{x+1}=\dfrac{2x-1}{x}$

(c) $x^2-x-12=0$ (d) $2x^2+4x+1=0$

(e) $x^4-3x^2+2=0$ (f) $3|x-4|=10$

(g) $2x(4-x)^{-1/2}-3\sqrt{4-x}=0$

9. Solve each inequality. Write your answer using interval notation.

(a) $-4<5-3x\leq 17$ (b) $x^2<2x+8$

(c) $x(x-1)(x+2)>0$ (d) $|x-4|<3$

(e) $\dfrac{2x-3}{x+1}\leq 1$

10. State whether each equation is true or false.

(a) $(p+q)^2=p^2+q^2$ (b) $\sqrt{ab}=\sqrt{a}\sqrt{b}$

(c) $\sqrt{a^2+b^2}=a+b$ (d) $\dfrac{1+TC}{C}=1+T$

(e) $\dfrac{1}{x-y}=\dfrac{1}{x}-\dfrac{1}{y}$ (f) $\dfrac{1/x}{a/x-b/x}=\dfrac{1}{a-b}$

ANSWERS TO DIAGNOSTIC TEST A: ALGEBRA

1. (a) 81 (b) -81 (c) $\frac{1}{81}$
(d) 25 (e) $\frac{9}{4}$ (f) $\frac{1}{8}$

2. (a) $6\sqrt{2}$ (b) $48a^5b^7$ (c) $\dfrac{x}{9y^7}$

3. (a) $11x-2$ (b) $4x^2+7x-15$
(c) $a-b$ (d) $4x^2+12x+9$
(e) $x^3+6x^2+12x+8$

4. (a) $(2x-5)(2x+5)$ (b) $(2x-3)(x+4)$
(c) $(x-3)(x-2)(x+2)$ (d) $x(x+3)(x^2-3x+9)$
(e) $3x^{-1/2}(x-1)(x-2)$ (f) $xy(x-2)(x+2)$

5. (a) $\dfrac{x+2}{x-2}$ (b) $\dfrac{x-1}{x-3}$
(c) $\dfrac{1}{x-2}$ (d) $-(x+y)$

6. (a) $5\sqrt{2}+2\sqrt{10}$ (b) $\dfrac{1}{\sqrt{4+h}+2}$

7. (a) $\left(x+\frac{1}{2}\right)^2+\frac{3}{4}$ (b) $2(x-3)^2-7$

8. (a) 6 (b) 1 (c) $-3, 4$
(d) $-1\pm\frac{1}{2}\sqrt{2}$ (e) $\pm 1, \pm\sqrt{2}$ (f) $\frac{2}{3}, \frac{22}{3}$
(g) $\frac{12}{5}$

9. (a) $[-4, 3)$ (b) $(-2, 4)$
(c) $(-2, 0)\cup(1, \infty)$ (d) $(1, 7)$
(e) $(-1, 4]$

10. (a) False (b) True (c) False
(d) False (e) False (f) True

If you have had difficulty with these problems, you may wish to consult the Review of Algebra on the website www.stewartcalculus.com.

B DIAGNOSTIC TEST: ANALYTIC GEOMETRY

1. Find an equation for the line that passes through the point $(2, -5)$ and
(a) has slope -3
(b) is parallel to the x-axis
(c) is parallel to the y-axis
(d) is parallel to the line $2x - 4y = 3$

2. Find an equation for the circle that has center $(-1, 4)$ and passes through the point $(3, -2)$.

3. Find the center and radius of the circle with equation $x^2 + y^2 - 6x + 10y + 9 = 0$.

4. Let $A(-7, 4)$ and $B(5, -12)$ be points in the plane.
(a) Find the slope of the line that contains A and B.
(b) Find an equation of the line that passes through A and B. What are the intercepts?
(c) Find the midpoint of the segment AB.
(d) Find the length of the segment AB.
(e) Find an equation of the perpendicular bisector of AB.
(f) Find an equation of the circle for which AB is a diameter.

5. Sketch the region in the xy-plane defined by the equation or inequalities.
(a) $-1 \leq y \leq 3$
(b) $|x| < 4$ and $|y| < 2$
(c) $y < 1 - \frac{1}{2}x$
(d) $y \geq x^2 - 1$
(e) $x^2 + y^2 < 4$
(f) $9x^2 + 16y^2 = 144$

ANSWERS TO DIAGNOSTIC TEST B: ANALYTIC GEOMETRY

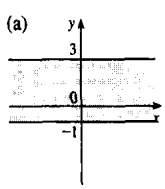
1. (a) $y = -3x + 1$ (b) $y = -5$
(c) $x = 2$ (d) $y = \frac{1}{2}x - 6$

2. $(x + 1)^2 + (y - 4)^2 = 52$

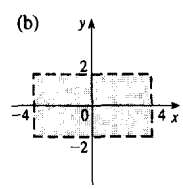
3. Center $(3, -5)$, radius 5

4. (a) $-\frac{4}{3}$
(b) $4x + 3y + 16 = 0$; x-intercept -4, y-intercept $-\frac{16}{3}$
(c) $(-1, -4)$
(d) 20
(e) $3x - 4y = 13$
(f) $(x + 1)^2 + (y + 4)^2 = 100$

5. (a)



(b)



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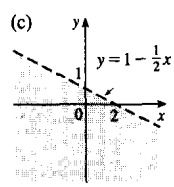

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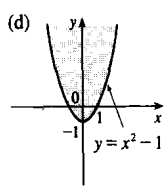

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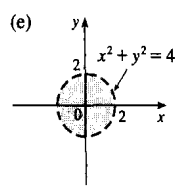

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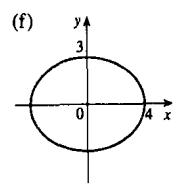


C DIAGNOSTIC TEST: FUNCTIONS

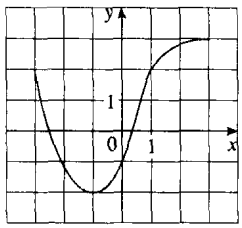


FIGURE FOR PROBLEM 1

1. The graph of a function f is given at the left.
(a) State the value of $f(-1)$.
(b) Estimate the value of $f(2)$.
(c) For what values of x is $f(x) = 2$?
(d) Estimate the values of x such that $f(x) = 0$.
(e) State the domain and range of f.

2. If $f(x) = x^3$, evaluate the difference quotient $\dfrac{f(2+h) - f(2)}{h}$ and simplify your answer.

3. Find the domain of the function.

(a) $f(x) = \dfrac{2x+1}{x^2 + x - 2}$ (b) $g(x) = \dfrac{\sqrt[3]{x}}{x^2 + 1}$ (c) $h(x) = \sqrt{4 - x} + \sqrt{x^2 - 1}$

4. How are graphs of the functions obtained from the graph of f?

(a) $y = -f(x)$ (b) $y = 2f(x) - 1$ (c) $y = f(x - 3) + 2$

5. Without using a calculator, make a rough sketch of the graph.

(a) $y = x^3$ (b) $y = (x + 1)^3$ (c) $y = (x - 2)^3 + 3$
(d) $y = 4 - x^2$ (e) $y = \sqrt{x}$ (f) $y = 2\sqrt{x}$
(g) $y = -2^x$ (h) $y = 1 + x^{-1}$

6. Let $f(x) = \begin{cases} 1 - x^2 & \text{if } x \leq 0 \\ 2x + 1 & \text{if } x > 0 \end{cases}$

(a) Evaluate $f(-2)$ and $f(1)$. (b) Sketch the graph of f.

7. If $f(x) = x^2 + 2x - 1$ and $g(x) = 2x - 3$, find each of the following functions.

(a) $f \circ g$ (b) $g \circ f$ (c) $g \circ g \circ g$

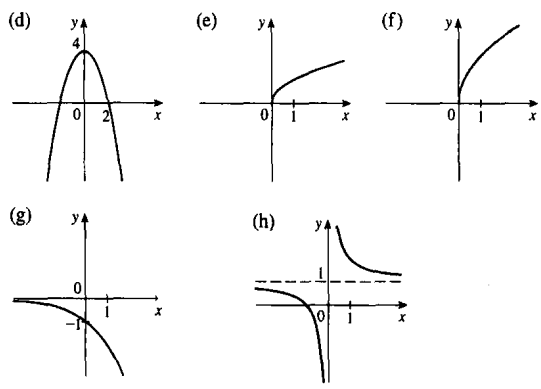
ANSWERS TO DIAGNOSTIC TEST C: FUNCTIONS

1. (a) -2 (b) 2.8
(c) $-3, 1$ (d) $-2.5, 0.3$
(e) $[-3, 3], [-2, 3]$

2. $12 + 6h + h^2$

3. (a) $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$
(b) $(-\infty, \infty)$
(c) $(-\infty, -1] \cup [1, 4]$

4. (a) Reflect about the x-axis
(b) Stretch vertically by a factor of 2, then shift 1 unit downward
(c) Shift 3 units to the right and 2 units upward



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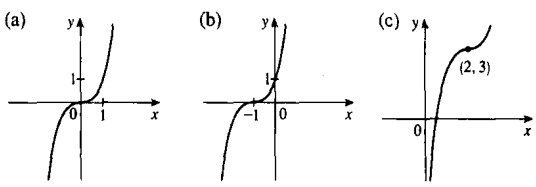
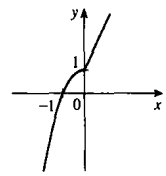


6. (a) $-3, 3$
(b)


7. (a) $(f \circ g)(x) = 4x^2 - 8x + 2$
(b) $(g \circ f)(x) = 2x^2 + 4x - 5$
(c) $(g \circ g \circ g)(x) = 8x - 21$

If you have had difficulty with these problems, you should look at Sections 1.1–1.2 of this book.

D DIAGNOSTIC TEST: TRIGONOMETRY

1. Convert from degrees to radians.
   (a) 300° (b) −18°
2. Convert from radians to degrees.
   (a) $5\pi/6$ (b) 2
3. Find the length of an arc of a circle with radius 12 cm if the arc subtends a central angle of 30°.
4. Find the exact values.
   (a) $\tan(\pi/3)$ (b) $\sin(7\pi/6)$ (c) $\sec(5\pi/3)$

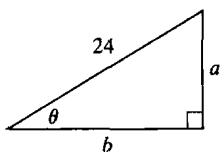
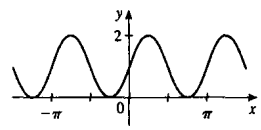


FIGURE FOR PROBLEM 5

5. Express the lengths a and b in the figure in terms of θ.
6. If $\sin x = \frac{1}{3}$ and $\sec y = \frac{5}{4}$, where x and y lie between 0 and $\pi/2$, evaluate $\sin(x + y)$.
7. Prove the identities.
   (a) $\tan\theta \sin\theta + \cos\theta = \sec\theta$
   (b) $\dfrac{2\tan x}{1 + \tan^2 x} = \sin 2x$
8. Find all values of x such that $\sin 2x = \sin x$ and $0 \leq x \leq 2\pi$.
9. Sketch the graph of the function $y = 1 + \sin 2x$ without using a calculator.

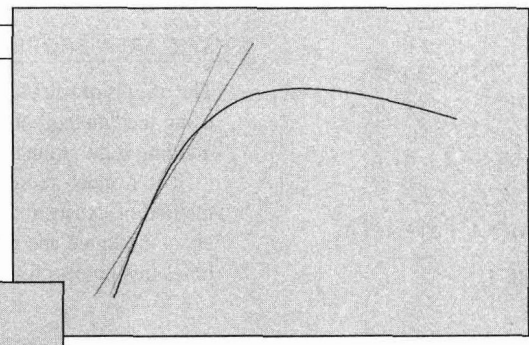
ANSWERS TO DIAGNOSTIC TEST D: TRIGONOMETRY

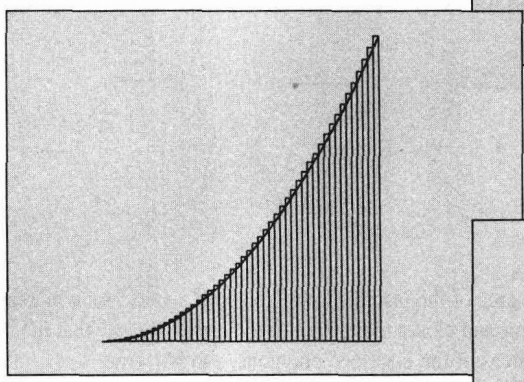
1. (a) $5\pi/3$ (b) $-\pi/10$
2. (a) 150° (b) $360/\pi \approx 114.6°$
3. 2π cm
4. (a) $\sqrt{3}$ (b) $-\frac{1}{2}$ (c) 2
5. (a) $24\sin\theta$ (b) $24\cos\theta$
6. $\frac{1}{15}(4 + 6\sqrt{2})$
8. $0, \pi/3, \pi, 5\pi/3, 2\pi$
9. 

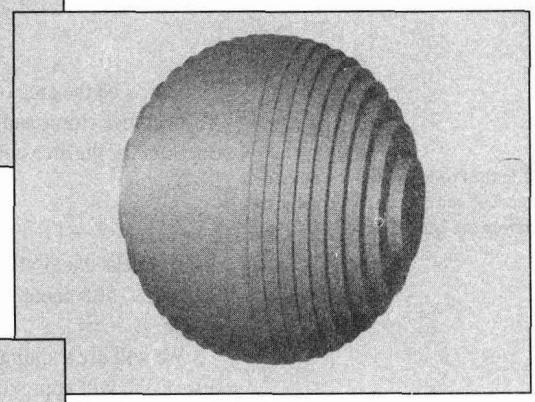
English-Chinese Key Terms

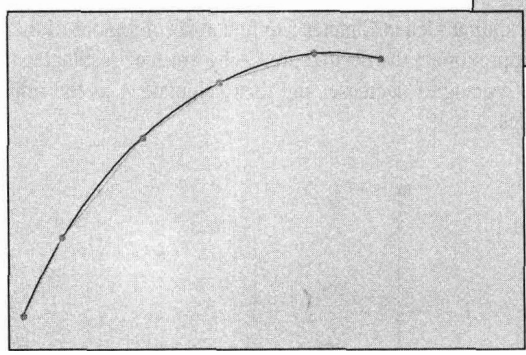
Algebra 代数	Plane 平面
Calculator 计算器	Slope 斜率
Expression 表达式	Function 函数
Exponent 指数	Domain 定义域
Expand 展开	Range 值域
Inequality 不等式	Trigonometry 三角学
Geometry 几何	Polynomials 多项式

A PREVIEW OF CALCULUS









Calculus is fundamentally different from the mathematics that you have studied previously: calculus is less static and more dynamic. It is concerned with change and motion; it deals with quantities that approach other quantities. For that reason it may be useful to have an overview of the subject before beginning its intensive study. Here we give a glimpse of some of the main ideas of calculus by showing how the concept of a limit arises when we attempt to solve a variety of problems.