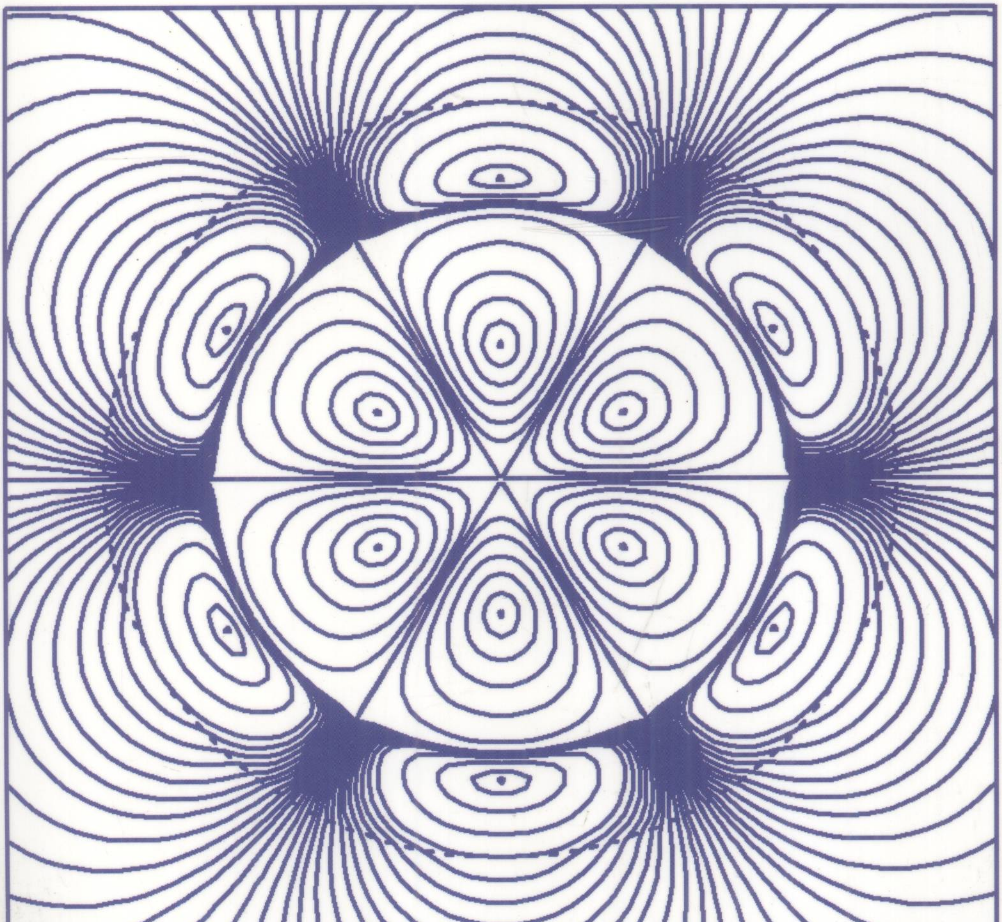
PHYSICS TEXTBOOK

Jacques Bures

WILEY-VCH

Guided Optics

Optical Fibers and All-fiber Components



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Guided Optics

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"Celui qui aime apprendre est bien près du savoir"
Confucius

Preface

What could appear simpler than the free space propagation of an homogeneous and monochromatic electromagnetic plane wave? The electric field **E** and the magnetic field **H**, intimately coupled by the Maxwell equations, seem to behave so nicely: not only are the vectors **E** and **H** proportional and perpendicular to each other in all points of space, they also lie in a plane which is normal to the direction of propagation. These fields are linearly polarized and the continuum of solutions resulting from Helmholtz's scalar wave equation is sufficient to give a complete description of the propagation of this optical wave.

Unfortunately, even in the vacuum, this ideal wave, which extends uniformly throughout space, does not truly exist. Neither does its geometrical optics analogue, the infinitely thin ray of light. The optical wave is physically inhomogeneous and, to treat it correctly, one must therefore decompose the field using a spectrum of homogeneous plane waves, where each of these waves obeys the Helmholtz equation.

The complexity increases when we consider guided waves because the fields **E** and **H** dissipate considerably. Interference of light arises in the directions normal to the waveguide interfaces and results in discrete solutions that are invariant in the direction of propagation: these are the guided modes. Longitudinal components e_z and h_z appear and they are in quadrature with respect to the transverse components $\mathbf{E}_t$ and $\mathbf{H}_t$, which are no longer proportional, nor perpendicular to each other. The lines of polarization in the cross section of the waveguide are no longer straight; their diverse features sometimes leave an artistic impression. Since the scalar wave equation is no longer valid, we must therefore use vectorial wave equations; which are much more general, but whose solutions are considerably more complicated. Finally, additional difficulties present themselves when invariance in the direction of propagation (translation symmetry) is no longer valid. Such is the case with fiber splices and fiber components like tapers, Bragg gratings and couplers.

It is within this context of subtleties that I have elaborated this book. It is addressed to all graduate students in optics, but particularly to the students from the fiber optics research group of the Department of Physics Engineering of École Polytechnique de Montréal. During my career as a professor and teacher, I discovered that some students, even the brilliant ones, still struggled

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at the beginning of their graduate studies with some of the general concepts of guided waves and fiber components like couplers, tapered fibers and fiber Bragg gratings.

This is therefore a fundamental book with some high-level refinements, however we limit ourselves to guided modes within the scope of linear optics. We begin with Maxwell equations for dielectric media. The resulting vectorial equations are solved analytically and exactly in the case of one-dimensional planar waveguides and multiple-cladding step-index optical fibers having circular symmetry. In all other cases, we rely on purely numerical methods. A large part is devoted to the theory of optical fiber components and the modeling of their behavior. Thus, we develop the fundamental notions of perturbed waveguides, mode coupling, local modes, supermodes, and coupling between fibers. Apart from a few rare exceptions, we present fully detailed demonstrations of the equations; this sometimes brings greater weight to the text, but it also brings greater clarity. A multitude of figures illustrate the theory and the experimental results. Furthermore, a few indispensable appendices were added, such as: definitions, properties and integrals of Bessel functions and modified Bessel functions; as well as the definitions of the absolute refraction indices of the dielectric media used in the numerical calculations. Finally, the last chapter presents a series of solved problems.

I am very grateful to the students who, through the fruits of their research, have significantly contributed in the development of our fiber optics lab. I would particularly like to thank Dr. François Gonthier, Ms. Isabelle Vaillancourt, as well as Mrs. Patrick Orsini and Denis Perron who have graciously given me the permission to reproduce certain figures and results from their theses. I would also like to thank all those who have helped me with my teaching efforts, especially my colleague professor Suzanne Lacroix, with whom I have had and continue to have many fruitful discussions.

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August 2008

Symbols, Operators, and Coordinate Systems

$*$: complex conjugate
$i = \sqrt{-1}$
Re, Im: real and imaginary parts of a complex number
$\mathbf{J}$: current density
σ: charge density
ω: angular frequency
k: wavenumber in the vacuum
λ: wavelength in the vacuum
c: speed of light in the vacuum
n: refractive index of a medium; n_{co} (core), n_{cl} (cladding), n_{e} (external)
ε: dielectric permittivity (ε_0 vacuum permittivity)
μ: magnetic permeability (μ_0 vacuum permeability)
$\mathbf{r}$: position vector
$\hat{\mathbf{n}}$: unit vector normal to a surface
$\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, $\hat{\mathbf{k}}$ or $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$: unit vector of the cartesian components
x, y, z: cartesian coordinates
$\hat{\mathbf{r}}$, $\hat{\boldsymbol{\varphi}}$, $\hat{\mathbf{z}}$: unit vectors of the cylindrical polar components
r, ϕ, z: cylindrical polar coordinates
$\mathbf{E}$: electric field vector
$\mathbf{e}_j$: electric field vector of a mode j
$\hat{\mathbf{e}}_j$: normalized electric field vector of a mode j
$\mathbf{E}_{\mathrm{t}}$, $\mathbf{e}_{\mathrm{t}}$, $\hat{\mathbf{e}}_{\mathrm{t}}$: transverse electric field vectors
$\mathbf{H}$: magnetic field vector
$\mathbf{h}_j$: magnetic field vector of a mode j
$\hat{\mathbf{h}}_j$: normalized magnetic field vector of a mode j
$\mathbf{H}_{\mathrm{t}}$, $\mathbf{h}_{\mathrm{t}}$, $\hat{\mathbf{h}}_{\mathrm{t}}$: transverse magnetic field vectors
$\mathbf{S}$: Poynting vector
e_z, h_z: longitudinal components of electric and magnetic field vectors $\mathbf{e}$ and $\mathbf{h}$
e_x, e_y or e_r, e_ϕ: Cartesian or polar transverse components of electric field vector $\mathbf{e}$
h_x, h_y or h_r, h_ϕ: Cartesian or polar transverse components of magnetic field vector $\mathbf{h}$

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e_t, h_t: amplitudes of transverse electric and magnetic field vectors $\mathbf{e}_t$ and $\mathbf{h}_t$
β: propagation constant along the z-axis
$a_j = b_j \exp(i\beta_j z)$: modal amplitude of a mode j (invariant cross-section waveguide)
$a_j(z) = b_j(z) \exp\left\{i \int_0^z \beta_j(z')\,dz'\right\}$: modal amplitude of a mode j (variable cross-section waveguide)
N_j: normalization constant of a mode j
ℓ, m: subscript numbers of the linearly polarized scalar $LP_{\ell m}$ modes
ν, m: subscript numbers of the vector modes TE_{0m}, TM_{0m}, $HE_{\nu m}$, and $EH_{\nu m}$
ρ or ρ_{co}: core radius of an optical fiber, ρ_{cl}: intermediate cladding radius of an optical fiber
U, W, V: modal parameters and the normalized frequency
$\Psi_{\ell m}(r)$: radial field of a linearly polarized scalar $LP_{\ell m}$ mode
$\hat{\Psi}_{\ell m}(r)$: normalized radial field of a linearly polarized scalar $LP_{\ell m}$ mode
$\boldsymbol{\nabla}$: gradient operator, $\boldsymbol{\nabla}_t$: transverse gradient operator
∇^2: scalar Laplacian operator, ∇_t^2: transverse scalar Laplacian operator
$\boldsymbol{\nabla}^2$: vector Laplacian operator, $\boldsymbol{\nabla}_t^2$: transverse vector Laplacian operator
δ_d: Dirac delta distribution.

The Operators

List of the Operators in Cartesian Components Applied to:

$$\begin{cases} \text{A scalar } \Psi(x, y, z) \\ \text{A vector } \mathbf{A}(x, y, z) = \mathbf{A}_t + \hat{\mathbf{z}}A_z = \hat{\mathbf{x}}A_x(x, y, z) \\ \qquad + \hat{\mathbf{y}}A_y(x, y, z) + \hat{\mathbf{z}}A_z(x, y, z) \end{cases}$$

- Gradient operator ∇

$$\text{gradient: } \nabla\Psi(x, y, z) = \nabla_t\Psi + \hat{\mathbf{z}}\frac{\partial\Psi}{\partial z} = \hat{\mathbf{x}}\frac{\partial\Psi}{\partial x} + \hat{\mathbf{y}}\frac{\partial\Psi}{\partial y} + \hat{\mathbf{z}}\frac{\partial\Psi}{\partial z}$$

$$\text{divergence: } \nabla \cdot \mathbf{A}(x, y, z) = \nabla_t \cdot \mathbf{A}_t + \frac{\partial A_z}{\partial z} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\text{curl: } \nabla \wedge \mathbf{A}(x, y, z) = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$

$$\text{and} \quad \nabla_t \wedge \mathbf{A}_t = \hat{\mathbf{z}}\left\{\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right\}$$

- Scalar Laplacian operator ∇^2

$$\nabla^2\Psi(x, y, z) = \nabla_t^2\Psi + \frac{\partial^2\Psi}{\partial z^2} = \frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2}$$

- Vector Laplacian operator $\boldsymbol{\nabla}^2$

$$\boldsymbol{\nabla}^2\mathbf{A}(x, y, z) = \boldsymbol{\nabla}^2\mathbf{A} = \hat{\mathbf{x}}(\nabla^2 A_x) + \hat{\mathbf{y}}(\nabla^2 A_y) + \hat{\mathbf{z}}(\nabla^2 A_z)$$

$$\boldsymbol{\nabla}^2\mathbf{A}(x, y, z) = \boldsymbol{\nabla}_t^2\mathbf{A} + \frac{\partial^2\mathbf{A}}{\partial z^2}$$

$$\boldsymbol{\nabla}_t^2\mathbf{A}(x, y, z) = \boldsymbol{\nabla}_t^2\mathbf{A} = \hat{\mathbf{x}}(\nabla_t^2 A_x) + \hat{\mathbf{y}}(\nabla_t^2 A_y) + \hat{\mathbf{z}}(\nabla_t^2 A_z)$$

List of the operators in cylindrical polar components applied to

$$\begin{cases} \text{A scalar } \Psi(r,\phi,z) \\ \text{A vector } \mathbf{A}(r,\phi,z) = \mathbf{A}_t + \hat{\mathbf{z}}A_z = \hat{\mathbf{r}}A_r(r,\phi,z) \\ \qquad + \hat{\boldsymbol{\varphi}}A_\phi(r,\phi,z) + \hat{\mathbf{z}}A_z(r,\phi,z) \end{cases}$$

- Gradient operator ∇

$$\text{Gradient:}\ \nabla\Psi(r,\phi,z) = \nabla_t\Psi + \hat{\mathbf{z}}\frac{\partial\Psi}{\partial z} = \hat{\mathbf{r}}\frac{\partial\Psi}{\partial r} + \frac{\hat{\boldsymbol{\varphi}}}{r}\frac{\partial\Psi}{\partial\phi} + \hat{\mathbf{z}}\frac{\partial\Psi}{\partial z}$$

$$\text{Divergence:}\ \nabla\cdot\mathbf{A}(r,\phi,z) = \nabla_t\cdot\mathbf{A}_t + \frac{\partial A_z}{\partial z} = \frac{1}{r}\frac{\partial(rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\text{Curl:}\ \nabla\wedge\mathbf{A}(r,\phi,z) = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{r}} & r\hat{\boldsymbol{\varphi}} & \hat{\mathbf{z}} \\ \partial/\partial r & \partial/\partial\phi & \partial/\partial z \\ A_r & rA_\phi & A_z \end{vmatrix}$$

$$\text{and}\quad \nabla_t\wedge\mathbf{A}_t = \frac{\hat{\mathbf{z}}}{r}\left\{\frac{\partial(rA_\phi)}{\partial r} - \frac{\partial A_r}{\partial\phi}\right\}$$

- Scalar Laplacian operator ∇^2

$$\nabla^2\Psi(r,\phi,z) = \nabla_t^2\Psi + \frac{\partial^2\Psi}{\partial z^2}$$

$$\nabla_t^2\Psi(r,\phi,z) = \frac{\partial^2\Psi}{\partial r^2} + \frac{1}{r}\frac{\partial\Psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\Psi}{\partial\phi^2}$$

- Vector Laplacian operator $\boldsymbol{\nabla}^2$

$$\boldsymbol{\nabla}^2\mathbf{A}(r,\phi,z) = \hat{\mathbf{r}}\left\{\nabla^2 A_r - \frac{2}{r^2}\frac{\partial A_\phi}{\partial\phi} - \frac{A_r}{r^2}\right\} + \hat{\boldsymbol{\varphi}}\left\{\nabla^2 A_\phi + \frac{2}{r^2}\frac{\partial A_r}{\partial\phi} - \frac{A_\phi}{r^2}\right\} + \hat{\mathbf{z}}(\nabla^2 A_z)$$

$$\boldsymbol{\nabla}^2\mathbf{A}(r,\phi,z) = \boldsymbol{\nabla}_t^2\mathbf{A} + \frac{\partial^2\mathbf{A}}{\partial z^2}$$

$$\boldsymbol{\nabla}_t^2\mathbf{A}(r,\phi,z) = \hat{\mathbf{r}}\left\{\nabla_t^2 A_r - \frac{2}{r^2}\frac{\partial A_\phi}{\partial\phi} - \frac{A_r}{r^2}\right\} + \hat{\boldsymbol{\varphi}}\left\{\nabla_t^2 A_\phi + \frac{2}{r^2}\frac{\partial A_r}{\partial\phi} - \frac{A_\phi}{r^2}\right\} + \hat{\mathbf{z}}(\nabla_t^2 A_z)$$

Relations between the Cartesian (x, y, z) and Cylindrical Polar (r, ϕ, z) Coordinate Systems

Between the Coordinates, the Vector Components, and the Derivatives of a Scalar

$$\begin{cases} x = r\cos\phi \\ y = r\sin\phi \end{cases} \quad \begin{cases} A_x = A_r\cos\phi - A_\phi\sin\phi \\ A_y = A_r\sin\phi + A_\phi\cos\phi \end{cases}$$

$$\begin{cases} \dfrac{\partial\Psi}{\partial x} = \cos\phi\dfrac{\partial\Psi}{\partial r} - \dfrac{\sin\phi}{r}\dfrac{\partial\Psi}{\partial\phi} \\ \dfrac{\partial\Psi}{\partial y} = \sin\phi\dfrac{\partial\Psi}{\partial r} + \dfrac{\cos\phi}{r}\dfrac{\partial\Psi}{\partial\phi} \end{cases}$$

$$\begin{cases} r = \sqrt{x^2 + y^2} \\ \phi = \tan^{-1}(y/x) \end{cases} \quad \begin{cases} A_r = A_x\cos\phi + A_y\sin\phi \\ A_\phi = -A_x\sin\phi + A_y\cos\phi \end{cases}$$

$$\begin{cases} \dfrac{\partial\Psi}{\partial r} = \cos\phi\dfrac{\partial\Psi}{\partial x} + \sin\phi\dfrac{\partial\Psi}{\partial y} \\ \dfrac{1}{r}\dfrac{\partial\Psi}{\partial\phi} = -\sin\phi\dfrac{\partial\Psi}{\partial x} + \cos\phi\dfrac{\partial\Psi}{\partial y} \end{cases}$$

Between the Unit Vectors

$$\begin{cases} \hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\varphi}}\sin\phi \\ \hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\varphi}}\cos\phi \end{cases} \quad \begin{cases} \hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi \\ \hat{\boldsymbol{\varphi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi \end{cases}$$

$$\begin{cases} \hat{\mathbf{r}}\wedge\hat{\boldsymbol{\varphi}} = \hat{\mathbf{z}} \\ \hat{\mathbf{x}}\wedge\hat{\mathbf{y}} = \hat{\mathbf{z}} \end{cases} \quad \begin{cases} \dfrac{\partial\hat{\mathbf{r}}}{\partial r} = 0, \quad \dfrac{\partial\hat{\mathbf{r}}}{\partial\phi} = \hat{\boldsymbol{\varphi}} \\ \dfrac{\partial\hat{\boldsymbol{\varphi}}}{\partial r} = 0, \quad \dfrac{\partial\hat{\boldsymbol{\varphi}}}{\partial\phi} = -\hat{\mathbf{r}} \end{cases}$$

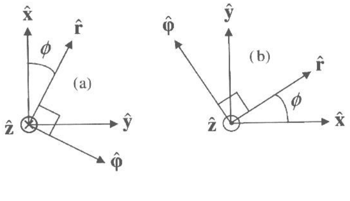


Fig. S1 Unit vectors $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{r}}$, $\hat{\boldsymbol{\varphi}}$ in the cross-section plane for: (a) $\hat{\mathbf{z}}$ going into the page and: (b) $\hat{\mathbf{z}}$ coming out of the page. ϕ is always the angle between $\hat{\mathbf{x}}$ and $\hat{\mathbf{r}}$.

Miscellaneous

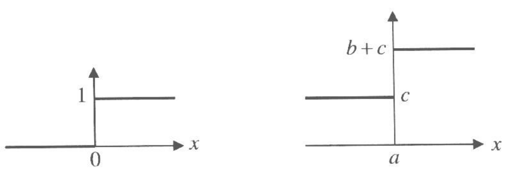
Inverse Matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\Delta}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad \text{with} \quad \Delta = ad - bc$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}^{-1} = \frac{1}{\Delta}\begin{bmatrix} (ei-fh) & (ch-bi) & (bf-ce) \\ (gf-di) & (ai-cg) & (cd-af) \\ (dh-eg) & (bg-ah) & (ae-bd) \end{bmatrix}$$

$$\text{with} \quad \Delta = a(ei-hf) + b(gf-di) + c(dh-eg)$$

Step Functions or Heaviside Functions



$$H(x) = \begin{cases} 0 & \text{when } x < 0, \\ 1 & \text{when } x > 0. \end{cases} \qquad bH(x-a)+c = \begin{cases} c & \text{when } x < a, \\ c+b & \text{when } x > a. \end{cases}$$

Fig. S2 Illustration of the step functions $H(x)$ and $bH(x-a)+c$.

Dirac Delta Distributions

$$\delta_d(x) = \frac{\mathrm{d}H(x)}{\mathrm{d}x} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{-ikx}\,\mathrm{d}k = \lim_{\mu\to\infty}\int_{-\infty}^{\infty} \frac{\mu}{\sqrt{\pi}}\mathrm{e}^{-\mu^2x^2}\,\mathrm{d}x,$$

$$\delta_d(-x) = \delta_d(x), \quad \delta_d(ax) = \frac{1}{|a|}\delta_d(x),$$

$$\delta_d(x, y, z) = \delta_d(x)\delta_d(y)\delta_d(z),$$

$$\delta_d(x^2 - a^2) = \frac{1}{2|a|}\{\delta_d(x-a) + \delta_d(x+a)\}.$$

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Note that these distributions only have meaning under the integration sign:

$$\int_{-\infty}^{+\infty} f(x)\delta_d(x-a)\,\mathrm{d}x = f(a), \quad \int_{-\infty}^{+\infty} \delta_d(x-a)\delta_d(x-b)\,\mathrm{d}x = \delta_d(a-b),$$

$$\int_{-\infty}^{\infty} \delta_d(x)\mathrm{e}^{ikx}\,\mathrm{d}x = 1, \quad \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta_d(x)\delta_d(y)\delta_d(z)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = 1.$$

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