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Managing Editors: M. Beckmann and W. Krelle

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P R E F A C E

The 8th Symposium on Operations Research took place from August 22 - August 25, 1983 at the University of Karlsruhe.

About 3oo participants from 28 countries all over the world took the opportunity to present and discuss their recent results in their fields of research.

As Optimization and Control Theory, Mathematical Economics and Statistics, Game Theory and Graph Theory and their applications play an increasingly important role within the area of applied mathematics, the organizers could state with great satisfaction that this symposium had resulted as an efficient platform of the exchange of new ideas in the above mentioned fields. So, the aim of strengthening the mutual understanding of the different fields of research had been reached by this conference.

This proceedings volume mainly contains the lectures of the invited speakers in order to present the state of art. A few smaller contributions which are of special interest have been added.

We owe special thanks to all the participants of the conference, to the contributors of this volume and to the referees for their advice. We also appreciated the excellent cooperation of Springer Verlag.

We gratefully acknowledge the sponsorship and financial support received by the separately mentioned public and private institutions whose generous and unselfish help was a substantial contribution to organize this conference in a time of extremely hard financial restrictions.

Also, we particularly appreciate the efforts Mrs. I. Haag-Smith, Mrs. C. Forler, Mr. Dipl.Math. P. Recht and Mr. K. Wieder made in helping to organize the symposium.

D. Pallaschke G. Hammer

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PART I

OPTIMIZATION THEORY

A METHOD FOR LINEARLY CONSTRAINED MINIMIZATION PROBLEMS

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Abstract : We give a method for computing points that minimize a differentiable function on a compact polyhedral set. This method takes into account the polyhedral structure as it was done by Zangwill [6] in the convex-simplex method, generalizes the simplex method and that of Bazaraa, Goode and Rardin [2], and finally can be interpreted as being the method of Frank and Wolfe [4] in which to find the descent direction, instead of solving a linear program completely, one perform only the first step of the simplex method.

INTRODUCTION

Let $\mathbb{R}^N$ be the usual vector-space of real N-uples with the usual inner product denoted by $(.,.)$. In this paper P is a nonempty compact polyhedral set of $\mathbb{R}^N$, f is a real-valued function defined on $\mathbb{R}^N$ continuously differentiable and $\mathcal{P}$ is the linearly constrained minimization problem stated as :

$$\min (f(x) \mid x \in P).$$

For computing stationary points of problem $\mathcal{P}$ we propose a method which attempts to operate within the linear-simplex method structure. This method then appears as a same type of method as the convex-simplex method of Zangwill [6]. It is however, different and has the advantage of being less technical with regards to the Zangwill method. It has also a simple geometrical interpretation which makes it more understandable and more open to other improvements. Also in the case where f is convex an implementable line-search is proposed which is not the case in the Zangwill method. Moreover, if $f(x) = (c,x)$ this method will coincide with the simplex method (this is also true in the case of the convex simplex method) ; if $f(x) = ||x||^2$ it will be almost the same as the algorithm given by Bazaraa, Goode, Rardin [2].

Finally, this method could be interpreted as being the method of Frank and Wolfe [4], in which to find the descent direction, instead of solving a linear program completely we perform only the 1st step of the simplex method using the extremal point found in the previous iteration.

This method is presented under its geometric form in the first part. In the second part, this method is modified to avoid the cycling phenomena as the one we can meet in the simplex method in presence of degenerate extremal points.

Let $\varepsilon \geq 0$; for the following $\{\varepsilon_n\}$ is a sequence of reals such that :

$$\forall n \quad 0 \leq \varepsilon_n \leq \varepsilon \quad , \lim_{n \to \infty} \varepsilon_n = 0 .$$

and A_ε is the ε-optimal set of $\mathcal{P}$, that is :

$$A_\varepsilon = \{x \in P : f(x) \leq \inf (f(u) \mid u \in P) + \varepsilon\}.$$

I DESCRIPTION OF THE GEOMETRIC METHOD

For the following $\mathcal{V}(P)$ denotes the set of vertices of P. For each $y \in P$ $\mathcal{E}(y)$ denotes the collection of edges emanating from y. Then for each edge G in $\mathcal{E}(y)$ there exist a vector $\alpha(G,y) \in \mathbb{R}^N$ and a scalar $\theta(G,y) > 0$ such that z belongs to G if, and only if there exists $\theta \in [0,\theta(G,y)]$ such that :

$$z = y + \theta\alpha(G,y).$$

In this case $y + \theta(G,y)\alpha(G,y)$ is the vertex adjacent to y and which belongs to G.

We recall first a classical theorem which is the key of the convergence of the algorithm. The proof of this theorem can be found for example in [1] (theorem 1.11 chapter 1) or in [2] (lemma 1).

<u>Theorem 1.1</u> Let $y \in \mathcal{V}(P)$, C(y) the conical hull of $\mathcal{E}(y)$ that is :

$$C(y) = \{v : v = y + \sum_{G \in \mathcal{E}(y)} \lambda_G \, \alpha(G,y) \quad , \quad \lambda_G \geqslant 0 \quad \forall G \in \mathcal{E}(y)\} .$$

Then C(y) contains P.

Now in this first section we shall <u>assume</u> that we can effectively calculate for each $y \in \mathcal{V}(P)$, each $G \in \mathcal{E}(y)$ the vector $\alpha(G,y)$ and the scalar $\theta(G,y)$.

This assumption is in particular satisfied when P is given in the standard form and when all the vertices of P are non degenerate. In this case $\alpha(G,y)$ is obtained by the simplex tableau associated to y and $\theta(G,y)$ is obtained from this tableau. With this assumption we can define the algorithm as follows.

<u>Algorithm</u> : <u>Initialization step</u> Let n = 1, let $x_1 = y_1 \in \mathcal{V}(P)$ and go to the main step.

<u>Main step</u> Step 1 : let

$$\partial_n = \min((\nabla f(x_n), \alpha(G,y_n)) \mid G \in \mathcal{E}(y_n)) \qquad 1.1$$

$$I_n = \{G \in \mathcal{E}(y_n) : \partial_n = (\nabla f(x_n), \alpha(G,y_n))\} \qquad 1.2$$

If $\partial_n \geqslant 0$ stop ; x_n is a "good" point. Otherwise y_{n+1} is given by :

$$y_{n+1} = y_n + \theta(G,y_n)\,\alpha(G,y_n) \quad \text{with } G \in I_n \qquad 1.3$$

and go to step 2

<u>Step 2</u> : Let

$$\varphi_n(\alpha) = f(x_n + \alpha(y_{n+1} - x_n))$$