STRESS WAVES IN NON-ELASTIC SOLIDS

W. K. NOWACKI
WARSAW, POLAND

PERGAMON PRESS

O347.4
N1
7961228

STRESS WAVES IN NON-ELASTIC SOLIDS

by

W. K. NOWACKI

Translated by

ZBIGNIEW OLESIAK

E7961228

PERGAMON PRESS
OXFORD · NEW YORK · TORONTO · SYDNEY
PARIS · FRANKFURT

U. K.	Pergamon Press Ltd., Headington Hill Hall, Oxford OX3 0BW, England
U. S. A.	Pergamon Press Inc., Maxwell House, Fairview Park, Elmsford, New York 10523, U.S.A.
CANADA	Pergamon of Canada Ltd., 75 The East Mall, Toronto, Ontario, Canada
AUSTRALIA	Pergamon Press (Aust.) Pty. Ltd., 19a Boundary Street, Rushcutters Bay, N.S.W. 2011, Australia
FRANCE	Pergamon Press SARL, 24 rue des Ecoles, 75240 Paris, Cedex 05, France
FEDERAL REPUBLIC OF GERMANY	Pergamon Press GmbH, 6242 Kronberg Taunus, Pferdstrasse 1, Federal Republic of Germany

Copyright © 1978 Pergamon Press Ltd.

All Rights Reserved. No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means: electronic, electrostatic, magnetic tape, mechanical, photocopying, recording or otherwise, without permission in writing from the publishers

First English Edition 1978

Library of Congress Cataloging in Publication Data

Nowacki, Wojciech Krzysztof.
Stress waves in non-elastic solids.

Bibliography: p.
1. Elastic solids. 2. Stress waves. 3. Visco-plasticity. I. Title.
QC191.N68 1977 531'.33 76-52418
ISBN 0-08-021294-8

This is a translation of the Polish book Zagadnienia Falowe w Teorii Plastycznosci *published by* Państwowe Wydawnictwo Naukowe, Warsaw.

Printed in Great Britain by William Clowes & Sons, Limited London, Beccles and Colchester

STRESS WAVES IN NON-ELASTIC SOLIDS

Other Pergamon Titles of Interest

BARTENEV, G. M. and ZUYEV, Yu. S.
Strength and Failure of Visco-elastic Materials

BELYAYEV, N. M.
Problems in Strength of Materials

BRITVEC, S. J.
The Stability of Elastic Systems

CALLADINE, C. R.
Engineering Plasticity

CHEUNG, Y. K.
Finite Strip Method in Structural Analysis

CHRISTIAN, J. W.
The Theory of Transformations in Metals and Alloys, 2nd edition

DUGDALE, D. S.
Elements of Elasticity

HENDRY, A. W.
Elements of Experimental Stress Analysis (SI edition)

HERRMANN, G. and PERRONE, N.
Dynamic Response of Structures

HULL, D.
Introduction to Dislocations, 2nd edition

ILYUSHIN, A. A. and LENSKII, V. S.
Strength of Materials

JAEGER, L. G.
Cartesian Tensors in Engineering Science

JAEGER, L. G.
Elementary Theory of Elastic Plates

KOVACS, I. and ZSOLDOS, L.
Dislocations and Plastic Deformation

SAADA, A. S.
Elasticity: Theory and Applications

WASHIZU, K.
Variational Methods in Elasticity and Plasticity, 2nd edition

NOWACKI: STRESS WAVES IN NON-ELASTIC SOLIDS
ISBN: 0 08 021294 8

Technical Editor:
Dr S R REID
University Engineering Department, Cambridge, England

此为试读，需要完整PDF请访问：www.ertongbook.com

PREFACE

The dynamics of inelastic continua is nowadays a domain of intense study. Society, aiming at its own welfare and safety but aware of limited natural resources, requires reliable answers regarding the mechanical behaviour of solids and the performance of structures under transient agencies. Impacts, blasts, collisions, thermal shocks, and irradiations constitute current hazards of modern technology. Assessments of damage and failure due to the propagation of stress waves, evaluations of permanent deformation and energy absorption under impact, methods of estimation of pressure transmitted through protective structures, calculations of structural reliability under transient random charges, are of importance for the development of technology and for a safe utilization of its products.

The last decade is associated with significant advances in the mechanical sciences, in particular as regards the inelastic behaviour of materials and structures. Mathematical models of complex material response were formulated within the framework of non-linear continuum mechanics and justified in often sophisticated experiments. Mechanisms of inelastic deformation were explored to a large extent and couplings between mechanical and other fields were studied. Methods of solving boundary value problems for the differential equations governing the dynamics of inelastic solids were developed and various problems of a direct technological interest regarding structural strength and the propagation of waves in elastic–plastic and rate sensitive materials were solved. Advanced specialized monographs are thus needed to facilitate an access to the accumulated knowledge and to its utilization in engineering service to society.

The present book largely contributes towards meeting such a demand in the domain of continuum dynamics. Dr. Nowacki has succeeded in producing both an original and a comprehensive presentation of principles, methods, and solutions regarding the propagation of waves in elastic–plastic and viscoplastic solids.

This monograph is comprehensive since it exposes wave propagation problems for the considered types of material response starting with discussions of constitutive relations, justifies the hypotheses introduced in specialized theories and the simplifications made in the analysis of particular problems, presents both analytical and numerical methods of solving problems, and gives a large number of solutions to specific problems of wave propagation in inelastic solids.

The book is original both in its outline and in its contents. It includes a number of contributions Dr. Nowacki has made to the study of wave propagation in elastic–plastic and viscoplastic continua. There is a competent exposition of the mathematical questions relating to hyperbolic differential equations, a presentation of analytical and numerical techniques resulting in effective solutions of problems concerning plane, cylindrical and spherical stress waves and thermal stress waves, and discussions of two-dimensional waves, the whole being supplemented by solved problems. All this makes the book useful both in study and in design.

By its contents and its competent and lucid presentation of problems of inelastic waves the book constitutes a significant and useful contribution to the domain of continuum dynamics. It has no equivalent counterpart in the existing literature and it will doubtless be appreciated by both specialists and students because of its originality and pertinence.

Warsaw, 13 *April*, 1976 A. SAWCZUK

INTRODUCTION

This book is devoted to wave problems in the theory of plasticity. Such problems arise in cases in which intensive dynamic loads acting on the elements of a structure are big enough to produce plastic deformation of the elements. To date a considerable number of contributions and monographs have been published in this field. The first papers referring to these problems appeared in the forties and the theory rapidly developed in the sixties. Many problems, particularly the one-dimensional ones, have been investigated fully. A few monographs exist presenting the theory of impact in continuous and discrete systems in a general way. Here we should quote, first of all, the monograph by Kolsky [71], published in 1953, next the book by Goldsmith [46] which appeared in 1960, and those by Rakhmatulin and Demianov [125] and Cristescu [34], published in 1961 and 1967 respectively. In these books the reader can find the general theory of impact in continuous plastic bodies. Since the publication of these books a number of problems have been solved which are significant from the standpoint of engineering practice. Numerous papers on the dynamics of inelastic structures have appeared. In connection with the rapid development of computational techniques a great number of methods have been devised for the numerical integration of quasi-linear and semi-linear partial differential equations. These investigations have been accompanied by the development of the pertinent analytical methods of solving initial and boundary value problems for inelastic bodies. They mainly refer to the problems of stress wave propagation in the cases of complicated stress states, the waves generated by multi-parameter loading, three-dimensional waves, and, finally, to thermal stress waves.

This book is confined to those wave problems which are such that they can be described by a system of hyperbolic partial differential equations of the first order, which are quasi-linear or semi-linear. A survey of the literature is given in the field of stress wave propagation in elastic/viscoplastic media and in particular the papers of Polish authors are discussed.

The book is arranged in the following way. The first chapter has been devoted to the fundamental equations of the dynamics of inelastic media. The dynamical properties of materials (metals and soils) are discussed offering an account of the most representative theories of plasticity and viscoplasticity. Chapter II considers the basic definitions of discontinuity surfaces and the conditions which have to be satisfied across these surfaces. Simultaneously, certain mathematical fundamentals have been given, referring to systems of differential equations, quasi-linear and semi-linear, of the first order. Also initial and boundary value problems for hyperbolic equations have been formulated. The remaining chapters have been devoted to methods of solving stress wave propagation problems, namely, one-dimensional plane waves, spherical and cylindrical waves, longitudinal–transverse waves, waves in beams and plates, and plane two-dimensional stress waves. Wave propagation problems for elastic–plastic and elastic/viscoplastic media have been treated in detail, as well as the most important problem of shock waves in metals and soils. The last chapter deals with thermal wave propagation problems, relating to waves generated by thermal shocks applied to the boundary of the media under consideration.

Wave propagation problems in strings and membranes are not discussed in this book. The reader is referred for an exhaustive treatment to the monographs by Rakhmatulin and Demianov [125] and by Cristescu [34]. Since the time these books appeared the subject has not been changed except for some insignificant modifications. The theory of impact in inelastic solids, in which wave phenomena are not taken into account, have also been neglected in this book. These problems have been discussed in a number of monographs, for example in [118] (chapter V) and in [47] (chapter XV, where the theory of impact in discrete mechanical systems is discussed).

I am much indebted and thankful to my colleague Dr. Bogdan Raniecki from the Institute of Fundamental Technical Research of the Polish Academy of Sciences for his valuable suggestions and numerous discussions during the preparation of the book.

7961228

CONTENTS

此为试读，需要完整PDF请访问：www.ertongbook.com

PUBLISHER'S NOTE

In order to make this volume available as economically and as rapidly as possible much of the mathematics has been reproduced directly from the original Polish edition. Due to technical problems, however, it was not possible to match the typography of the new updated material with the original, and certain characters within the new text matter will appear different from those in the reproduced matter. The variations comprise

ν which appears as υ in the original,
ϵ which appears as ε in the original,
g which appears as g in the original;

no distinction is intended and it is hoped that they will in no way distract the reader.

CHAPTER I

FUNDAMENTAL EQUATIONS OF THE DYNAMICS OF INELASTIC MEDIA

1. Dynamical properties of materials

The dynamical properties of materials will be briefly presented based on the results of experiments. The reader can find more details of the experiments in the monographs by Rakhmatulin and Demianov [125], Kolsky [71], Cristescu [34], and Perzyna [114] and [118].

The investigation of the behaviour of materials under dynamic conditions has taken place in several different directions. The determination of the dynamic characteristic (constitutive relationship) constitutes the primary direction of the investigations. Also, much research has been done in order to find the plastic strain distribution along the specimens tested and to investigate the influence of transverse motion during the propagation of longitudinal waves in a test piece which produces the effect of wave dispersion. The object of numerous experimental investigations has been to determine the influence of temperature and that of radiation on the behaviour of metals under dynamic conditions.

In the case of static processes, the mechanical properties of material are described by the stress–strain diagram. In the case of the dynamic loadings, however, the stress–strain relation is influenced by the strain rate. The influence of a number of other phenomena is also observed, e.g. temperature, radiation, etc.

The fundamental properties of metals and soils will be presented in this section.

1.1. DYNAMICAL PROPERTIES OF METALS

The experimental investigations of Clark and Duwez [25], Manjoine [84], Hauser, Simmons and Dorn [51], Campbell and Ferguson [22], Marsh and Campbell [88], Lindholm [76]–[78], and of many other authors reveal that much higher stress is necessary to reach the yield limit in an impact loading compared with that for a slow one. In the case of a number of practically important materials (e.g. high-carbon steels) a relationship explicitly independent of the strain rate [125] $\sigma = \sigma(\epsilon)$, obtained during dynamic loading of a specimen, can be used to characterize the dynamic behaviour of the materials. This relationship essentially differs from the static characteristic. The dynamic and static characteristics for a specimen made of mild steel are presented in Fig. 1. Curve 1 denotes the static case, while 2 indicates the dynamic one. Curve 2 has been drawn using the method due to Rakhmatulin [125], based on the measurement of plastic deformation of the initial cross-section of the specimen under the effect of the impact as a function of the impact speed. The yield limit of the material increases for dynamic loading of the specimen. The character of the curve is determined by the type of material. It has been found in many experimental investigations that those metals with a distinct yield limit are particularly sensitive to strain rate. A good example of this behaviour of exhibiting high sensitivity to strain rate is that of mild steel and pure iron. The influence of strain rate on the change of yield limit for mild steel was the subject of investigations in [25], [84], [76]–[78], [88], [24], and in other papers. For example, Clark and Duwez [25] found that for mild steel

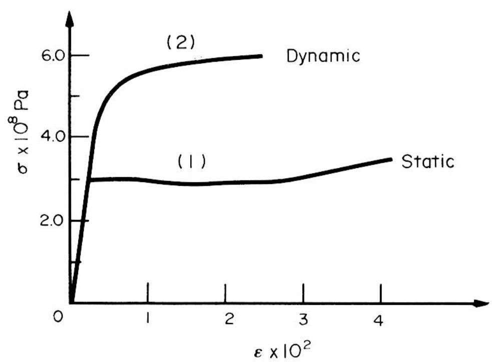

Fig. 1

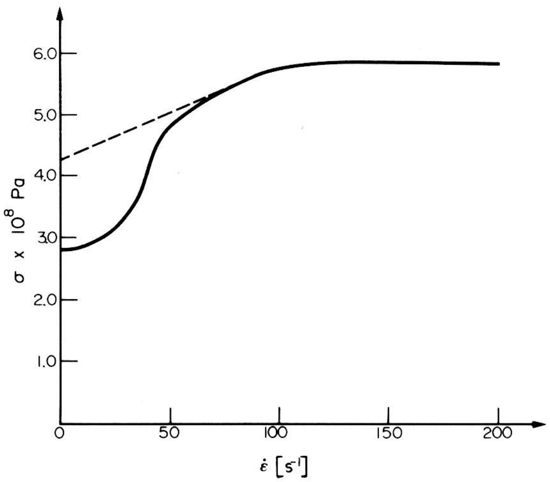

Fig. 2

(0.22% carbon) the yield limit increases with the increase of strain rate (Fig. 2) from about 2.71×10^8 Pa (the static yield limit) to a value of about 5.76×10^8 Pa for a strain rate of $\dot{\epsilon} \approx 200\ s^{-1}$. The yield limit increases up to the moment when it joins the curve representing the change of the conventional strength limit (dashed line in Fig. 2). The conventional strength limit increases in the range of strain rates from 0 to $\approx 200\ s^{-1}$.

The results of numerous experiments have shown that the yield limit for mild steel in dynamic loading situations can reach a value which is 2–3 times greater than that for static loading processes. It has also been noted that in a dynamic loading process the strain-hardening effect decreases in comparison with that for static loading.

Most of the investigations connected with the determination of dynamic material characteristics were performed under conditions of uniaxial tension or compression or for pure shear. Only a few experiments have been performed so far investigating the material characteristics in complex loading conditions. The investigations of Lindholm [76]–[78] belong to the pioneer ones in this field. The results of these experiments refer to aluminium and steel specimens under uniaxial tension and shear. The second stress tensor invariant against the second strain tensor invariant is shown in Fig. 3 for various strain rates [76]–[78]. The experiments were performed for a range of strain rates from $10^3\ s^{-1}$ to $4 \times 10^{-3}\ s^{-1}$. It is clear from the diagram that for higher strain rates the greater values of the stress intensity $\sqrt{J_2}$ correspond to a definite value of the strain intensity $\sqrt{I_2}$ = const. Lindholm [77] has performed identical experiments for steel specimens, obtaining similar results.

There already exists an extensive literature concerning the influence of strain rate as well as of temperature on the stress–strain relations. Each of these effects is considered separately. It has been found experimentally that at low temperatures the lower yield limit of metals does not depend on the strain rate, while at elevated temperatures even very small changes of strain rate cause a significant decrease or increase in the yield limit [22], [24].

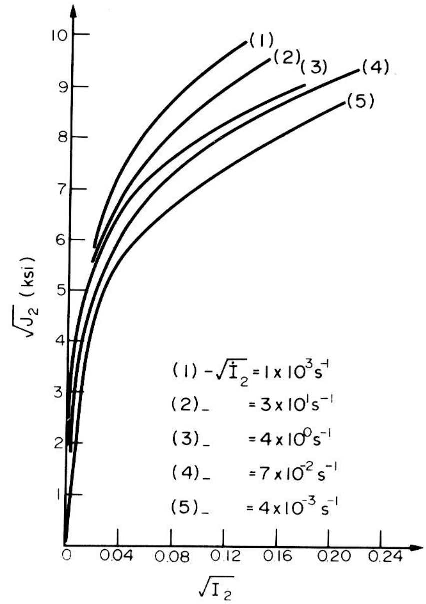

Fig. 3

The effect of radiation on the plastic properties of metals has also been investigated. As a rule the radiation of specimens leads to an increase of both yield limit and strength limit. The influence of strain rate is different for radiated metal than for unradiated. A radiated metal is considerably more sensitive to a change of strain rate [119].

1.2. DYNAMICAL PROPERTIES OF SOILS

In the case of quasi-static loading, soil settlement is produced by fluid seeping into and filling the cavities in the soil skeleton. Then the entire loading is transmitted by the soil skeleton and the soil deformation is entirely connected with the displacement of the soil skeleton grains. In the case of dynamic loading the soil behaves like a more homogeneous medium; the pressure in the fluid and air filling the soil skeleton is close to the pressure acting on the soil, particularly when the soil is very wet.

Compressibility is an important property of soils. For small loads of the order $10^4 - 5 \times 10^4$ Pa, the soil can be treated as an incompressible medium. The compressibility of soils differs significantly from that of metals, water, air, etc. When the stresses compressing a specimen increase the density of soil can increase significantly and the associated dilatational strain is irreversible due to the displacements of the soil grains and their crushing. During unloading this irreversible character of the processes mentioned is demonstrated by very slight changes of the density. It is noteworthy that in the case of dynamic loading the displacement of the soil grains due to their inertia lags behind the increase of pressure. This can be observed at the instant when the pressure attains its maximum value when the loading begins to develop. Though the pressure decreases a diminishing of the specimen volume still takes place during the first stage of unloading. This property for sand of different initial water content is shown in Fig. 4. The irreversibility of the dilatational compression decreases with the increase of water content and is caused by the elasticity of water filling the pores of the skeleton.

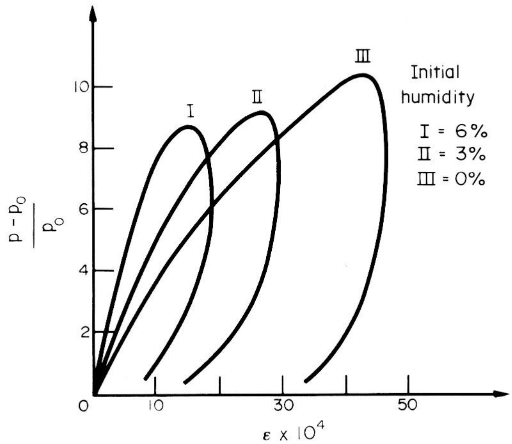

Fig. 4

Recent investigations have revealed that soil possesses rheological properties and is sensitive to change in the displacement rate. This sensitivity varies with the kind of soil. The papers of Grigorian [48], of Rakhmatulin, Sagomonian, and Alekseyev [126], and of other authors are devoted to the dynamic characteristics of soils. The influence of displacement-rate effects on soil behaviour is discussed in [126].

2. Basic theories of plasticity

The physical relationships of the basic theories of plasticity will now be presented including the strain theory of plasticity, the bilinear theory, and the theory of plastic flow. The rate effect on the stress–strain relations is not taken into account in these theories; nevertheless, they are frequently applied to the dynamic problems of plasticity due to the fairly good approximations that the theories provide for a definite class of cases.

2.1. STRAIN THEORY OF PLASTICITY

The constitutive equations of the theory of small elastic-plastic strains (equations of Nádai–Hencky–Iliushin) constitute a generalization of the physical relationships of the theory of elasticity and can be treated as a certain kind of extrapolation beyond the elastic state. The following three postulates are assumed in this theory, the first two being exactly the postulates of the theory of elasticity, namely:

(1) The principal directions of the stress tensor coincide with the principal directions of the strain tensor.

(2) Mean normal stress is proportional to the mean strain; the proportionality coefficient is the same in both elastic and plastic states.

(3) The stress intensity is a function of only the strain intensity, and should be determined for each material experimentally, i.e.

$$\sigma_i = 2m(\varepsilon_i)\,\varepsilon_i, \tag{2.1}$$

where $m(\epsilon_i)$ is a function of strain intensity only, and σ_i and ϵ_i denote the stress and strain intensities, respectively, and are defined by the formulae

$$\sigma_i = \left(\frac{3}{2} s_{ij} s_{ij}\right)^{1/2}, \qquad \varepsilon_i = \left(\frac{3}{2} e_{ij} e_{ij}\right)^{1/2}.$$

The system of equations for the strain theory takes the form [151]:

$$\tilde{s}_{ij} = \tilde{e}_{ij}, \qquad \sigma_i = 2m(\varepsilon_i)\,\varepsilon_i, \qquad \sigma_{kk} = 3K\varepsilon_{kk}, \tag{2.2}$$

where $\tilde{s}_{ij}$ and $\tilde{e}_{ij}$ are the normalized deviatoric stress and strain tensors $K = \frac{1}{3}(2\mu + 3\lambda)$ and λ and μ denote the Lamé constants. After some algebra we obtain from (2.2) the constitutive equations

$$\sigma_{ij} = 2m(\varepsilon_i)\,\varepsilon_{ij} + \frac{1}{3}\left[3K - 2m(\varepsilon_i)\right]\varepsilon_{kk}\,\delta_{ij}. \tag{2.3}$$

Function $m(\epsilon_i)$ is determined on the basis of experimental data from relation (2.1). The relationship $\sigma_i = f(\epsilon_i)$ does not differ much from the relationship $\sigma = f(\epsilon)$ (Fig. 5) obtained for a uniaxial stress state. Function $f(\epsilon_i)$ can be obtained from function $f(\epsilon)$ by a change of scale of the coordinates (Fig. 5). The character of the diagram remains the same. In a uniaxial state of stress we obtain $\sigma_i = \sigma$, $\epsilon_i = (1 + \bar{\nu})\epsilon$, where $\bar{\nu}$ denotes the coefficient of transverse shrinkage.

The states described by (2.3) correspond to a loading process, i.e. to a process in which the stress intensity is an increasing function of time, i.e. for $d\epsilon_i/dt > 0$. If σ_i after reaching a certain value, e.g. $\sigma_i = \sigma_i^*$ (Fig. 5) at a point B, starts decreasing then unloading occurs. Then $d\epsilon_i/dt < 0$ and the unloading follows the straight line BC, parallel to Hooke's straight line OA.

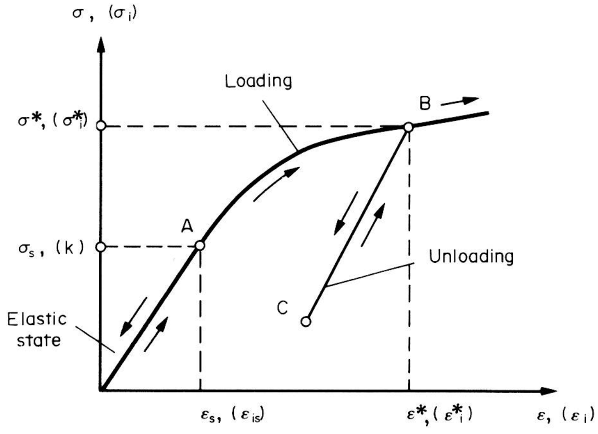

Fig. 5

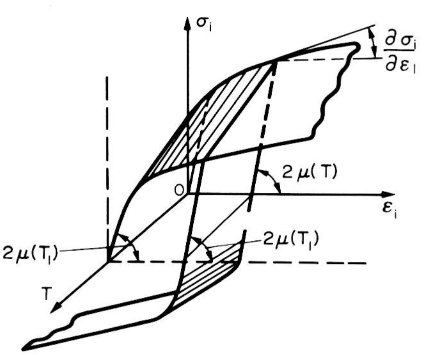

Fig. 6

此为试读，需要完整PDF请访问：www.ertongbook.com