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V. I. Arnol'd (Ed.)

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Bifurcation Theory and Catastrophe Theory

动力系统 V
分歧理论和突变理论

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《国外数学名著系列》(影印版)序

要使我国的数学事业更好地发展起来，需要数学家淡泊名利并付出更艰苦地努力。另一方面，我们也要从客观上为数学家创造更有利的发展数学事业的外部环境，这主要是加强对数学事业的支持与投资力度，使数学家有较好的工作与生活条件，其中也包括改善与加强数学的出版工作。

从出版方面来讲，除了较好较快地出版我们自己的成果外，引进国外的先进出版物无疑也是十分重要与必不可少的。从数学来说，施普林格(Springer)出版社至今仍然是世界上最具权威的出版社。科学出版社影印一批他们出版的好的新书，使我国广大数学家能以较低的价格购买，特别是在边远地区工作的数学家能普遍见到这些书，无疑是对推动我国数学的科研与教学十分有益的事。

这次科学出版社购买了版权，一次影印了 23 本施普林格出版社出版的数学书，就是一件好事，也是值得继续做下去的事情。大体上分一下，这 23 本书中，包括基础数学书 5 本，应用数学书 6 本与计算数学书 12 本，其中有些书也具有交叉性质。 这些书都是很新的，2000 年以后出版的占绝大部分，共计 16 本，其余的也是 1990 年以后出版的。这些书可以使读者较快地了解数学某方面的前沿，例如基础数学中的数论、代数与拓扑三本，都是由该领域大数学家编著的“数学百科全书”的分册。对从事这方面研究的数学家了解该领域的前沿与全貌很有帮助。按照学科的特点，基础数学类的书以“经典”为主，应用和计算数学类的书以“前沿”为主。这些书的作者多数是国际知名的大数学家，例如《拓扑学》一书的作者诺维科夫是俄罗斯科学院的院士，曾获“菲尔兹奖”和“沃尔夫数学奖”。这些大数学家的著作无疑将会对我国的科研人员起到非常好的指导作用。

当然，23 本书只能涵盖数学的一部分，所以，这项工作还应该继续做下去。更进一步，有些读者面较广的好书还应该翻译成中文出版，使之有更大的读者群。

总之，我对科学出版社影印施普林格出版社的部分数学著作这一举措表示热烈的支持，并盼望这一工作取得更大的成绩。

王　元

2005 年 12 月 3 日

List of Editors, Authors and Translators

Editor-in-Chief

R.V. Gamkrelidze, Russian Academy of Sciences, Steklov Mathematical Institute, ul. Vavilova 42, 117966 Moscow, Institute for Scientific Information (VINITI), ul. Usievicha 20a, 125219 Moscow, Russia

Consulting Editor

V.I. Arnol'd, Steklov Mathematical Institute, ul. Vavilova 42, 117966 Moscow, Russia

Authors

V.S. Afrajmovich, N. Novgorod State University, pl. Gagarina 23, 603078 N. Novgorod, Russia

V.I. Arnol'd, Steklov Mathematical Institute, ul. Vavilova 42, 117966 Moscow, Russia

Yu.S. Il'yashenko, Department of Mathematics and Mechanics, Moscow State University, 119899 Moscow, Russia

L.P. Shil'nikov, Institute for Applied Mathematics and Cybernetics, ul. Ul'janova 10, 603005 Nizhnij Novgorod, Russia

Translator

N.D. Kazarinoff†

Translator's Preface

In translating this volume, I am happy to thank Y.-H. Wan and James Boa for much help on technical points and P. Ashwin for the final check of Part I. I am particularly thankful to G. Wassermann for his careful reading of and many excellent suggestions for the translation of Part II.

N.D. Kazarinoff

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I. Bifurcation Theory

V.I. Arnol'd, V.S. Afrajmovich,
Yu. S. Il'yashenko, L.P. Shil'nikov

Translated from the Russian
by N.D. Kazarinoff

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Preface

The word "bifurcation" means "splitting into two". "Bifurcation" is used to describe any sudden change that occurs while parameters are being smoothly varied in any system: dynamical, ecological, etc. Our survey is devoted to the bifurcations of phase portraits of differential equations – not only to bifurcations of equilibria and limit cycles, but also to perestroikas of the phase portraits of systems in the large and, above all, of their invariant sets and attractors. The statement of the problem in this form goes back to A.A. Andronov.

Connections with the theory of bifurcations penetrate all natural phenomena. The differential equations describing real physical systems always contain parameters whose exact values are, as a rule, unknown. If an equation modeling a physical system is structurally unstable, that is, if the behavior of its solutions may change qualitatively through arbitrarily small changes in its right-hand side, then it is necessary to understand which bifurcations of its phase portrait may occur through changes of the parameters.

Often model systems seem to be so complex that they do not admit meaningful investigation, above all because of the abundance of the variables which occur. In the study of such systems, some of the variables that change slowly in the course of the process described are, as a rule, assumed to be constant. The resulting system with a smaller number of variables can then be investigated. However, it is frequently impossible to consider the individual influences of the discarded terms in the original model. In this case, the discarded terms may be looked upon as typical perturbations, and, accordingly, the original model can be described by means of bifurcation theory applied to the reduced system.

Reformulating the well-known words of Poincaré on periodic solutions, one may say that bifurcations, like torches, light the way from well-understood dynamical systems to unstudied ones. L.D. Landau, and later E. Hopf, using this idea of bifurcation theory, offered a heuristic description of the transition from laminar to turbulent flow as the Reynolds number increases. In Landau's scenario this transition was accomplished through bifurcations of tori of steadily growing dimensions. Later on when the zoo of dynamical systems and their bifurcations had significantly grown, many papers appeared, describing – mainly at a physical level – the transition from regular (laminar) flow to chaotic (turbulent) flow. The chaotic behavior of the 3-dimensional model of Lorenz for convective motions has been explained with the aid of a chain of bifurcations. This explanation is not included in the present survey since, to save space, bifurcations of systems with symmetry have not been included. Lorenz's system is centrally symmetric.

The theory of relaxation oscillations, which deals with systems in which the parameters slowly change with time (these parameters are called slow variables), closely adjoins the theory of bifurcations in which parameters do not change with time. In "fast-slow" systems of relaxation oscillations, a slowness parameter enters that characterizes the speed of change of the slow variables. When this parameter is zero, a fast-slow system transforms into a family studied in the theory of bifurcations, but at a nonzero value of the parameter specific phenomena arise which are sometimes called dynamical bifurcations.

In this survey, systematic use is made of the theory of singularities. The solutions to many problems of bifurcation theory (mostly of local ones) consist of presenting and investigating a so-called principal family – a kind of topological normal form for families of the class studied. The theory of singularities helps to guess at, and partially to investigate, principal families. This theory also describes the theory of bifurcations of equilibrium states, singularities of slow surfaces, slow motions in the theory of relaxation oscillations, etc.

We also note that finitely smooth normal forms of local families of differential equations are especially useful in the theory of nonlocal bifurcations. On one hand, these normal forms substantially simplify the presentation and investigation of bifurcations, and also simplify and clarify the proof and analysis of the results obtained. On the other hand, the nonlocal theory of bifurcations helps to select problems from the theory of normal forms that are important for applications. In our opinion, at the present time, the connection between the theory of normal forms and the nonlocal theory of bifurcations is not used often enough.

This survey includes, along with what is known, a series of new results, some of these are known to the authors through private communications. [Added in translation: The results mentioned below were new when the Russian text was written (1985). Now most of them have been published. The additional list of references is given after the main one and numbered.] Among these are eight new topics. The first is a complete investigation of bifurcations from equilibria in generic two-parameter families of vector fields on the plane with two intersecting invariant curves (the so-called reduced problem for two purely imaginary pairs, Sect. 4.5 and Sect. 4.6 of Chap. 1 (see Żołądek (1987)). The second is the construction of finitely smooth normal forms and functional moduli of the C^1-classification of local families of vector fields and diffeomorphisms (Yu.S. Il'yashenko and S.Yu. Yakovenko, Sect. 5.7–5.10 of Chap. 2 (see Il'yashenko and Yakovenko [3*, 4*])). The third is the construction of a topological invariant of vector fields with a trajectory homoclinic to a saddle with complex eigenvalues (Sect. 5.6 of Chap. 3). The fourth is the description of a generic two-parameter deformation of a vector field with two homoclinic curves at a saddle, in which the bifurcation diagram of the deformation contains a continuum of components. (D.V. Turaev and L.P. Shil'nikov [9*], Sect. 7.2 of Chap. 3). The fifth result is the definition of a statistical limit set as a possible candidate for the concept of a physical attractor (Sect. 8.2 of Chap. 3 (Il'yashenko [2*])). The sixth one is the description of connections between the theory of implicit equations and relaxation oscillations, and the normalization of slow motions for fast-slow systems with one or two slow variables (see Arnol'd's theorem in Sect. 2.2–2.7 of Chap. 4 and the related paper by Davidov [1*]). The seventh result is normalization of fast-slow equations, and the explicit form and investigation of systems of first approximation (Sect. 3.2–3.5 of Chap. 4; see the related paper by Teperin [8*]). The eighth and last one is the investigation of the delayed loss of stability in generic fast-slow systems as a pair of eigenvalues of a stable singular point of a fast equation crosses the imaginary axis (the birth of a cycle as a dynamical bifurcation (A.I. Nejshtadt, § 4 of Chap. 4); see [6*, 7*]). We also point here to a conjecture on the bifurcations in generic multiple parameter families of vector fields on the plane that is closely related to Hilbert's 16th problem (Sect. 2.8 of Chap. 3).

Our survey, inevitably, is incomplete. We did not include in it the comparatively few works on local bifurcations in three-parameter families and on nonlocal bifurcations in two-parameter families; some relevant citations are, however, given in the References. In describing nonlocal bifurcations we limited ourselves to only those things which happen on the boundary of the set of Morse-Smale systems. The theory of such bifurcations is substantially complete, although it is not very well known; it is mostly due to works of the Gor'kij school, which often have been published in sources that are hard to obtain. That part of the boundary of the set of Morse-Smale systems on which a countable set of nonwandering trajectories arise is not yet fully explored; but Sect. 7 of Chap. 3 is devoted to this problem. For reasons of consistency of style we often formulate known results in a form different from that in which they first appeared.

Chap. 1 and 2 were written by V.I. Arnol'd and Yu.S. Il'yashenko. Chap. 3, in its final version, was written by V.S. Afrajmovich and Yu.S. Il'yashenko with the participation of V.I. Arnol'd and L.P. Shil'nikov. Sect. 1.6 of Chap. 2 was written by V.S. Afrajmovich. Sects. 1 and 2 of Chap. 4 were written by V.I. Arnol'd, Sect. 3, except for Sect. 3.7, by Yu.S. Il'yashenko. Sect. 3.7 was written by N.Kh. Rozov, Sect. 4 by A.I. Nejshtadt, Sect. 5 by A.K. Zvonkin; the authors sincerely thank them. The authors do not claim that the list of References is complete. In its organization we followed the same principles as in the survey by Arnol'd and Il'yashenko (1985). The symbol ▲ denotes the end of some formulations.