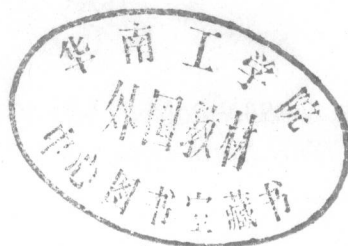


0175
F229
V.15

7961715
5

0175

COLLOQUIA MATHEMATICA
SOCIETATIS JÁNOS BOLYAI, 15.



DIFFERENTIAL EQUATIONS

Edited by:

M. FARKAS



E7961725



E7951715



NORTH-HOLLAND PUBLISHING COMPANY
AMSTERDAM — OXFORD — NEW YORK

© BOLYAI JÁNOS MATEMATIKAI TÁRSULAT

Budapest, Hungary, 1977



ISBN Bolyai: 963 8021 19 5

ISBN North-Holland: 0 7204 0496 7

Joint edition published by

JÁNOS BOLYAI MATHEMATICAL SOCIETY

and

NORTH-HOLLAND PUBLISHING COMPANY

Amsterdam—Oxford—New York

In the U.S.A. and Canada:

NORTH-HOLLAND PUBLISHING COMPANY

52 Vanderbilt Avenue

New York, N.Y. 10017

Technical Editor:

A. SZÉP

Printed in Hungary

ÁFÉSZ, VÁC

Sokszorosító üzeme

PREFACE

A Colloquium on Differential Equations was organized by the Bolyai János Mathematical Society at Keszthely (Hungary) in 1974. Research in the field of differential equations has become nowadays so extensive that it is impossible even to touch upon the main directions of development in the limitations of such a colloquium. While in the past in Hungary only a few mathematicians cultivated the subject (among whom the name of J. Egerváry is to be mentioned), recently the situation has changed and considerable interest has been shown towards this field of study also in this country. The aim of the organizing committee was to gather the mathematicians in Hungary as well as several colleagues from abroad dealing with, or interested in, differential equations. It was an honour to greet several excellent experts of this matter at Keszthely.

This volume contains 33 papers presented at the colloquium. Most of the papers deal with ordinary and functional differential equations. Topics treated in one or more papers are: stability, oscillation, local equivalence, asymptotic behaviour, boundary value problems, existence, uniqueness and continuation problems. The few papers on partial differential equations are of classical nature.

Differential equations have always been closely related to applied mathematics. Accordingly, some papers originate from, or are connected with, Hamiltonian mechanics, hydrodynamics, gyroscope problems etc.

In some papers algorithms are described for solving different boundary or initial value problems; a part of them involves experiences on high speed computers.

A number of papers deal with functional differential equations. We refer to S.D. Norkin's paper where a brief account of the results presented at the colloquium in this branch is given.

We wish to thank Miss M. Meszlényi and Mrs. Zs. Monostori for their excellent typing of the manuscript and for the enormous amount of greatly diverse work that had to be done in preparing this volume.

The Editor

CONTENTS

PREFACE	3
CONTENTS	5
SCIENTIFIC PROGRAM	8
LIST OF PARTICIPANTS	12
A.A. Abramov – E.S. Birger – N.B. Konyukhova – V.I. Ulyanova, Numerical segregation of the bounded solutions for systems of ordinary differential equations	17
Yu.A. Arkhangelskiĭ, Construction of periodic solutions for the Euler – Poisson equations by means of power series expansion containing a small parameter	27
D.D. Bainov – M.M. Konstantinov, On a boundary value problem with a generalized boundary condition for systems of super neutral differential equations	51
D.D. Bainov – S.D. Milusheva, On the application of the averaging method for certain classes of equations with delay	63
I. Bartsch, On the most general solution of some boundary value problems for the differential equation $\sum_{k=0}^n c_k (D_1 + aD_2)^k U = 0$	81
E.M. Bruins, The differential equations of quantum mechanics ...	91
T.A. Čanturia, On singular solutions of nonlinear system of ordinary differential equations	107
M.L. Cartwright – H.P.F. Swinnerton-Dyer, The boundedness of solutions of systems of differential equations	121
K. Deimling, On existence and uniqueness for Cauchy's problem in infinite dimensional Banach spaces	131
I.F. Dorofeev, On stable algorithms concerning numerical differen- tiation and the numerical solution of some inverse problems for equations with deviating argument	143
S. Fučík, Periodic solutions of generalized Liénard equation with with forcing term	155



J. Gergely , Numerical solution of a nonlinear boundary value problem	171
D. Greenspan , Conservative discrete models with computer examples of nonlinear phenomena in solids and fluids	179
D. Greenspan — D. Schultz , Simplification and improvement of a numerical method for Navier — Stokes problems	201
L.J. Grimm — L.M. Hall , Extensions of Lettenmeyer's theorem ...	223
P.S. Gromova , On the stability of solutions of differential-difference equations in the critical case	231
L.V. Guščo , On the boundedness of the solutions of $2n$ -dimensional systems	243
I. Győri , On asymptotically ordinary functional differential equations	257
L. Hatvani — A. Krámlí , A condition for non-asymptotical stability	269
J. Hegedűs , On a two-sided iterative method	277
G. Jank , Investigating the solutions of certain elliptic differential equations by complex analytic methods	291
J. Jarník — J. Kurzweil , On Ryabov's special solutions of functional differential equations	303
E. Kozakiewicz , On the asymptotic behaviour of positive solutions of two differential inequalities with retarded argument	309
P. Marušiak , Sufficient conditions for the oscillation of nonlinear delay differential equations	321
B. Mehri , On the conditions for the non-oscillatory solutions of non-linear third order differential equations	331
S.B. Norkin , Decomposition of the space of solutions of retarded differential equations and the oscillation of solutions	335
L. Pintér , Oscillatory properties of certain second order differential equations	347
L.E. Reizin's , Local topological equivalence of differential equations	355
A. Schmidt , Linear differential equations with constant coefficients in two independent variables	365

J. Terjéki , On the existence of the solutions of functional-differential equations	373
M. Tvrdý , Linear functional-differential operators: Normal solvability and adjoints	379
Ju.A. Ved' , An asymptotic problem for differential equations with retarded argument	391
J. Walter , Methodical remarks on the notion of a saturated solution of the differential equation $x' = f(t, x)$	409

SCIENTIFIC PROGRAM

September 2. Monday

Session A

- 15.00 – 16.00 M.L. Cartwright – H.P.F. Swinnerton-Dyer: Boundedness theorems for some second order differential equations
- 16.15 – 16.35 J. Kurzweil – J. Jarník: On Ryabov's special solutions of functional differential equations
- 16.40 – 17.00 M. Tvrdý: Boundary value type problems for functional differential equations and their adjoints
- 17.30 – 17.50 P.S. Gromova: Conditions for the stability of solutions of differential-difference equations in some singular case (in Russian)
- 17.55 – 18.15 J. Terjéki: Remarks on the existence of the solutions of functional-differential equations
- 18.20 – 18.40 M.M. Konstantinov – D.D. Bainov: On a boundary value problem with generalized boundary conditions for systems of differential equations of neutral type (in Russian)

Session B

- 16.15 – 16.35 I. Bihari: Behaviour of the 0 paths in the neighbourhood of a saddle point
- 16.40 – 17.00 L. Hatvani: Application of the method of Ljapunov-functions for investigating bounded stability (in Russian)
- 17.30 – 17.50 G. Sonnevend: On a class of nonlinear control processes
- 18.20 – 18.40 L. Reizin's: Local topological equivalence of differential equations (in Russian)

September 3. Tuesday

Session A

- 9.30 – 10.30 J. Szarski: Strong maximum principle for non linear parabolic differential-functional inequalities
- 10.45 – 11.45 D. Greenspan: Conservative discrete models and a new particle theory of fluids
- 15.30 – 15.50 J. Walter: Methodological remarks on the notion of the saturated solutions of the differential equation $x' = f(t, x)$
- 15.55 – 16.15 K. Karták: On modified iterations and existence theorems
- 16.20 – 16.40 A. Schmidt: Über gewisse lineare Differentialgleichungen
- 16.45 – 17.05 G. Jank: Funktionentheoretische Untersuchungen von Lösungen gewisser elliptischer Differentialgleichungen
- 17.30 – 17.50 D. Schultz – D. Greenspan: Simplification and improvement of a numerical method for Navier – Stokes problems
- 17.55 – 18.15 J. Moravčík: Some oscillatory and asymptotic properties of linear differential equations of order n ($n \geq 3$) of 3-rd kind (in Russian)
- 18.20 – 18.40 L.N. Fadeeva: On a certain class of non-linear boundary-value problems

Session B

- 15.30 – 15.50 I. Györi: On asymptotically ordinary functional-differential equations (in Russian)
- 15.55 – 16.15 R.J. Grimm: Some extensions of Lettenmeyer's theorem on singularities
- 16.20 – 16.40 P. Marušiak: Oscillation of solutions of delay differential equations



- 16.45 – 17.05 E. Kozakiewicz: Über das asymptotische Verhalten der positiven Lösungen einiger Differentialgleichungen mit nacheilendem Argument
- 17.30 – 17.50 S.D. Milusheva – D.D. Bainov: Application of the averaging method for systems of integro-differential equations of standard type with discontinuous right-hand side (in Russian)
- 17.55 – 18.15 J. Hegedüs: On two-sided estimates for the solutions of certain ordinary differential equations (in Russian)
- 18.20 – 18.40 Ju.A. Ved': Some asymptotic problems for differential equations with delayed argument (in Russian)

September 5. Thursday

Session A

- 9.30 – 10.30 A. Pleijel: Limit types of symmetric ordinary differential equations
- 10.45 – 11.45 K. Deimling: On existence and uniqueness for ordinary differential equations in Banach spaces
- 15.30 – 15.50 K. Schmitt – P. Volkmann: Randwert-Probleme für gewöhnliche Differentialgleichungen zweiter Ordnung in konvexen Teilmengen eines Banachraumes
- 15.55 – 16.15 L. Pintér: Oscillatory properties of certain ordinary second order nonlinear differential equations
- 16.20 – 16.40 Á. Elbert: On the distance between zeros of solutions of certain second order linear differential equations
- 16.45 – 17.05 B. Mehri: On the conditions for the oscillation and non-oscillation of solutions of nonlinear third order differential equations
- 17.30 – 17.50 I. Fenyő: On the smallest roots of solutions of fourth order differential equations

17.55 – 18.15 I. Bartsch: Über die Differentialgleichung

$$\sum_{k=1}^n c_k (D_1 + aD_2)^k U = 0$$

18.20 – 18.40 P. Soltész: On differential equations of infinite order

Session B

15.30 – 15.50 L. Simon: On approximation of solutions of elliptic boundary problems in unbounded domains

15.55 – 16.15 E.M. Bruins: The differential equations of quantum mechanics

16.20 – 16.40 J. Gergely: Numerical solutions of a nonlinear boundary value problem

16.45 – 17.05 J. Kačur: Application of Rothe's method to nonlinear evolution equations

17.30 – 17.50 V.M. Petkov: Sur les systemes hyperboliques a caracteristiques de multiplicité constants

17.55 – 18.15 I.F. Dorofeev: On the stable algorithms for numerical solutions of some inverse problems for an equation with delay (in Russian)

18.20 – 18.40 L. Gerencsér: A second order optimization technique for constrained variational problems

September 6. Friday

Session A

9.30 – 10.30 F.V. Atkinson: The limit-point/limit-circle classification of second-order operators

10.45 – 11.45 S.B. Norkin: On k -oscillation of systems with one degree of freedom (in Russian)

LIST OF PARTICIPANTS

- ATKINSON, F.V., 35 Green Ridge, Brighton, Sussex, England
BAINOV, D., Ul. Oboriste 23, Sofia-4, Bulgaria
BALLA, KATALIN, MTA SzTAKI, 1052 Budapest, 112, Pf. 63, Hungary
BÁRÁNY, I., BME Gépészmérnöki Kar, Matematika Tsz., 1111 Budapest, Stoczek u. 2, Hungary
BARTSCH, INGEBORG, 25 Rostock, Universitätsplatz 1, GDR
BENKÓ, S., MTA SzTAKI, 1502 Budapest, 112, Pf. 63, Hungary
BIHARI, I., MTA Matematikai Kutató Intézet, 1053 Budapest, Reáltanoda u. 13-15, Hungary
BLEYER, A., BME Villamosmérnöki Kar, Matematika Tsz., 1111 Budapest, Stoczek u. 2, Hungary
BRUINS, E.M., Joh. Verhulststraat 185, Amsterdam-Z 1, The Netherlands
CARTWRIGHT, MARY LUCY, 38 Sherlock Close, Cambridge CB3 OHP, England
DEIMLING, K., Math. Seminar der Universität, D-23 Kiel, Olshausen Str. 40-60, GFR
DETKI, J., Szabadka (24000), J.E. Tanitya 66, Yugoslavia
DÉVÉNYI, D., 1036 Budapest, Korvin Ottó u. 63, Hungary
DOROFEEV, I.F., Univ. Druzby Nar. im. Patr. Lumumbu, Moscow B-302, Ul. Ordzonikidze, USSR
DRASNY, JÓZSEFNÉ, MTA SzTAKI, 1052 Budapest, 112, Pf. 63, Hungary
DURIKOVIČ, V., Dept. of Math., Analysis I A Komensky Univ., Matematický pavilon, Mlinská dolina 816 31, Bratislava, Czechoslovakia
ELBERT, Á., MTA Matematikai Kutató Intézet, 1053 Budapest, Reáltanoda u. 13-15, Hungary
FARKAS, ÉVA, MTA KFKI, 1121 Budapest, Konkoly Thege út 6, Hungary
FARKAS, MIKLÓSNÉ, 1123 Budapest, Alkotás u. 11, Hungary
FAZZÁN, H.R., 1121 Budapest, Konkoly Thege u. 6, Hungary

FENYŐ, I., 1125 Budapest, Istenehyi út 48/a, Hungary

FRIGERIO, A., via Donatello 16, 35100 Padova, Italy

FRITZ, JÓZSEFNÉ, BME Gépészmérnöki Kar, Matematika Tsz., H. ép.,
1111 Budapest, Stoczek u. 2, Hungary

FRIVALDSZKY, S., KGM-ISzSzI, 1394 Budapest, Pf. 356, Hungary

FUČÍK, S., Dept. of Math., Ana. Faculty of Math-Ph. Charles Univ., 83
Sokolovská, 18600 Prague, Czechoslovakia

GERENCSÉR, L., MTA SzTAKI, 1052 Budapest, 112, Pf. 63, Hungary

GERGELY, J., MTA SzTAKI, 1052 Budapest, 112, Pf. 63, Hungary

GREENSPAN, D., c/o Dept. Computer Sciences, Univ. Wisconsin,
Madison, Wisconsin 53706, USA

GRIMM, L.J.E., Department of Mathematics, University of Missouri,
Rolla, Missouri 65401, USA

GROMOVA, P.S., Univ. Druzby Nar. im. Patr. Lumumbu, Moscow B-302,
ul. Ordzonikidze, USSR

GYŐRI, I., 6723 Szeged, Tarján 620 ép. B. I/5, Hungary

GYURKOVICS, ÉVA, 6720 Szeged, Somogyi Béla u. 7, Hungary

HATVANI, L., 6720 Szeged, Aradi vértanúk tere 1, Bolyai Intézet,
Hungary

HEGEDŰS, CS., MTA KFKI, 1121 Budapest, Konkoly Thege út 6,
Hungary

HEGEDŰS, J., 6720 Szeged, Aradi vértanúk tere 1, Bolyai Intézet,
Hungary

HIBBEY, L., Energiagazdálkodási Intézet, 1251 Budapest, Pf. 15, Hungary

JANK, G., A-8010 Graz, Brockmannng. 9/III, Austria

JARNÍK, J., Žitná 25, 11567 Praha 1, Czechoslovakia

JELITAI, Á., 1043 Budapest, Munkásotthon u. 34, Hungary

JUHÁSZ, F., MTA SzTAKI, 1052 Budapest, 112, Pf. 63, Hungary

KAČUR, J., Bratislava 16, Department of Mathematics, Czechoslovakia

KARTÁK, V., VSCHT, Math. Dept., Praha – Dejvice 1905, Czecho-
slovakia

KOZAKIEWICZ, E., Humboldt Univ. zu Berlin, 108 Berlin, Unter den
Linden 6, GDR

KRISZTINKOVICS, F., 2100 Gödöllő, Nyitor tér 3/c, Hungary

- LÖKÖS, ÁGNES, 1111 Budapest, Lágymányosi u. 14/a, V. em. 17,
Hungary
- LUSZCZKI, Z., 61860 Poznan, ul. Za Grobla 6/8, Poland
- MAKAI, E., MTA Matematikai Kutató Intézet, 1053 Budapest, Reál-
tanoda u. 13-15, Hungary
- MÁLYUSZ, K., 1011 Budapest, Batthyány u. 26, Hungary
- MARUŠIAK, P., Katedra Matematiky, Marxa a Engelsa 15, 010 88 Zilina,
Czechoslovakia
- MEHRI, B., Dept. of Mathematics, University of Arya-Mehr, Tehran, Iran
- MESKÓ, ATTILÁNÉ, MTA SzTAKI, 1052 Budapest, 112, Pf. 63,
Hungary
- MILE, KÁROLYNÉ, 1042 Budapest, Viola u. 3, Hungary
- MILUSHEVA, S., Dede-Agacs No. 11, Sofia, Bulgaria
- MOLNÁRKA, GY., ELTE TTK Num. és Gépi Mat. Tsz., 1088 Budapest,
Múzeum krt. 6-8, Hungary
- MORAVCIK, J., Marxa a Engelsa 25, 010 88 Zilina, Czechoslovakia
- MOSON, P., 1036 Budapest, Árpád fejedelem u. 52, Hungary
- OLAS, A., Inst. Fund. Techn. Research, Warsaw, ul. Swietokrzyska 21,
ZUM, Poland
- PETKOV, V.M., P.O. Box 373, Sofia, Bulgaria
- PINTÉR, L., 6720 Szeged, Aradi vértanúk tere 1, Bolyai Intézet, Hungary
- PLEIJEL, A.V.C., Stora Mossens Backe 14, 161 37 Bromma, Sweden
- PÁGER, T., 1123 Budapest, Alkotás u. 39/a, Hungary
- RADZISZEWSKI, B., Inst. Fund. Techn. Research, PAN, Warsaw,
Swietokrzyska 21, ZUM, Poland
- RENNER, G., MTA SzTAKI, 1502 Budapest, 112, Pf. 63, Hungary
- SCHMIDT, A., 25 Rostock, Schliemannstr. 40, GDR
- SCHNEIDER, M., 1 Berlin 33, Rohffsstr. 6, West-Berlin GDR
- SCHULTZ, D.H., The University of Wisconsin-Milwaukee, Dept. of
Mathematics, Milwaukee, Wisconsin 53201, USA
- SCHÜTTE, ILSE, Humboldt Univ. zu Berlin Sect. Math., 108 Berlin,
Unter den Linden 6, GDR
- SIMON, L., 1117 Budapest, Baranyai u. 17, V. 4, Hungary
- SOLTÉSZ, P., 1083 Budapest, Tömő u. P/1, Hungary
- SONNEVEND, G., MTA SzTAKI, 1502 Budapest, 112, Pf. 63, Hungary

- SZADKOWSKI, A., Inst. Fund. Techn. Research PAN, Warsaw, ul.
Swietokrzyska 21, ZUM, Poland
- SZARSKI, J., ul. Kochanowskiego 1115, Kraków, Poland
- SZEPESVÁRI, I., MTA SzTAKI, 1052 Budapest, 112, Pf. 63, Hungary
- SZILÁRD, K., MTA Matematikai Kutató Intézet, 1053 Budapest, Reáltanoda u. 13-15, Hungary
- SZÉP, A., MTA Matematikai Kutató Intézet, 1053 Budapest, Reáltanoda u. 13-15, Hungary
- TARNAY, GY., MTA SzTAKI, 1052 Budapest, 113, Pf. 63, Hungary
- TERJÉKI, J., 5094 Tiszajenő, Külsőjenő 21, Hungary
- TVRDÝ, M., Matematický ústav ČSAV, Žitná 25, 11567 Praha 1, Czechoslovakia
- VILLARI, G., Istituto di Matematica Applicata, Università di Firenze, viale Morgagni 44, 50134 Firenze, Italy
- VIGASSY, J., MTA KFKI, 1525 Budapest, Konkoly Thege út 6, Hungary
- VOLKMANN, P., Universität Karlsruhe (TH), Mathematisches Institut I, 75 Karlsruhe 1, GFR
- WALTER, J., 51 Aachen, Templergraben 55, GFR
- ZIBOLEN, E., BME Gépészmérnöki Kar, Matematika Tsz., 1111 Budapest, Stoczek u. 2, Hungary
- ZOCCARATO, L., Collegio Universitario Antoniarum, via Donatello 16, 35100 Padova, Italy

