WAVE PROPAGATION IN ELASTIC SOLIDS

BY

J. D. ACHENBACH
The Technological Institute,
Northwestern University, Evanston, Illinois

NORTH-HOLLAND PUBLISHING COMPANY
AMSTERDAM · NEW YORK · OXFORD

Library of Congress Catalog Card Number: 72-79720
North-Holland ISBN for the series: 0 7204 2350 3
North-Holland ISBN for this volume: 0 7204 2367 8
American Elsevier ISBN: 0 444 10465 8

First edition 1973

Second printing 1976

Publishers:

NORTH-HOLLAND PUBLISHING COMPANY
AMSTERDAM • NEW YORK • OXFORD

Sole distributors for the U.S.A. and Canada:

AMERICAN ELSEVIER PUBLISHING COMPANY, INC.
52 VANDERBILT AVENUE
NEW YORK, N.Y. 10017

Printed in Great Britain

WAVE PROPAGATION IN ELASTIC SOLIDS

NORTH-HOLLAND SERIES IN

APPLIED MATHEMATICS AND MECHANICS

VOLUME 16

NORTH-HOLLAND PUBLISHING COMPANY
AMSTERDAM · NEW YORK · OXFORD

To Marcia

PREFACE

The propagation of mechanical disturbances in solids is of interest in many branches of the physical sciences and engineering. This book aims to present an account of the theory of wave propagation in *elastic* solids. The material is arranged to present an exposition of the basic concepts of mechanical wave propagation within a one-dimensional setting and a discussion of formal aspects of elastodynamic theory in three dimensions, followed by chapters expounding on typical wave propagation phenomena, such as radiation, reflection, refraction, propagation in waveguides, and diffraction. The treatment necessarily involves considerable mathematical analysis. The pertinent mathematical techniques are, however, discussed at some length.

I hope that the book will serve a dual purpose. In addition to being a reference book for engineers and scientists in the broad sense, it is also intended to be a textbook for graduate courses in elastic wave propagation. As a text the book should be suitable for students who have completed first-year graduate courses in mechanics and mathematics. To add to its utility as a textbook each chapter is supplemented by a set of problems, which provide a useful test of the reader's understanding, as well as further illustrations of the basic ideas.

The book was developed from notes for a course offered to graduate students at Northwestern University. In the spring of 1969 a substantial part of the text was prepared in the form of typewritten notes for a series of lectures, while I was a visiting member of the faculty at the University of California in La Jolla. I am pleased to record my thanks for that opportunity. I also wish to express my gratitude to the Rector Magnificus of the Technological University of Delft and the Trustees of the Ir. Cornelis Gelderman Fund for inviting me to act as visiting professor in the department of mechanical engineering at the Technological University, in 1970–1971. While I was in Delft the larger part of the manuscript was completed. A sabbatical leave from Northwestern University during that period is gratefully acknowledged.

For the material of Chapters 5 and 6 I should like to acknowledge my indebtedness to the lectures and publications of Professor R. D. Mindlin. Substantial parts of Chapter 3 are based on the dissertation of Professor A. T. de Hoop, and on the work of Professor E. Sternberg. I am also indebted to many colleagues who read chapters of the book, and who provided me with their constructive criticism. Needless to say, I alone am responsible for errors of fact and logic.

A special word of thanks goes to Mrs. Ruth H. Meier who for many years provided excellent secretarial assistance, and who typed and retyped most of the manuscript as the material was arranged and rearranged.

Let me close with the wish that this book may convey some of the fascinating aspects of wave propagation as a phenomenon, and that it may have done justice to the elegance of the mathematical methods that have been employed.

J. D. A.

TABLE OF CONTENTS

INTRODUCTION

The propagation of mechanical disturbances

The local excitation of a medium is not instantaneously detected at positions that are at a distance from the region of excitation. It takes time for a disturbance to propagate from its source to other positions. This phenomenon of propagation of disturbances is well known from physical experience, and some illustrative examples immediately come to mind. Thus an earthquake or an underground nuclear explosion is recorded in another continent well after it has occurred. The report of a distant gun is heard after the projectile has arrived, because the velocity of disturbances in air, i.e., the speed of sound, is generally smaller than the velocity of the projectile. More familiar manifestations of the propagation of disturbances are waves in a rope or propagating ripples on the surface of water. These examples illustrate mechanical wave motions or mechanical wave propagation.

Mechanical waves originate in the forced motion of a portion of a deformable medium. As elements of the medium are deformed the disturbance is transmitted from one point to the next and the disturbance, or wave, progresses through the medium. In this process the resistance offered to deformation by the consistency of the medium, as well as the resistance to motion offered by inertia, must be overcome. As the disturbance propagates through the medium it carries along amounts of energy in the forms of kinetic and potential energies. Energy can be transmitted over considerable distances by wave motion. The transmission of energy is effected because motion is passed on from one particle to the next and not by any sustained bulk motion of the entire medium. Mechanical waves are characterized by the transport of energy through motions of particles about an equilibrium position. Thus, bulk motions of a medium such as occur, for example, in turbulence in a fluid are not wave motions.

Deformability and inertia are essential properties of a medium for the transmission of mechanical wave motions. If the medium were not deformable any part of the medium would immediately experience a disturbance in the form of an internal force or an acceleration upon application of a localized excitation. Similarly, if a hypothetical medium were without inertia there would be no delay in the displacement of particles and the transmission of the disturbance from particle to particle would be effected instantaneously to the most distant particle. Indeed, in later chapters it will be shown analytically that the velocity of propagation of a mechanical disturbance always assumes the form of the square root of the ratio of a parameter defining the resistance to deformation and a parameter defining the inertia of the medium. All real materials are of course deformable and possess mass and thus all real materials transmit mechanical waves.

The inertia of a medium first offers resistance to motion, but once the medium is in motion inertia in conjunction with the resilience of the medium tends to sustain the motion. If, after a certain interval the externally applied excitation becomes stationary, the motion of the medium will eventually subside due to frictional lossess and a state of static deformation will be reached. The importance of dynamic effects depends on the relative magnitudes of two characteristic times: the time characterizing the external application of the disturbance and the characteristic time of transmission of disturbances across the body.

Suppose we consider a solid body subjected to an external disturbance $F(t)$ applied at a point P. The purpose of an analysis is to compute the deformation and the distribution of stresses as functions of the spatial coordinates and time. If the greatest velocity of propagation of disturbances is c, and if the external disturbance is applied at time $t = 0$, the disturbed regions at times $t = t_1$ and $t = t_2$ are bounded by spheres centered at the point P, with radii ct_1 and ct_2, respectively. Thus the whole of the body is disturbed at time $t = r/c$, where r is the largest distance within the body measured from the point P. Now suppose that the significant changes in $F(t)$ take place over a time t_a. It can then be stated that dynamic effects are of importance if t_a and r/c are of the same order of magnitude. If $t_a \gg r/c$, the problem is quasistatic rather than dynamic in nature and inertia effects can be neglected. Thus for bodies of small dimensions a wave propagation analysis is called for if t_a is small. If the excitation source is removed the body returns to rest after a certain time. For excitation sources that are applied and removed, the effects of wave motion are important if the time interval of application is of the same order of magnitude as a characteristic time of transmission of a disturbance across the body. For bodies of finite dimensions this is the case for loads of explosive origins or for impact loads. For sustained external disturbances the effects of wave motions need be considered if the externally applied disturbances are rapidly changing with time, i.e., if the frequency is high.

In mathematical terms a traveling wave in one dimension is defined by an expression of the type $f = f(x-ct)$, where f as a function of the spatial coordinate x and the time t represents a disturbance in the values of some physical quantity. For mechanical waves f generally denotes a displacement, a particle velocity or a stress component. The function $f(x-ct)$ is called a simple wave function, and the argument $x-ct$ is the phase of the wave function. If t is increased by any value, say Δt, and simultaneously x is increased by $c\Delta t$, the value of $f(x-ct)$ is clearly not altered. The function $f(x-ct)$ thus represents a disturbance advancing in the positive x-direction with a velocity c. The velocity c is termed the phase velocity. The propagating disturbance represented by $f(x-ct)$ is a special wave in that the shape of the disturbance is unaltered as it propagates through the medium.

Continuum mechanics

Problems of the motion and deformation of substances are rendered amenable to mathematical analysis by introducing the concept of a continuum or continuous medium. In this idealization it is assumed that properties averaged over a very small element, for example, the mean mass density, the mean displacement, the mean interaction force, etc., vary continuously with position in the medium, so that we may speak about *the* mass density, *the* displacement and *the* stress, as functions of position and time. Although it might seem that the microscopic structure of real materials is not consistent with the concept of a continuum, the idealization produces very useful results, simply because the lengths characterizing the microscopic structure of most materials are generally much smaller than any lengths arising in the deformation of the medium. Even if in certain special cases the microstructure gives rise to significant phenomena, these can be taken into account within the framework of the continuum theory by appropriate generalizations.

The analysis of disturbances in a medium within the context of the continuum concept belongs to the time-tested discipline of continuum mechanics. In achieving the traditional objective of determining the motion and deformation generated by external excitations the analysis passes through two major stages. In the first stage the body is idealized as a continuous medium and the physical phenomena are described in mathematical terms by introducing appropriate mathematical abstractions. Completion of this