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AN INTRODUCTION FOR SCIENTISTS AND ENGINEERS

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Turbulence

For Henri and Mars

Preface

Turbulence is all around us. The air flowing in and out of our lungs is turbulent, as is the natural convection in the room in which you sit. Glance outside; the wind which gusts down the street is turbulent, and it is turbulence that disperses the pollutants, which belch from the rear of motor cars, saving us from asphyxiation. Turbulence controls the drag on cars, aeroplanes, and bridges, and it dictates the weather through its influence on large-scale atmospheric and oceanic flows. The liquid core of the earth is turbulent, and it is this turbulence that maintains the terrestrial magnetic field against the natural forces of decay. Even solar flares are a manifestation of turbulence, since they are triggered by vigorous motion on the surface of the sun. It is hard not to be intrigued by a subject which pervades so many aspects of our lives.

Yet curiosity can so readily give way to despair when the budding enthusiast embarks on serious study. The mathematical description of turbulence is complex and forbidding, reflecting the profound difficulties inherent in describing three-dimensional, chaotic processes.

This is a textbook and not a research monograph. Our principle aim is to bridge the gap between the elementary, heuristic accounts of turbulence to be found in undergraduate texts, and the more rigorous, if daunting, accounts given in the many excellent monographs on the subject. Throughout we seek to combine the maximum of physical insight with the minimum of mathematical detail.

Turbulence holds a unique place in the field of classical mechanics. Despite the fact that the governing equations have been known since 1845, there is still surprisingly little we can predict with relative certainty. The situation is reminiscent of the state of electromagnetism before it was transformed by Faraday and Maxwell. A myriad of tentative theories have been assembled, often centred around particular experiments, but there is not much in the way of a coherent theoretical framework.[1] The subject tends to consist of an uneasy mix of semi-empirical laws and deterministic but highly simplified cartoons,

[1] One difference between turbulence and nineteenth century electromagnetism is that the latter was eventually refined into a coherent theory, whereas it is unlikely that this will ever occur in turbulence.

bolstered by the occasional rigorous theoretical result. Of course, such a situation tends to encourage the formation of distinct camps, each with its own doctrines and beliefs. Engineers, mathematicians, and physicists tend to view turbulence in rather different ways, and even within each discipline there are many disparate groups. Occasionally religious wars break out between the different camps. Some groups emphasize the role of coherent vortices, while others downplay the importance of such structures and advocate the use of purely statistical methods of attack. Some believe in the formalism of fractals or chaos theory, others do not. Some follow the suggestion of von Neumann and try to unlock the mysteries of turbulence through massive computer simulations, others believe that this is not possible. Many engineers promote the use of semi-empirical models of turbulence; most mathematicians find that this is not to their taste. The debate is often vigorous and exciting and has exercised some of the finest twentieth century minds, such as L.D. Landau and G.I. Taylor. Any would-be author embarking on a turbulence book must carefully pick his way through this minefield, resigned to the fact that not everyone will be content with the outcome. But this is no excuse for not trying; turbulence is of immense importance in physics and engineering, and despite the enormous difficulties of the subject, significant advances have been made.

Roughly speaking, texts on turbulence fall into one of two categories. There are those that focus on the turbulence itself and address such questions as: where does turbulence come from, what are its universal features, to what extent is it deterministic? On the other hand, we have texts whose primary concern is the influence of turbulence on practical processes, such as drag, mixing, heat transfer, and combustion. Here the main objective is to parameterize the influence of turbulence on these processes. The word *modelling* appears frequently in such texts. Applied mathematicians and physicists tend to be concerned with the former category, while engineers are necessarily interested in the latter. Both are important, challenging subjects.

On balance, this text leans slightly towards the first of these categories. The intention is to provide some insight into the physics of turbulence and to introduce the mathematical apparatus which is commonly used to dissect turbulent phenomena. Practical applications, alas, take a back seat. Evidently such a strategy will not be to everyone's taste. Nevertheless, it seems natural when confronted with such a difficult subject, whose pioneers adopted both rigorous and heuristic means of attack, to step back from the practical applications and try and describe, as simply as possible, those aspects of the subject which are now thought to be reasonably well understood.

Our choice of material has been guided by the observation that the history of turbulence has, on occasions, been one of heroic initiatives which promised much yet delivered little. So we have applied the filter of time and chosen to emphasize those theories, both rigorous and heuristic, which look like they might be a permanent feature of the turbulence landscape. There is little attempt to document the latest controversies, or those findings whose significance is still unclear. We begin, in Chapters 1–5, with a fairly traditional introduction to the subject. The topics covered include: the origins of turbulence, boundary layers, the log-law for heat and momentum, free-shear flows, turbulent heat transfer, grid turbulence, Richardson's energy cascade, Kolmogorov's theory of the small scales, turbulent diffusion, the closure problem, simple closure models, and so on. Mathematics is kept to a minimum and we presuppose only an elementary knowledge of fluid mechanics and statistics. (Those statistical ideas which are required, are introduced as and when they are needed in the text.) Chapters 1–5 may be appropriate as background material for an advanced undergraduate or introductory postgraduate course on turbulence.

Next, in Chapters 6–8, we tackle the somewhat refined, yet fundamental, problem of homogeneous turbulence. That is, we imagine a fluid that is vigorously stirred and then left to itself. What can we say about the evolution of such a complex system? Our discussion of homogeneous turbulence differs from that given in most texts in that we work mostly in real space (rather than Fourier space) and we pay as much attention to the behaviour of the large, energy-containing eddies, as we do to the small-scale structures.

Perhaps it is worth explaining why we have taken an unconventional approach to homogeneous turbulence, starting with our slight reluctance to embrace Fourier space. The Fourier transform is conventionally used in turbulence because it makes certain mathematical manipulations easier and because it provides a simple (though crude) means of differentiating between large and small-scale processes. However, it is important to bear in mind that the introduction of the Fourier transform produces no new information; it simply represents a transfer of information from real space to Fourier space. Moreover, there are other ways of differentiating between large and small scales, methods that do not involve the complexities of Fourier space. Given that turbulence consists of eddies (blobs of vorticity) and not waves, it is natural to ask why we must invoke the Fourier transform at all. Consider, for example, grid turbulence. We might picture this as an evolving vorticity field in which vorticity is stripped off the bars of the grid and then mixed to form a seething tangle of vortex tubes and sheets. It is hard to picture a Fourier mode being stripped off the bars of the grid! It is the view of this author that, by and large, it is preferable to work in real space, where the relationship between mathematical representation and physical reality is, perhaps, a little clearer.

The second distinguishing feature of Chapters 6–8 is that equal emphasis is given to both large and small scales. This is a deliberate attempt to redress the current bias towards small scales in monographs on homogeneous turbulence. Of course, it is easy to see how such an imbalance developed. The spectacular success of Kolmogorov's theory of the small eddies has spurred a vast literature devoted to verifying (or picking holes in) this theory. Certainly it cannot be denied that Kolmogorov's laws represent one of the milestones of turbulence theory. However there have been other success stories too. In particular, the work of Landau, Batchelor, and Saffman on the large-scale structure of homogeneous turbulence stands out as a shining example of what can be achieved through careful, physically motivated analysis. So perhaps it is time to redress the balance, and it is with this in mind that we devote part of Chapter 6 to the dynamics of the large-scale eddies. Chapters 6–8 may be suitable as background material for an advanced postgraduate course on turbulence, or act as a reference source for professional researchers.

The final section of the book, Chapters 9 and 10, covers certain special topics rarely discussed in introductory texts. The motivation here is the observation that many geophysical and astrophysical flows are dominated by the effects of body forces, such as buoyancy, Coriolis and Lorentz forces. Moreover, certain large-scale flows are approximately two-dimensional and this has led to a concerted investigation of two-dimensional turbulence over the last few years. We touch on the influence of body forces in Chapter 9 and two-dimensional turbulence in Chapter 10.

There is no royal route to turbulence. Our understanding of it is limited and what little we do know is achieved through detailed and difficult calculation. Nevertheless, it is hoped that this book provides an introduction which is not too arduous and which allows the reader to retain at least some of that initial sense of enthusiasm and wonder.

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P.A. Davidson
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It remains to call attention to the chief outstanding difficulty of our subject.

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