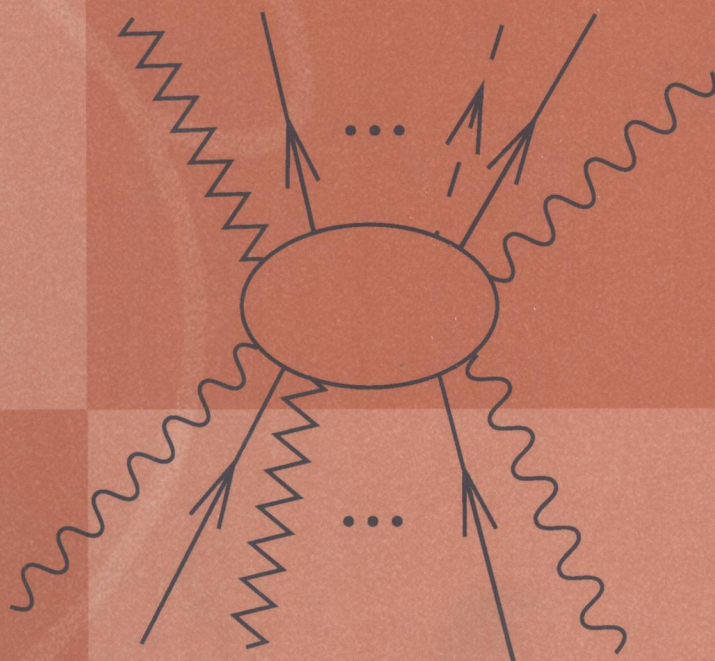


F. Schwabl

Advanced Quantum Mechanics

Second Edition



Springer

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Franz Schwabl

Advanced Quantum Mechanics

Translated by Roginald Hilton and Angela Lahee

Second Edition

With 79 Figures, 4 Tables, and 103 Problems



Springer



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The true physics is that which will, one day,
achieve the inclusion of man in his wholeness
in a coherent picture of the world.

Pierre Teilhard de Chardin

To my daughter Birgitta

Preface to the Second Edition

In the new edition, supplements, additional explanations and cross references have been added at numerous places, including new formulations of some of the problems. Figures have been redrawn and the layout has been improved. In all these additions I have attempted not to change the compact character of the book. The present second English edition is identical to the current German third edition. The proofs were read by E. Bauer, E. Marquard-Schmitt und T. Wollenweber. Special thanks go to them and to Dr. R. Hilton for comments on some of the formulations and to Mrs. Jörg-Müller for general supervision. I would like to thank all colleagues and students who have made suggestions to improve the book, as well as the publisher, Dr. Thorsten Schneider and Mrs. J. Lenz for the excellent cooperation.

Munich, May 2003

F. Schwabl

Preface to the First Edition

This textbook deals with advanced topics in the field of quantum mechanics, material which is usually encountered in a second university course on quantum mechanics. The book, which comprises a total of 15 chapters, is divided into three parts: I. Many-Body Systems, II. Relativistic Wave Equations, and III. Relativistic Fields. The text is written in such a way as to attach importance to a rigorous presentation while, at the same time, requiring no prior knowledge, except in the field of basic quantum mechanics. The inclusion of all mathematical steps and full presentation of intermediate calculations ensures ease of understanding. A number of problems are included at the end of each chapter. Sections or parts thereof that can be omitted in a first reading are marked with a star, and subsidiary calculations and remarks not essential for comprehension are given in small print. It is not necessary to have read Part I in order to understand Parts II and III. References to other works in the literature are given whenever it is felt they serve a useful purpose. These are by no means complete and are simply intended to encourage further reading. A list of other textbooks is included at the end of each of the three parts.

In contrast to Quantum Mechanics I, the present book treats relativistic phenomena, and classical and relativistic quantum fields.

Part I introduces the formalism of second quantization and applies this to the most important problems that can be described using simple methods. These include the weakly interacting electron gas and excitations in weakly interacting Bose gases. The basic properties of the correlation and response functions of many-particle systems are also treated here.

The second part deals with the Klein-Gordon and Dirac equations. Important aspects, such as motion in a Coulomb potential are discussed, and particular attention is paid to symmetry properties.

The third part presents Noether's theorem, the quantization of the Klein-Gordon, Dirac, and radiation fields, and the spin-statistics theorem. The final chapter treats interacting fields using the example of quantum electrodynamics: S-matrix theory, Wick's theorem, Feynman rules, a few simple processes such as Mott scattering and electron-electron scattering, and basic aspects of radiative corrections are discussed.

The book is aimed at advanced students of physics and related disciplines, and it is hoped that some sections will also serve to augment the teaching material already available.

This book stems from lectures given regularly by the author at the Technical University Munich. Many colleagues and coworkers assisted in the production and correction of the manuscript: Ms. I. Wefers, Ms. E. Jörg-Müller, Ms. C. Schwierz, A. Vilfan, S. Clar, K. Schenk, M. Hummel, E. Wefers, B. Kaufmann, M. Bulenda, J. Wilhelm, K. Kroy, P. Maier, C. Feuchter, A. Wonhas. The problems were conceived with the help of E. Frey and W. Gasser. Dr. Gasser also read through the entire manuscript and made many valuable suggestions. I am indebted to Dr. A. Lahee for supplying the initial English version of this difficult text, and my special thanks go to Dr. Roginald Hilton for his perceptive revision that has ensured the fidelity of the final rendition.

To all those mentioned here, and to the numerous other colleagues who gave their help so generously, as well as to Dr. Hans-Jürgen Kölsch of Springer-Verlag, I wish to express my sincere gratitude.

Munich, March 1999

F. Schwabl



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