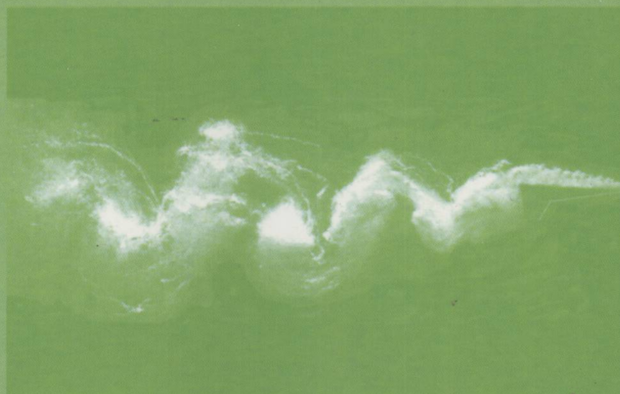

Fundamentals of Multiphase Flow

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FUNDAMENTALS OF MULTIPHASE FLOW

This book is targeted to graduate students and researchers at the cutting edge of investigations into the fundamental nature of multiphase flows. It is intended as a reference book for the basic methods used in the treatment of multiphase flows. The subject of multiphase flows encompasses a vast field, a host of different technological contexts, a wide spectrum of different scales, a broad range of engineering disciplines, and a multitude of different analytical approaches. The aim of *Fundamentals of Multiphase Flow* is to bring much of this fundamental understanding together into one book, presenting a unifying approach to the fundamental ideas of multiphase flows. The book summarizes those fundamental concepts with relevance to a broad spectrum of multiphase flows. It does not pretend to present a comprehensive review of the details of any one multiphase flow or technological context; references to such reviews are included where appropriate.

Christopher E. Brennen is professor of Mechanical Engineering in the Faculty of Engineering and Applied Science at the California Institute of Technology. He has published over 200 referred articles and is especially well known for his research on cavitation, turbomachinery flows, as well as multiphase flows. He is the author of *Cavitation and Bubble Dynamics* and *Hydrodynamics of Pumps* and has edited several other works.

Preface

The subject of multiphase flows encompasses a vast field, a host of different technological contexts, a wide spectrum of different scales, a broad range of engineering disciplines, and a multitude of different analytical approaches. Not surprisingly, the number of books dealing with the subject is voluminous. For the student or researcher in the field of multiphase flow this broad spectrum presents a problem for the experimental or analytical methodologies that might be appropriate for his/her interests can be widely scattered and difficult to find. The aim of the present text is to try to bring much of this fundamental understanding together into one book and to present a unifying approach to the fundamental ideas of multiphase flows. Consequently the book summarizes those fundamental concepts with relevance to a broad spectrum of multiphase flows. It does not pretend to present a comprehensive review of the details of any one multiphase flow or technological context, although reference to books providing such reviews is included where appropriate. This book is targeted at graduate students and researchers at the cutting edge of investigations into the fundamental nature of multiphase flows; it is intended as a reference book for the basic methods used in the treatment of multiphase flows.

I am deeply grateful to all my many friends and fellow researchers in the field of multiphase flows whose ideas fill these pages. I am particularly indebted to my close colleagues Allan Acosta, Ted Wu, Rolf Sabersky, Melany Hunt, Tim Colonius, and the late Milton Plesset, all of whom made my professional life a real pleasure. This book grew out of many years of teaching and research at the California Institute of Technology. It was my privilege to have worked on multiphase flow problems with a group of marvelously talented students, including Hojin Ahn, Robert Bernier, Abhijit Bhattacharyya, David Braisted, Charles Campbell, Steven Ceccio, Luca d'Agostino, Fabrizio d'Auria, Mark Duttweiler, Ronald Franz, Douglas Hart, Steve Hostler, Gustavo Joseph, Joseph Katz, Yan Kuhn de Chizelle, Sanjay Kumar, Harri Kytomaa, Zhenhuan Liu, Beth McKenney, Sheung-Lip Ng, Tanh Nguyen, Kiam Oey, James Pearce, Garrett Reisman, Y.-C. Wang, Carl Wassgren, Roberto Zenit Camacho, and Steve Hostler. To them I owe a special debt. Also, Cecilia Lin devoted many selfless hours to the preparation of the illustrations.

A substantial fraction of the introductory material in this book is taken from my earlier book entitled "Cavitation and Bubble Dynamics" by Christopher Earls Brennen, © 1995 by Oxford University Press, Inc. It is reproduced here by permission of Oxford University Press, Inc.

The original hardback edition was dedicated to my mother, Muriel M. Brennen, whose love and encouragement inspired me throughout my life. This paperback edition is dedicated to another very special woman, my wife, Barbara, who gave me new life and love beyond measure.

Christopher Earls Brennen
California Institute of Technology
December 2003.

Nomenclature

Roman Letters

a	Amplitude of wavelike disturbance
A	Cross-sectional area or cloud radius
$\mathcal{A}$	Attenuation
b	Power law index
Ba	Bagnold number, $\rho_S D^2 \dot{\gamma}/\mu_L$
c	Concentration
c	Speed of sound
c_κ	Phase velocity for wavenumber κ
c_p	Specific heat at constant pressure
c_s	Specific heat of solid or liquid
c_v	Specific heat at constant volume
C	Compliance
C	Damping coefficient
C_D	Drag coefficient
C_{ij}	Drag and lift coefficient matrix
C_L	Lift coefficient
C_p	Coefficient of pressure
C_{pmin}	Minimum coefficient of pressure
d	Diameter
d_j	Jet diameter
d_o	Hopper opening diameter
D	Particle, droplet or bubble diameter
D	Mass diffusivity
D_m	Volume (or mass) mean diameter
D_s	Sauter mean diameter
$D(T)$	Determinant of the transfer matrix $[T]$
$\mathcal{D}$	Thermal diffusivity
e	Specific internal energy

$\mathcal{E}$	Rate of exchange of energy per unit volume
f	Frequency in hertz
f	Friction factor
f_L, f_V	Liquid and vapor thermodynamic quantities
F_i	Force vector
Fr	Froude number
$\mathcal{F}$	Interactive force per unit volume
g	Acceleration due to gravity
g_L, g_V	Liquid and vapor thermodynamic quantities
G_{Ni}	Mass flux of component N in direction i
G_N	Mass flux of component N
h	Specific enthalpy
h	Height
H	Height
H	Total head, $p^T/\rho g$
He	Henry's law constant
Hm	Haberman–Morton number, normally $g\mu^4/\rho S^3$
i, j, k, m, n	Indices
i	Square root of -1
I	Acoustic impulse
$\mathcal{I}$	Rate of transfer of mass per unit volume
j_i	Total volumetric flux in direction i
j_{Ni}	Volumetric flux of component N in direction i
j_N	Volumetric flux of component N
$\grave{k}$	Polytropic constant
k	Thermal conductivity
k	Boltzmann's constant
k_L, k_V	Liquid and vapor quantities
K	Constant
K^*	Cavitation compliance
Kc	Keulegan–Carpenter number
K_{ij}	Added mass coefficient matrix
K_n, K_s	Elastic spring constants in normal and tangential directions
Kn	Knudsen number, $\lambda/2R$
$\mathcal{K}$	Frictional constants
ℓ	Typical dimension
ℓ_t	Turbulent length scale
L	Inertance
$\mathcal{L}$	Latent heat of vaporization
m	Mass
$\dot{m}$	Mass flow rate
m_G	Mass of gas in bubble

m_p	Mass of particle
M	Mach number
M^*	Mass flow gain factor
M_{ij}	Added mass matrix
$\mathcal{M}$	Molecular weight
Ma	Martinelli parameter
n	Number of particles per unit volume
$\dot{n}$	Number of events per unit time
n_i	Unit vector in the i direction
$N(R)$, $N(D)$, $N(v)$	Particle size distribution functions
N^*	Number of sites per unit area
Nu	Nusselt number
p	Pressure
p^T	Total pressure
p_a	Radiated acoustic pressure
p_G	Partial pressure of gas
p_s	Sound pressure level
P	Perimeter
Pe	Peclet number, usually WR/α_C
Pr	Prandtl number, $\rho \nu c_p/k$
q	General variable
q_i	Heat flux vector
Q	General variable
$\mathcal{Q}$	Rate of heat transfer or release per unit mass
$\mathcal{Q}_\ell$	Rate of heat addition per unit length of pipe
r, r_i	Radial coordinate and position vector
r_d	Impeller discharge radius
R	Bubble, particle or droplet radius
R_k^*	Resistance of component, k
R_B	Equivalent volumetric radius, $(3\tau/4\pi)^{\frac{1}{3}}$
R_e	Equilibrium radius
Re	Reynolds number, usually $2WR/\nu_C$
$\mathcal{R}$	Gas constant
s	Coordinate measured along a streamline or pipe centerline
s	Laplace transform variable
s	Specific entropy
S	Surface tension
S_D	Surface of the disperse phase
St	Stokes number
Str	Strouhal number
t	Time

t_c	Binary collision time
t_u	Relaxation time for particle velocity
t_T	Relaxation time for particle temperature
T	Temperature
T	Granular temperature
T_{ij}	Transfer matrix
u_i	Velocity vector
u_{Ni}	Velocity of component N in direction i
u_r, u_θ	Velocity components in polar coordinates
u_s	Shock velocity
u^*	Friction velocity
U, U_i	Fluid velocity and velocity vector in absence of particle
U_∞	Velocity of upstream uniform flow
v	Volume of particle, droplet or bubble
V, V_i	Absolute velocity and velocity vector of particle
V	Volume
$\mathcal{V}$	Control volume
$\dot{V}$	Volume flow rate
w	Dimensionless relative velocity, W/W_∞
W, W_i	Relative velocity of particle and relative velocity vector
W_∞	Terminal velocity of particle
W_p	Typical phase separation velocity
W_t	Typical phase mixing velocity
We	Weber number, $2\rho W^2 R/S$
$\mathcal{W}$	Rate of work done per unit mass
x, y, z	Cartesian coordinates
x_i	Position vector
x	Mass fraction
$\mathcal{X}$	Mass quality
z	Coordinate measured vertically upward

Greek Letters

α	Volume fraction
β	Volume quality
γ	Ratio of specific heats of gas
$\dot{\gamma}$	Shear rate
Γ	Rate of dissipation of energy per unit volume
δ	Boundary layer thickness
δ_d	Damping coefficient
δm	Fractional mass
δ_T	Thermal boundary layer thickness
δ_2	Momentum thickness of the boundary layer

δ_{ij}	Kronecker delta: $\delta_{ij} = 1$ for $i = j$; $\delta_{ij} = 0$ for $i \neq j$
ϵ	Fractional volume
ϵ	Coefficient of restitution
ϵ	Rate of dissipation of energy per unit mass
ζ	Attenuation or amplification rate
η	Bubble population per unit liquid volume
θ	Angular coordinate or direction of velocity vector
θ	Reduced frequency
θ_w	Hopper opening half-angle
κ	Wavenumber
κ	Bulk modulus of compressibility
κ_L, κ_G	Shape constants
λ	Wavelength
λ	Mean free path
λ	Kolmogorov length scale
Λ	Integral length scale of the turbulence
μ	Dynamic viscosity
μ^*	Coulomb friction coefficient
ν	Kinematic viscosity
ν	Mass-based stoichiometric coefficient
ξ	Particle loading
ρ	Density
σ	Cavitation number
σ_i	Inception cavitation number
σ_{ij}	Stress tensor
σ_{ij}^D	Deviatoric stress tensor
$\Sigma(T)$	Thermodynamic parameter
τ	Kolmogorov time scale
τ_i	Interfacial shear stress
τ_n	Normal stress
τ_s	Shear stress
τ_w	Wall shear stress
ψ	Stokes stream function
ψ	Head coefficient, $\Delta p^T / \rho \Omega^2 r_d^2$
ϕ	Velocity potential
ϕ	Internal friction angle
ϕ	Flow coefficient, $j / \Omega r_d$
$\phi_L^2, \phi_G^2, \phi_{L0}^2$	Martinelli pressure gradient ratios
φ	Fractional perturbation in bubble radius
ω	Radian frequency
ω_a	Acoustic mode frequency
ω_i	Instability frequency
ω_n	Natural frequency

ω_{m}	Cloud natural frequencies
ω_{m}	Manometer frequency
ω_{p}	Peak frequency
Ω	Rotating frequency (radians per second)

Subscripts

On any variable, Q:

Q_{o}	Initial value, upstream value or reservoir value
Q_1, Q_2, Q_3	Components of Q in three Cartesian directions
Q_1, Q_2	Values upstream and downstream of a component or flow structure
Q_∞	Value far from the particle or bubble
Q_*	Throat values
Q_{A}	Pertaining to a general phase or component, A
Q_{b}	Pertaining to the bulk
Q_{B}	Pertaining to a general phase or component, B
Q_{B}	Value in the bubble
Q_{C}	Pertaining to the continuous phase or component, C
Q_{c}	Critical values and values at the critical point
Q_{D}	Pertaining to the disperse phase or component, D
Q_{e}	Equilibrium value or value on the saturated liquid/vapor line
Q_{e}	Effective value or exit value
Q_{G}	Pertaining to the gas phase or component
Q_i	Components of vector Q
Q_{ij}	Components of tensor Q
Q_{L}	Pertaining to the liquid phase or component
Q_{m}	Maximum value of Q
Q_{N}	Pertaining to a general phase or component, N
Q_{O}	Pertaining to the oxidant
Q_r	Component in the r direction
Q_{s}	A surface, system or shock value
Q_{S}	Pertaining to the solid particles
Q_{V}	Pertaining to the vapor phase or component
Q_{w}	Value at the wall
Q_θ	Component in the θ direction

Superscripts and Other Qualifiers

On any variable, Q:

Q', Q'', Q^*	Used to differentiate quantities similar to Q
$\bar{Q}$	Mean value of Q or complex conjugate of Q

$\grave{Q}$	Small perturbation in Q
$\tilde{Q}$	Complex amplitude of oscillating Q
$\dot{Q}$	Time derivative of Q
$\ddot{Q}$	Second time derivative of Q
$\hat{Q}(s)$	Laplace transform of $Q(t)$
$\check{Q}$	Coordinate with origin at image point
δQ	Small change in Q
$\mathrm{Re}\{Q\}$	Real part of Q
$\mathrm{Im}\{Q\}$	Imaginary part of Q

Notes

Notation

The reader is referred to Section 1.1.3 for a more complete description of the multiphase flow notation employed in this book. Note also that a few symbols that are only used locally in the text have been omitted from the above lists.

Units

In most of this book, the emphasis is placed on the nondimensional parameters that govern the phenomenon being discussed. However, there are also circumstances in which we shall utilize dimensional thermodynamic and transport properties. In such cases the International System of Units will be employed using the basic units of mass (kg), length (m), time (s), and absolute temperature (K).

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