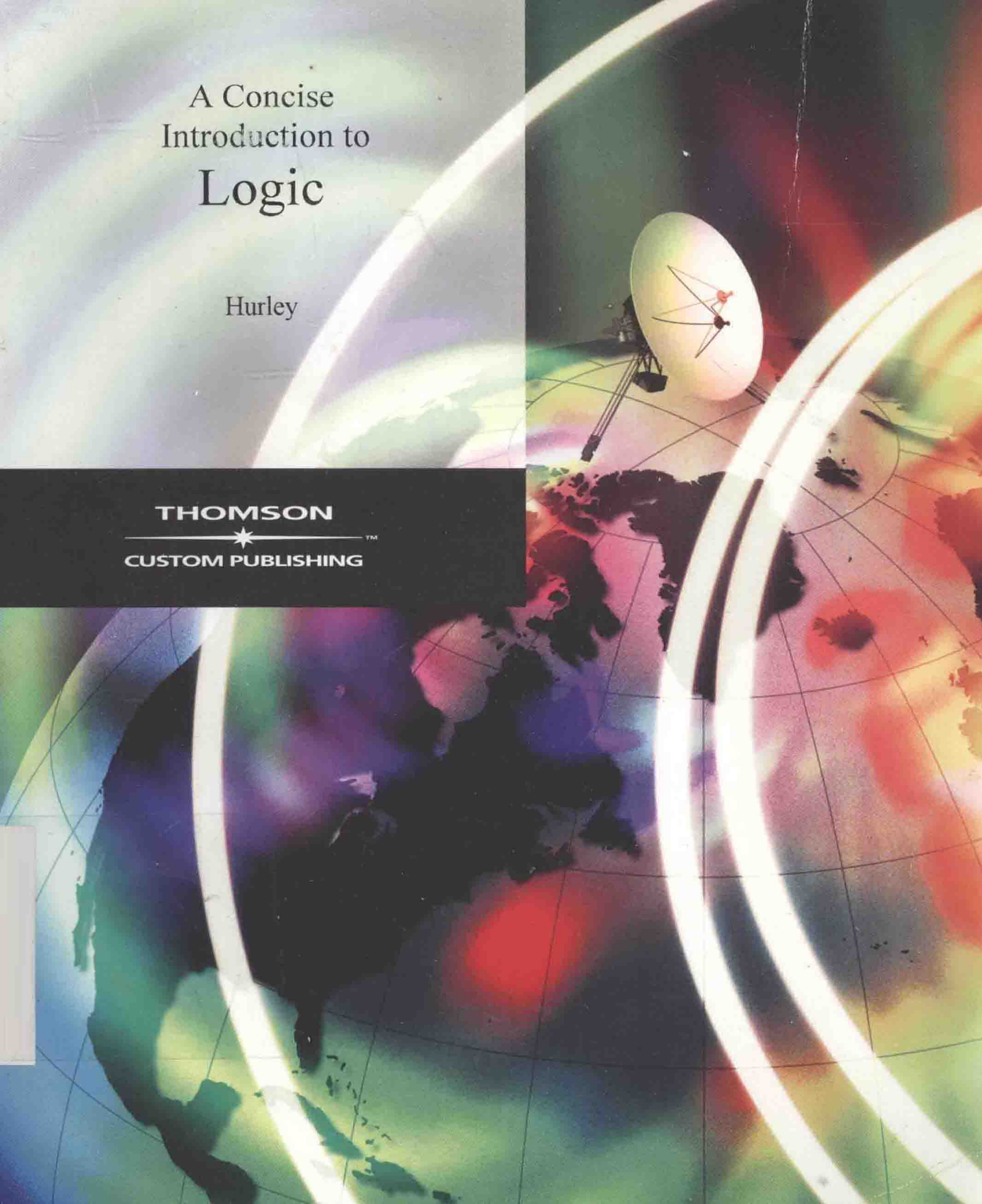
A Concise Introduction to Logic

Hurley



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A Concise Introduction to Logic

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Thomson Learning GmbH
Königswinterer Strasse 418
53227 Bonn, Germany

Thomson Learning Editores
Campos Eliseos 385, Piso 7
Col. Polanco
11560 México D.F. México

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ISBN 0-534-72056-0

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Custom Contents

Linear Equations 1

You have studied three different branches of mathematics: arithmetic, algebra, and geometry. Algebra involves equations and inequalities, while geometry involves shapes and graphs. **Analytic geometry** is a bridge between algebra and geometry. It emphasizes a correspondence between equations and graphs; every equation has a graph, and most graphs have equations. This correspondence is a fruitful one; it allows algebra problems to be attacked with the tools of geometry in addition to the tools of algebra.

Business analysts and economists use a great deal of analytic geometry in their fields. In this chapter we will investigate some of the mathematics used in business and economics. In particular, we will investigate the Central State University's Business Club and its attempt to make money selling T-shirts at football games. This enterprise involves the careful determination of the quantity of shirts to order as well as of the shirts' sales price. Ordering too many shirts or charging too much would result in the club's buying shirts they can't sell, and ordering too few or charging too little would result in not having enough to sell; in either event, the club would lose revenue.

1.0 LINES AND THEIR EQUATIONS

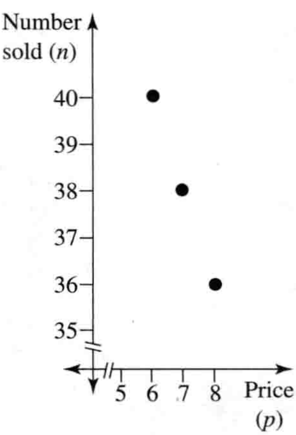
Cartesian Coordinates

Stuart sells sunglasses from a stand at Venice Beach. After experimenting with prices, he discovered (not surprisingly) that the more he charges, the less he sells. For several days Stuart charged \$6 per pair. He kept records on the number of pairs sold and found that he sold an average of 40 pairs per day at that price. This and similar data are given in Figure 1.1.

FIGURE 1.1

Price	Number sold
\$6	40
\$7	38
\$8	36

FIGURE 1.2



Stuart's data can be illustrated graphically by drawing two perpendicular number lines, where the horizontal number line represents price and the vertical number line represents number sold, as shown in Figure 1.2. Notice that both number lines have breaks; the price is always above 5, and the number sold is always above 35.

The upper-left point in Figure 1.2 is directly above 6 on the horizontal number line, so it corresponds to a price of \$6. It is also directly across from 40 on the vertical number line, so it corresponds to 40 pairs sold. If we let p refer to price and n refer to number sold, the upper-left point could be labeled $p = 6$, $n = 40$. A more traditional way of labeling this point is to write $(p, n) = (6, 40)$. This is called an **ordered pair,** because it is a pair of numbers written in a certain order. The order is important; if we write $(40, 6)$, we get the incorrect statement that at a price of \$40, 6 pairs are sold.

The number 6 is called the ***p*-coordinate** of the ordered pair, and the number 40 is called the ***n*-coordinate.** The system of graphing is called **Cartesian coordinates,** in honor of René Descartes, a mathematician and philosopher. (Oddly, while Descartes explored the relationship between algebra and geometry, he neither invented nor utilized the system that bears his name.)

There are two different traditions of the use of letters in this type of situation:

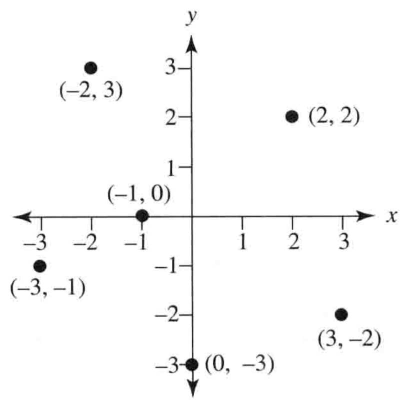
- Use x and y
- Use letters that refer to the quantity being measured, as p refers to price and n refers to number sold above

Descartes started the former tradition; he used letters from the end of the alphabet to represent variables. This tradition is usually adhered to in algebra classes. However, in an application like this one, the latter tradition can serve as a valuable memory aid.

When the "x and y" tradition is followed, we have ***x*-coordinates** and ***y*-coordinates** rather than p-coordinates and n-coordinates. The horizontal axis is called the ***x*-axis,** and the vertical axis is called the ***y*-axis.** In the above discussion, we used p and n rather than x and y, respectively, so the horizontal axis is the p-axis, and the vertical axis is the n-axis. The two axes meet where both p and n are 0; this point, (0, 0), is called the **origin.**

In Figure 1.3, we show x- and y-axes that include both positive and negative values (the negative values are on the left end of the x-axis and on the lower end of the y-axis). The upper-left point corresponds to the ordered pair $(x, y) = (-2, 3)$, since it is above -2 on the x-axis and across from 3 on the y-axis.

FIGURE 1.3



Slope

A line's steepness is measured by its slope. **Slope** (usually denoted by the letter m) is the ratio of the **rise** (the change in y) to the **run** (the change in x):

$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

EXAMPLE 1

a. Calculate the slope of the line between the ordered pairs $(p, n) = (6, 40)$ and $(p, n) = (7, 38)$ from Stuart's sales data.

b. Calculate the slope of the line between the ordered pairs $(p, n) = (7, 38)$ and $(p, n) = (8, 36)$ from Stuart's sales data.

c. Determine what these slopes measure in the context of the problem.

d. Use the slope to predict the number of sunglasses that will sell at \$9. What is this prediction based on?

Solution Here our ordered pairs are (p, n), rather than (x, y). Thus,

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } n}{\text{change in } p}$$

a. In moving from the point $(p, n) = (6, 40)$ to the point $(p, n) = (7, 38)$, p increases from 6 to 7, so p changes by $7 - 6 = 1$. Similarly, n decreases from 40 to 38, so n changes by $38 - 40 = -2$. Thus, the slope is

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } n}{\text{change in } p} = \frac{-2}{1} = -2$$

b. In moving from the point $(p, n) = (7, 38)$ to the point $(p, n) = (8, 36)$, p increases from 7 to 8, so p changes by 1. Similarly, n decreases from 38 to 36, so n changes by -2. Thus, the slope is

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } n}{\text{change in } p} = \frac{-2}{1} = -2$$

c. In the context of the problem, the 1 in the denominator represents a price increase of \$1, and the -2 in the numerator indicates that the number sold decreased by 2. The fact that the slopes are the same in parts (a) and (b) indicates that a price increase of \$1 *consistently* corresponds to a sales decrease of 2 pairs of sunglasses. In the context of the graph, equal slopes means that the steepness doesn't change; that is, the 3 points lie on a line.

d. A charge of \$9 per pair of sunglasses is a \$1 increase above an \$8 price. Each \$1 increase consistently corresponded to a sales decrease of 2 pairs, so sales should decrease to 34 pairs per day, if future sales are consistent with past sales. •

Since a change in y is calculated by subtracting y-values, and a change in x is calculated by subtracting x-values, we have the following formula.

Slope Formula
The **slope** m of the line passing through the points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$

In the preceding formula, each of the symbols is meant to be a memory aid. For example, y_2 means the y-coordinate of the second point, and x_1 means the x-coordinate of the first point.

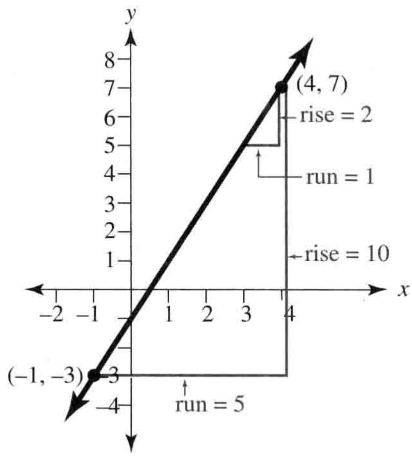
EXAMPLE 2 Find and interpret the slope of the line passing through the points $(-1, -3)$ and $(4, 7)$. Graph the points and the line, and show the rise and the run.

Solution Select $(-1, -3)$ as the first point [so $(x_1, y_1) = (-1, -3)$] and $(4, 7)$ as the second point [so $(x_2, y_2) = (4, 7)$], and substitute into the slope definition.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope definition} \\ &= \frac{7 - -3}{4 - -1} && \text{Substituting} \\ &= \frac{10}{5} = \frac{2}{1} = 2 \end{aligned}$$

This means that in moving from $(-1, -3)$ to $(4, 7)$, the y-value increases by 10 while the x-value increases by 5—in other words, the line rises 10 while it runs 5. And since 10/5 reduces to 2/1, the line rises 2 while it runs 1. The nonreduced slope (10/5) and the reduced slope (2/1) are illustrated in Figure 1.4.

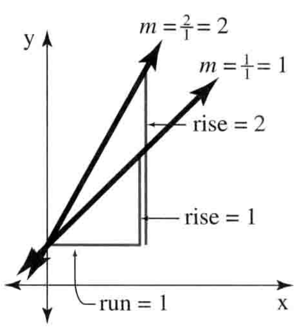
FIGURE 1.4



In Figure 1.5, we show two lines with different rises but the same runs. The steeper line has the larger rise and therefore the larger slope; the less steep line has the smaller rise and therefore the smaller slope.

If the slope of a line is positive (as in Figure 1.5), the line *rises* from left to right and the value of y *increases* as the value of x increases. If the slope of a line is negative (as in

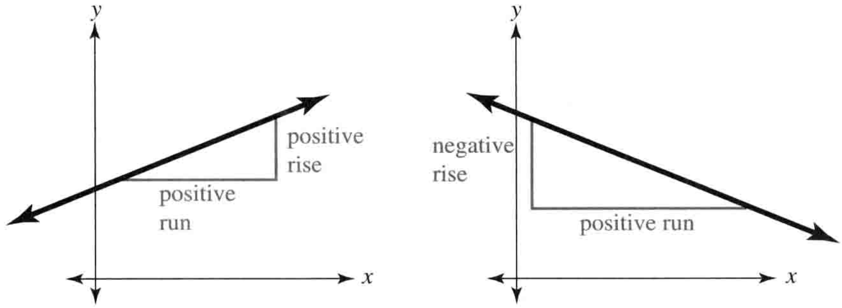
FIGURE 1.5



The steeper line has the larger slope; the less steep line has the smaller slope.

Example 1), the line *falls* from left to right and the value of y *decreases* as the value of x increases. These differences are illustrated in Figure 1.6.

FIGURE 1.6

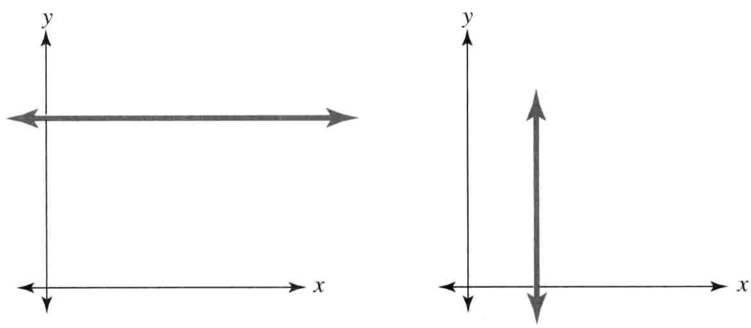


(a) Positive slope:
Line rises from left to right;
y increases as x increases

(b) Negative slope:
Line falls from left to right;
y decreases as x increases

There are two cases in which the slope of a line is something other than a positive or a negative number. A horizontal line has no rise, so its slope is $\frac{\text{rise}}{\text{run}} = \frac{0}{\text{run}} = 0$. A vertical line has no run, so its slope is $\frac{\text{rise}}{\text{run}} = \frac{\text{rise}}{0}$, which is undefined. This is shown in Figure 1.7.

FIGURE 1.7



(a) Horizontal line:
rise is 0, slope is 0

(b) Vertical line:
run is 0, slope is undefined

Finding the Equation of a Line

The Point-Slope Formula

As we saw in Example 1, different points on the same line yield the same slope; a line doesn't change its steepness. If (x_1, y_1) is one particular point on a line, then for *all other* points (x, y) on that line, the slope calculation yields the same result:

$$m = \frac{y - y_1}{x - x_1}$$

If we multiply both sides of this equation by $x - x_1$, we get an important formula.

$$m(x - x_1) = \frac{y - y_1}{x - x_1}(x - x_1)$$

$$m(x - x_1) = y - y_1 \qquad \text{Simplifying}$$

This is called the **point-slope formula,** because it is used to find the equation of a line through a point (x_1, y_1) with slope m.

Point-Slope Formula for a Line

A line through a point (x_1, y_1) with slope m has the equation

$$y - y_1 = m(x - x_1)$$

EXAMPLE 3 Find the equation of the line in Example 2.

Solution We found that the line through $(-1, -3)$ and $(4, 7)$ has slope 2. Substitute $(-1, -3)$ for (x_1, y_1) and 2 for m into the point-slope formula.

$$y - y_1 = m(x - x_1) \quad \text{Point-slope formula}$$
$$y - -3 = 2(x - -1) \quad \text{Substituting}$$
$$y + 3 = 2x + 2 \quad \text{Distributing}$$
$$y = 2x - 1 \quad \text{Simplifying}$$

✔ We can check our answer by verifying that the points $(-1, -3)$ and $(4, 7)$ satisfy our equation.
Substituting $(-1, -3)$ for (x, y):

$$y = 2x - 1 \quad \text{The line's equation}$$
$$-3 = 2(-1) - 1 \quad \text{Substituting}$$
$$-3 = -2 - 1 \quad \text{A true statement ✔}$$

Substituting $(4, 7)$ for (x, y):

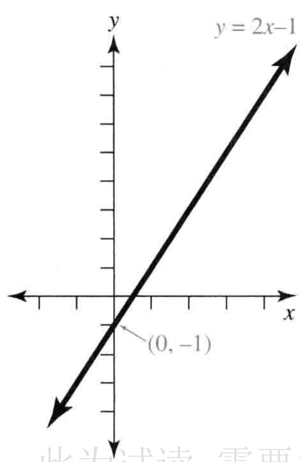
$$y = 2x - 1 \quad \text{The line's equation}$$
$$7 = 2 \cdot 4 - 1 \quad \text{Substituting}$$
$$7 = 8 - 1 \quad \text{A true statement ✔}$$

•

The Slope-Intercept Formula

In Example 3, the simplified equation of the line is $y = 2x - 1$. This is called a **linear equation,** because its graph is a line. The 2 in that equation represents the slope; to find out what the -1 represents, substitute 0 for x:

FIGURE 1.8



$$y = 2x - 1 \quad \text{The line's equation}$$
$$y = 2 \cdot 0 - 1 \quad \text{Substituting 0 for } x$$
$$y = -1 \quad \text{Simplifying}$$

This means that the line goes through the point $(0, -1)$. This point is on the y-axis, so it is called the line's ***y*-intercept** (or just **intercept**), as illustrated in Figure 1.8.

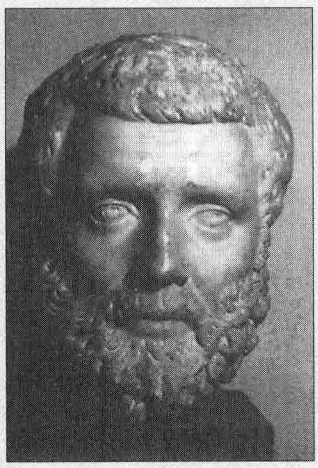
If a line's equation is found with the point-slope formula, that equation can always be simplified so that it is in the form $y = mx + b$. The equation $y = mx + b$ is called the **slope-intercept formula,** because it specifies the slope m and the intercept b. In the equation $y = 2x - 1$ [or $y = 2x + (-1)$], the slope is $m = 2$ and the intercept is $b = -1$.

Historical Note

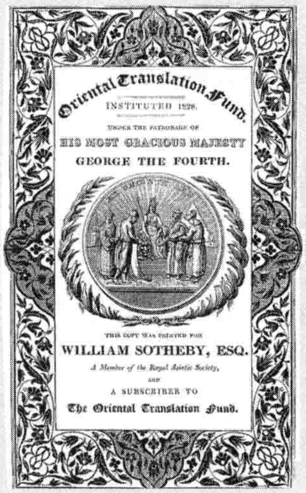
The Inventors of Analytic Geometry

Around 220 B.C., the Greek mathematician Apollonius of Perga wrote an eight-volume work in which he investigated various curves, including circles, ellipses, and parabolas. He used a very early form of algebra to analyze these curves, and his research marks the origin of analytic geometry. Apollonius's work was the standard reference on these curves for almost 2000 years, even though several volumes were lost.

Around 830 A.D., the Arab mathematician Mohammed ibn Musa al-Khowarizmi wrote *Al-Jabr w'al Muqabalah,* in which he discussed linear and quadratic equations. Much of the mathematical knowledge of medieval Europe was derived from the Latin translation of



Apollonius of Perga



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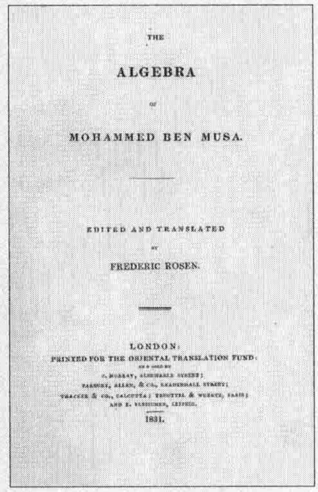

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MOHAMMED BEN MUSA.
EDITED AND TRANSLATED
BY
FREDERIC ROSEN.
LONDON:
PRINTED FOR THE ORIENTAL TRANSLATION FUND:
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Title pages of a translation of al-Khowarizmi's *Al-Jabr w'al Muqabalah*

Slope-Intercept Formula for a Line

A line with slope m and y-intercept b has equation

$$y = mx + b$$

Graphing a Line from Its Equation

The slope-intercept form of a line's equation is especially useful in graphing the line.

EXAMPLE 4

a. Rewrite the equation $x + 2y = 8$ in slope-intercept form.
b. Use part (a) to determine the line's slope and y-intercept.
c. Sketch the line's graph, showing the slope and y-intercept.

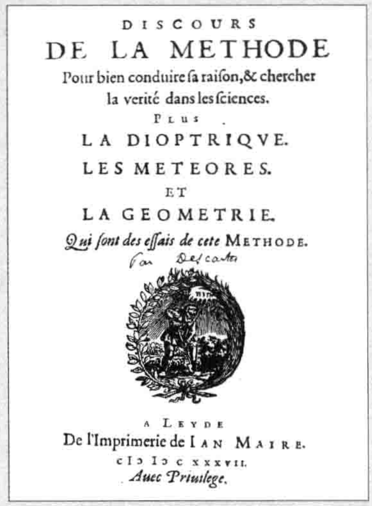
René Descartes

al-Khowarizmi's works. In fact, the word *algebra* comes from the title.

Around 1360, Parisian scholar Nicole Oresme drew graphs of the speed of a falling object. In his graphs he used vertical and horizontal lines (somewhat similar to our x- and y-axes). He called these lines *longitudo* and *latitudo,* which indicate the concept's origins in mapmaking. Oresme's work is one of the earliest appearances of the idea of x- and y-axes and the graphing of a variable quantity.

In 1637, the French mathematician, scientist, and philosopher René Descartes wrote a book on his philosophy of science. This book contained an appendix, *La géométrie,* in which he explored the relationship between algebra and geometry. Descartes's analytic geometry did not especially resemble our modern analytic geometry, which consists of ordered pairs, x- and y-axes, and a correspondence between algebraic equations and their graphs. Descartes used an x-axis, but he did not have a y-axis. Although he knew that an equation in two unknowns determines a curve, he had very little interest in sketching curves. Descartes's algebra, on the other hand, was more modern than that of any of his contemporaries. Algebra had been advancing steadily since the Renaissance, and it found its culmination in Descartes's *La géométrie.*

In 1629, eight years before Descartes wrote *La géométrie,* the French lawyer and amateur mathematician Pierre de Fermat attempted to recreate one of the lost works of Apollonius, using references to that work made by other Greek mathematicians. The restoration of lost works of antiquity was a popular pastime of the upper class at that time. Fermat applied al-Khowarizmi's algebra to Apollonius's analytic geometry, and in so doing created modern analytic geometry. Unfortunately, Fermat never published his work; it was released only after his death, almost 50 years after it was written.



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Title page of Descartes's *La géométrie*

Solution **a.** To rewrite the equation in slope-intercept form, solve the equation for y.

$$x + 2y = 8$$

$$2y = -x + 8 \qquad \text{Subtracting } x$$

$$y = \frac{1}{2}(-x + 8) \qquad \text{Multiplying by 1/2}$$

$$y = \frac{-1}{2}x + 4 \qquad \text{Distributing}$$

The equation is now in slope-intercept form.

b. By comparing the slope-intercept formula with the above equation, we can find our line's slope and y-intercept:

$$y = mx + b$$

$$y = \frac{-1}{2}x + 4$$

The slope is $m = -1/2$, and the y-intercept is $b = 4$.

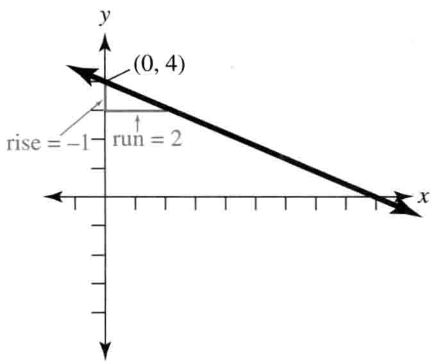
c. To sketch the graph of the line, first use the fact that the y-intercept is $b = 4$. This means that the line intercepts the y-axis at 4 [i.e., at the point $(0, 4)$]. Next, use the fact that the slope is $m = -1/2$.

$$m = \frac{-1}{2}$$

$$\frac{\text{rise}}{\text{run}} = \frac{-1}{2}$$

From the point $(0, 4)$, rise -1 (go 1 down) and run 2 (go 2 to the right). The graph is shown in Figure 1.9.

FIGURE 1.9



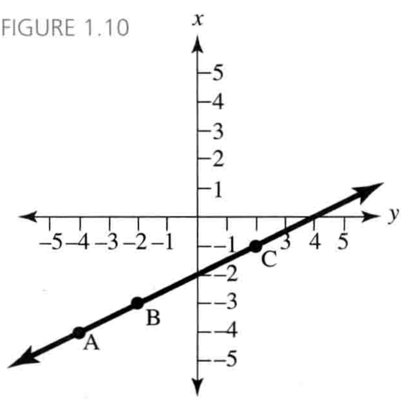
1.0

EXERCISES

1. a. Find the rise, the run, and the slope of the line between $(x_1, y_1) = (3, 7)$ and $(x_2, y_2) = (5, 11)$.
   b. Find the rise, the run, and the slope of the line between $(x_1, y_1) = (5, 11)$ and $(x_2, y_2) = (3, 7)$.
   c. What conclusion can you make regarding the computation of the slope of a line between two given points?
   d. Draw the line between $(3, 7)$ and $(5, 11)$. Show the rise and run.

2. Find the slope between the given points, as illustrated in Figure 1.10.
   a. The line through points A and B.
   b. The line through points B and C.
   c. The line through points A and C.

FIGURE 1.10



In Exercises 3–4:

a. Plot the points corresponding to the two given ordered pairs.
b. Find the slope of the line joining these two points.
c. Graph the line.
d. On your graph, show the rise and the run from the nonreduced slope, as well as the rise and the run from the reduced slope, as done in Example 2.

3. (1, 4) and (3, 10)

4. (1, −1) and (4, −7)

In Exercises 5–10:

a. Plot the points corresponding to the three given ordered pairs.
b. Find the slope of the line joining the points given by the first and second ordered pairs.
c. Find the slope of the line joining the points given by the second and third ordered pairs.
d. Are the three points in a line? Why or why not?

5. (1, 3), (2, 5), (3, 7)

6. (−2, 1), (0, 6), (2, 11)

7. (−5, −6), (−1, −9), (3, −11)

8. (−7, 5), (−3, 3), (0, 1)

9. (−8, −11), (−4, −8), (4, −2)

10. (3, 1), (−5, −7), (−9, −11)

In Exercises 11–14:

a. Find the slope of the line joining the points given by the two ordered pairs.
b. Use the slope to find a third point on that same line.
c. Draw the line. Show the two given points as well as the point found in part (b).
NOTE: A line consists of many points, so there are many different correct answers to part (b). For this reason, answers to part (b) are not given at the back of the book. Instead, you will check these answers in Exercises 19–22.

11. (1, 5) and (3, −3)

12. (2, −8) and (−3, 4)

13. (−6, −3) and (2, 4)

14. (−8, −4) and (−2, −9)

15. Jim and Barry's, a nationwide chain of ice cream stores, has found that it sells an average of 42,000 cones per summer day when it charges $1.30 each. When it increased the price to $1.40, average sales dropped to 41,000.
a. Express these data as two ordered pairs (p, n).
b. Find the slope of the line between these two ordered pairs.
c. Determine what this slope measures in the context of the problem.
d. Use the slope to predict the number of cones that will sell at $1.50.
e. Use the slope to predict the number of cones that will sell at $1.35.

16. The owners of Blondie's Pizzas found that they sell an average of 120 slices of pizza per day when they charge $1.10 each. When they decreased the price to $1.00, average sales increased to 140.
a. Express these data as two ordered pairs (p, n).
b. Find the slope of the line between these two ordered pairs.
c. Determine what this slope measures in the context of the problem.
d. Use the slope to predict the number of slices that will sell at $.90.
e. Use the slope to predict the number of slices that will sell at $1.05.

17. Why are the results of Exercises 15 and/or 16 parts (d) and (e) merely predictions and not guaranteed occurrences? What assumptions are these predictions based on?

18. a. Find the equation of the line in Exercise 1, using (3, 7) for (x_1, y_1).
b. Check your answer as shown in Example 3.
c. Find the equation of the line in Exercise 1, using (5, 11) for (x_1, y_1).
d. What conclusion can you make regarding the computation of the equation of a line between two given points?

In Exercises 19–22:

a. Find the equation of the line through the two given points.
b. Check your answer to part (a) by verifying that the two given points satisfy the equation, as shown in Example 3.
c. Check your answers to part (b) of Exercises 11–14 by verifying that the points you found there satisfy the equations.
NOTE: Answers are not given in the back of the book.

19. (1, 5) and (3, −3) *(see Exercise 11)*

20. (2, −8) and (−3, 4) *(see Exercise 12)*

21. (−6, −3) and (2, 4) *(see Exercise 13)*

22. (−8, −4) and (−2, −9) *(see Exercise 14)*

In Exercises 23–26:

a. Find the slope of the line through the two given points.
b. Find the equation of the line through the two given points. What is the line's intercept?
c. Check your answers to parts (a) and (b) by verifying that the two given points and the intercept satisfy the equation.
d. Graph the line whose equation was found in part (c). Show the two given points, the slope, and the intercept.
NOTE: Answers are not given in the back of the book.

23. (3, −5) and (8, −2) **24.** (−5, −7) and (1, 1)

25. (8, 11) and (12, 11) **26.** (3, 9) and (−2, 4)

27. Use the information given in Exercise 15 to do the following.
a. Find the equation of the line through the two given points, using the letters p and n rather than x and y, respectively.
b. Predict the number of cones that will sell at $1.45 each.
c. Graph the line for $p \geq 0$ and $n \geq 0$. Why must both p and n be greater than 0? (Discuss why n can be 0 as well as why n cannot be negative.)

28. Use the information given in Exercise 16 to do the following.
a. Find the equation of the line through the two given points, using the letters p and n rather than x and y, respectively.
b. Predict the number of pizzas that will sell at $1.25 each.
c. Graph the line for $p \geq 0$ and $n \geq 0$. Why must both p and n be greater than 0? (Discuss why n can be 0 as well as why n cannot be negative.)

29. Jim and Barry's, the chain of ice cream stores in Exercise 15, has found that the cost of making and selling 1000 cones per day at a typical store is $900, and the cost of making and selling 800 cones per day is $780.
a. Express these data as two ordered pairs (n, c), where n is the number sold and c is the corresponding cost of production.
b. Find the slope, and interpret it in the context of the problem.
c. Find the equation of the line through these two points, using the letters n and c rather than x and y, respectively.
d. Graph the line, for $n \geq 0$.
e. Find the intercept, and interpret it in the context of the problem.

30. Blondie's Pizzas, the pizza store in Exercise 16, has found that the cost of making and selling 1100 slices per day is $1100, and the cost of making and selling 1000 slices per day is $1030.
a. Express these data as two ordered pairs (n, c), where n is the number sold and c is the corresponding cost.
b. Find the slope, and interpret it in the context of the problem.
c. Find the equation of the line through these two points, using the letters n and c rather than x and y, respectively.
d. Graph the line, for $n \geq 0$.
e. Find the intercept, and interpret it in the context of the problem.

31. The average life expectancy of a newborn female in the United States in 1970 was 74.7 years; in 1990, the average was 78.8 years. (*Source:* National Center for Health Statistics)
a. Express these data as two ordered pairs (t, e), where t is the year and e is the average life expectancy.
b. Find the slope, and interpret it in the context of the problem.
c. Find the equation of the line through these two points, using the letters t and e rather than x and y, respectively.
d. Graph the line, for $t \geq 1900$.
e. Predict when the average life expectancy will be 85 years.
f. Predict the average life expectancy in the year 2000.
g. What assumptions are the calculations in (b) through (f) based on?

32. The average life expectancy of a newborn male in the United States in 1970 was 67.1 years; in 1990, the average was 72.0 years. (*Source:* National Center for Health Statistics)
a. Express these data as two ordered pairs (t, e), where t is the year and e is the average life expectancy.
b. Find the slope, and interpret it in the context of the problem.
c. Find the equation of the line through these two points, using the letters t and e rather than x and y, respectively.
d. Graph the line, for $t \geq 1900$.
e. Predict when the average life expectancy will be 85 years.
f. Predict the average life expectancy in the year 2000.
g. What assumptions are the calculations in (b) through (f) based on?

In Exercises 33–36:

a. Rewrite the equation in slope-intercept form.
b. Use part (a) to determine the line's slope and y-intercept.
c. Sketch the line's graph.

33. $3x + y = 9$ **34.** $4x - 2y = 16$

35. $5x + 7y = 35$ **36.** $4x - 6y = 12$

37. **a.** Sketch the line through the points (1, 3) and (2, 3).
b. Find the slope of the line.
c. Find the equation of the line.
d. List three other points on the line.
e. How could you have found the equation of the line without using any formulas?

38. **a.** Sketch the line through the points (2, 1) and (2, 3).
b. Find the slope of the line.
c. List three other points on the line.
d. Find the equation of the line.
HINT: See Exercise 37(e).

In Exercises 39–43, find (a) the slope and (b) the equation of the line through the two given points.

39. (5, 7) and (8, 7) **40.** (6, 2) and (6, 3)

41. (−3, 1) and (−3, 2) **42.** (1, −3) and (−2, −3)

43. (2, 11) and (2, 14)

44. **a.** On one set of axes, draw lines with the following slopes (where each line goes through the origin): 3, 2, 1, 1/2, 1/3, 0, −1/3, −1/2, −1, −2, −3. For each line, show the rise and the run.

b. Which line(s) is/are the steepest?
c. Which line(s) is/are the least steep?
d. If you were given the slopes of two different lines, how could you tell which line is steepest? How could you tell whether either line rises from left to right or falls from left to right?

45. Draw a line with the given slope and intercept, and give the equation of that line.
a. positive slope and positive intercept
b. positive slope and zero intercept
c. positive slope and negative intercept
d. zero slope and positive intercept
e. zero slope and zero intercept
f. zero slope and negative intercept
g. negative slope and positive intercept
h. negative slope and zero intercept
i. negative slope and negative intercept
j. undefined slope
HINT: See Exercises 38–43

46. Answer the following using complete sentences. Why are Cartesian coordinates named in honor of René Descartes? In honor of which mathematician would it be more accurate to name them?

Graphing Lines on a Graphing Calculator

As their name implies, graphing calculators can graph lines. They follow the "x and y" tradition discussed earlier; they don't accept letters that refer to the quantity being measured, such as p for price and n for number sold. Furthermore, the equation must be given in slope-intercept form (or some unsimplified versions of the slope-intercept form).

The Graphing Buttons on a TI Graphing Calculator

The graphing buttons on a TI graphing calculator are all at the top of the keypad, directly under the screen. The labels on these buttons vary a little from model to model, but their uses are the same. The button labels and their uses are listed in Figure 1.11.

FIGURE 1.11

	Use this button to tell the calculator:				
Calculator Model	**What to graph**	**What part of the graph to draw**	**To zoom in or out**	**To give the coordinates of a highlighted point**	**To draw the graph**
TI-82/83:	Y=	WINDOW	ZOOM	TRACE	GRAPH
TI-85/86: Labels on buttons	M1 F1	M2 F2	M3 F3	M4 F4	M5 F5
TI-85/86: Labels on screen in graphing mode*	y(x) =	RANGE or WIND	ZOOM	TRACE	GRAPH

*__TI-85/86 users:__ Your calculator is different from the other TI models in that its graphing buttons are labeled "F1" through "F5" ("M1" through "M5" when preceded by the 2nd button). The use of these buttons varies, depending on what you're doing with the calculator. When these buttons are active, their uses are displayed at the bottom of the screen.