

THE VNR
CONCISE
ENCYCLOPEDIA OF
MATHEMATICS SECOND
EDITION

W. Gellert · S. Gottwald
M. Hellwich · H. Kästner · H. Küstner

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Introduction

The great achievements of technology in all its forms, which deeply influence the life of every human being, have led to a widespread recognition of the importance of mathematics: everybody knows, or at least believes, that without mathematics these achievements in their entirety could not have come about. Interest in mathematics has therefore grown steadily, and with it the need for information about this science.

Now in many respects mathematics is an exceptional science, in particular, as regards the presentation of its problems and results. While in medicine, zoology, botany, geography and geology, or in languages, history, astronomy, a scholar, fully equipped with the knowledge of his time, can explain to a layman the majority of his problems and results, perhaps even his methods or the fundamental principles of his special interests, in such a way that he succeeds in conveying an impression of the contents of this field, in present-day chemistry and physics this is far more difficult – and in mathematics well-nigh impossible. Not only has the volume of results grown phenomenally, but the problems are so difficult to treat and lie so deep that even mathematicians can have no more than a superficial view of the whole of mathematics.

One tries to counteract the fragmentation of mathematics into many *special branches* by extracting as far as possible from various domains common features, which sometimes do not lie at all close to the surface, and by creating from them a new and even more abstract theory: in just this way new links are forged between at first sight widely diverging directions. This process can be regarded as a repeated abstraction: whereas the basic disciplines such as algebra and geometry have their origin in abstractions from everyday experience, one arrives at such a unifying theory by further abstractions, for example, from algebra and geometry: and under certain circumstances such abstracting processes can be repeatedly piled on top of one another. Here 'abstract' has to be understood in the literal meaning of the word as 'removing', as leaving aside everything inessential for the context in question or for a particular purpose; for example, ignoring colour in geometric figures, which may very well play a role in ornaments.

From all this it follows that it is quite impossible to give a layman even a glimpse of the whole of contemporary mathematics. Here a *layman* is not only one whose knowledge is limited to the normal contents of a school syllabus. Even a mathematician with a diploma or a B. Sc., even a teacher of mathematics, has to be regarded as a layman in many special branches. It is simply impossible to acquire specialized knowledge of all branches of mathematics in three or four years of study. Therefore this book cannot have the ambition of imparting knowledge in all special fields of mathematics – restriction is essential.

In its historical development mathematics first proceeded in quite a naive manner. It started out from the numbers 1, 2, 3, ... and from the intuitively obvious *figures* of geometry such as points, segments, lines, planes in space, angles, triangles, circles, etc.; gradually it ascended to more complex formations, with the realm of numbers and that of figures not developing as separate entities, but connected through the notion of *measuring*. It was in this development, progressing from the intuitively simple and obvious to more complicated problems, that mathematics was built up, for example, in Babylonia and Egypt; astonishing achievements were reached in astronomy, such as the prediction of lunar eclipses. But it was the Greeks who lifted mathematics to a completely new level of development when they felt compelled not always to forge ahead, but also to reflect: what is it that one does in pursuing mathematics? The result was that through them mathematics became a science in the present-day sense. On the one hand, they recognized that a *proof* consists in reducing a mathematical proposition to other known facts by the simplest logical conclusions, supported and made convincing sufficiently often by evidence or experience. On the other hand, they realized that such a reduction process cannot go on indefinitely but only as far as certain simplest properties of numbers or figures, which appear secure by virtue of intuition or experience.

In this way they compiled for the first time consciously a system of *fundamental facts*, for example, that there is precisely one straight line passing through two points, and they created the foundation of *logic*. Together these two features lead to a systematic build-up of geometry, rising from the simple to the complex.

For a long time this Euclidean geometry, apart from a few minor supplements, remained the model of a science. However, no comparable attempt was made for about two thousand years to