THE VNR CONCISE ENCYCLOPEDIA OF MATHEMATICS

SECOND EDITION

W. Gellert · S. Gottwald
M. Hellwich · H. Kästner · H. Küstner

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Editors

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Colours are used extensively to help the reader. Important definitions and groups of formulae are on a *yellow* background, examples on *blue*, and theorems on *red*. The course of more complicated calculations is indicated by red arrows. Also, in the illustrations in the text colours show up the essential features. Ample examples help to make general statements understandable. Frequently the numerical calculations have been arranged separately so that a problem can be read as an explanatory text, without reference to calculations, while the latter can be regarded as worked examples with explicit details. Physical units, which occur in some examples, are given in the SI-system, which is coming more and more into legal and practical use. Everyday examples are given in everyday units, both metric and others.

A systematic subdivision of the material, many brief section headings, and tables are meant to provide the reader with quick and reliable orientation. The detailed index to the book gives an easy access to specific questions.

In the plates at the end numerous photographs and colour plates help to make the material more vivid and provide interesting glimpses of the history of mathematics.

We thank the authors of the various chapters, specially to acceding to our request for generally understandable diction even at the risk of deviating from the usual terminology. Above all in the brief reports on special topics many an author has found it difficult to be content with mere indications about a topic in which he is an expert.

Our particular thanks are due to our advisors, Professor K. A. Hirsch, Queen Mary College, University of London, and Professor H. Reichardt, Section for Mathematics, Humboldt University of Berlin. They have worked untiringly for the improvement of the book and have helped to create a work which is a reliable source of information for every user and should convince everyone that mathematics is essentially a simple and learnable discipline.

The Editors and the Publishers

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Introduction

The great achievements of technology in all its forms, which deeply influence the life of every human being, have led to a widespread recognition of the importance of mathematics: everybody knows, or at least believes, that without mathematics these achievements in their entirety could not have come about. Interest in mathematics has therefore grown steadily, and with it the need for information about this science.

Now in many respects mathematics is an exceptional science, in particular, as regards the presentation of its problems and results. While in medicine, zoology, botany, geography and geology, or in languages, history, astronomy, a scholar, fully equipped with the knowledge of his time, can explain to a layman the majority of his problems and results, perhaps even his methods or the fundamental principles of his special interests, in such a way that he succeeds in conveying an impression of the contents of this field, in present-day chemistry and physics this is far more difficult – and in mathematics well-nigh impossible. Not only has the volume of results grown phenomenally, but the problems are so difficult to treat and lie so deep that even mathematicians can have no more than a superficial view of the whole of mathematics.

One tries to counteract the fragmentation of mathematics into many *special branches* by extracting as far as possible from various domains common features, which sometimes do not lie at all close to the surface, and by creating from them a new and even more abstract theory: in just this way new links are forged between at first sight widely diverging directions. This process can be regarded as a repeated abstraction: whereas the basic disciplines such as algebra and geometry have their origin in abstractions from everyday experience, one arrives at such a unifying theory by further abstractions, for example, from algebra and geometry: and under certain circumstances such abstracting processes can be repeatedly piled on top of one another. Here '*abstract*' has to be understood in the literal meaning of the word as 'removing', as leaving aside everything inessential for the context in question or for a particular purpose; for example, ignoring colour in geometric figures, which may very well play a role in ornaments.

From all this it follows that it is quite impossible to give a layman even a glimpse of the whole of contemporary mathematics. Here a *layman* is not only one whose knowledge is limited to the normal contents of a school syllabus. Even a mathematician with a diploma or a B. Sc., even a teacher of mathematics, has to be regarded as a layman in many special branches. It is simply impossible to acquire specialized knowledge of all branches of mathematics in three or four years of study. Therefore this book cannot have the ambition of imparting knowledge in all special fields of mathematics – restriction is essential.

In its historical development mathematics first proceeded in quite a naive manner. It started out from the *numbers* 1, 2, 3, ... and from the intuitively obvious *figures* of geometry such as points, segments, lines, planes in space, angles, triangles, circles, etc.; gradually it ascended to more complex formations, with the realm of numbers and that of figures not developing as separate entities, but connected through the notion of *measuring*. It was in this development, progressing from the intuitively simple and obvious to more complicated problems, that mathematics was built up, for example, in Babylonia and Egypt; astonishing achievements were reached in astronomy, such as the prediction of lunar eclipses. But it was the Greeks who lifted mathematics to a completely new level of development when they felt compelled not always to forge ahead, but also to reflect: what is it that one does in pursuing mathematics? The result was that through them mathematics became a sience in the present-day sense. On the one hand, they recognized that a *proof* consists in reducing a mathematical proposition to other known facts by the simplest logical conclusions, supported and made convincing sufficiently often by evidence or experience. On the other hand, they realized that such a reduction process cannot go on indefinitely but only as far as certain simplest properties of numbers or figures, which appear secure by virtue of intuition or experience.

In this way they compiled for the first time consciously a system of *fundamental facts*, for example, that there is precisely one straight line passing through two points, and they created the foundation of *logic*. Together these two features lead to a systematic build-up of geometry, rising from the simple to the complex.

For a long time this Euclidean geometry, apart from a few minor supplements, remained the model of a science. However, no comparable attempt was made for about two thousand years to