THE Development of Logic

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THE DEVELOPMENT OF LOGIC

Haec autem est dialectica, cui quidem omnis veritatis seu falsitatis discretio ita subiecta est ut omnis philosophiae principatum dux universae doctrinae atque regimen possideat. ABELARD

Lockius aliique qui spernunt non intelligunt. LEIBNIZ

Neque enim leges intellectui aut rebus damus ad arbitrium nostrum, sed tanquam scribae fideles ab ipsius naturae voce latas et prolatas excipimus et describimus. CANTOR

Inimicus Plato, sed magis inimica falsitas. TARSKI

PREFACE

As its name indicates, this book is an account of the growth of logic, rather than an attempt to chronicle all that past scholars, good and bad, have said about the science. For the sake of continuity, and in order to give historical perspective to our story, my wife and I have included some references to work which does not deserve to be remembered for its own sake; and occasionally we have allowed ourselves to indulge an antiquarian curiosity, when we thought that the result might be of some interest to others. But our primary purpose has been to record the first appearances of those ideas which seem to us most important in the logic of our own day. Such a programme is based on judgements of value, and we realize that our selection of material and still more our comments, especially in the later chapters, may seem eccentric to some readers. In defence of our undertaking we can only say that we have followed the plan which our interests suggested, and that we could not have written in any other way.

The idea of attempting a history of logic on these lines occurred to me first in 1947 when I was asked to give a lecture in Cambridge on the centenary of Boole's *Mathematical Analysis of Logic*. Part of that lecture survives in Chapter VI of this book, where it is reprinted from *Mind*, lvii (1948), by permission of the editor. During the next ten years I gave to the project all the time I could spare from teaching and other more urgent work, and by 1957 I had a draft which covered most of the field but in a very uneven fashion. Some of the material now contained in Chapter IX, § 3, was published under the title 'The Province of Logic' in *Contemporary British Philosophy*, Third Series (George Allen & Unwin, 1956), from which it is reprinted here by permission of the publishers, but much of what I had written seemed to me unsatisfactory. As might be expected, the earlier chapters, which I had put together quickly in an impressionistic style, were those in need of most revision, and I soon came to the conclusion that they would have to be completely rewritten on a larger scale. At this stage the Leverhulme Trustees gave me a grant to make possible two terms' special leave from my tutorial duties in Exeter College. I am very grateful for their generous help, which enabled me to finish the chapters now numbered IV, V, and VI. But I am afraid that even so I might have lost heart, if my wife had not at the same time agreed to take charge of the Greek part and then devoted to it not only a term of sabbatical leave but also most of her leisure during the next two and a half years. Apart from the concluding section, on the Stoic System of Inference Schemata, the first three chapters, as they stand, are her work. In addition she has helped me with advice about the treatment of many subjects in the later chapters.

We have to thank Mr. John Lemmon, Mr. Brian McGuinness, Dr. Lorenzo Minio-Paluello, Dr. Richard Walzer, and Professor Hao Wang for reading parts of the work and suggesting corrections or improvements. Professor Arthur Prior, who read the whole in typescript, gave us a great many useful comments, and we are very grateful for the generosity with which he has allowed us to profit from his wide knowledge of the history of logic. Although we have gladly accepted most of the advice we have received, we have sometimes persisted in going our own way, and none of our friends are to be held responsible for faults that remain.

W. K.

April 1960

For the fifth impression we have corrected some surviving mistakes and misprints, re-worded some passages in the hope of achieving greater clarity, and added some references. But the chief novelty is an appendix in which we translate the Latin quotations of Chapter IV. Many of these improvements, like those in earlier impressions, are due to the suggestions of readers, whom we thank for their kindness in writing to us.

W. K.

May 1971

CONTENTS

I

THE BEGINNINGS

1. *The Notion of Validity*

LOGIC is concerned with the principles of valid inference; and it is certain that men made inferences and criticized the inferences of others long before the time of Aristotle. This is not enough in itself to justify us in saying that there must have been a beginning of logic before the time of Aristotle; for men may perform various activities correctly (e.g. talk English) without formulating the rules for those activities explicitly. But it is clear from what we find in Plato and Aristotle and other sources that Greek philosophers had begun to discuss the principles of valid inference before Aristotle wrote those works which came to be known as the *Organon*. It is the purpose of this chapter to trace, as far as the evidence allows, the development of logical thought before Aristotle. This is not easy on the basis of the evidence alone, but it is possible to form reasonable conjectures about the origins of logical reflection and to show that these are supported to a certain extent by the evidence.

Since logic is not simply valid argument but the reflection upon principles of validity, it will arise naturally only when there is already a considerable body of inferential or argumentative material to hand. Not every type of discourse provokes logical inquiry. Pure story-telling or literary discourse, for example, does not provide a sufficient amount of argumentative material. It is those types of discourse or inquiry in which *proof* is sought or demanded that naturally give rise to logical investigation; for to prove a proposition is to infer it validly from true premisses. The conditions of proof are two: true premisses, or starting-points, and valid arguments. It is not easy to tell how soon it was realized that the two conditions are independent, but this was perfectly clear to Aristotle when he drew the distinction between apodeictic and dialectical reasoning in the *Topics*[1] and again in the *Prior Analytics*.[2] The latter passage is worth quoting in full because it throws light on the context in which the distinction was first drawn.

'The demonstrative premiss (ἀποδεικτικὴ πρότασις) differs from the dialectical, because the demonstrative is the assumption of one of

[1] *Topica*, i. 1 (100^{a}25–30).

[2] *An. Pr.* i. 1 (24^{a}22–24^{b}12).

a pair of contradictory propositions (for the man who demonstrates assumes something and does not ask a question), but the dialectical premiss is a question as to which of two contradictories is true. But this makes no difference to the fact that there is a syllogism in each case. Both the man who demonstrates and the man who asks a question reason assuming that some predicate belongs or does not belong to something. So that a syllogistic premiss is simply the affirmation or denial of some predicate of some subject, as we have said, but it is demonstrative if it is true and accepted because deduced from basic assumptions, while a dialectical premiss is for the enquirer a question as to which of two contradictories is true and for the reasoner the assumption of some plausible or generally held proposition.'

The distinction between demonstrative and dialectical argument is introduced here by reference to the activities in which, according to Aristotle, the statement of the premisses properly plays its part. The demonstrative premiss is laid down by a teacher in the course of developing his subject. It is the premiss of what Aristotle calls in the *De Sophisticis Elenchis* a didactic argument.[1] The dialectical premiss, on the other hand, is one adopted in debate for the sake of argument. From the logical point of view, however, the important distinction is that the demonstrative premiss is true and necessary, while the dialectical premiss need not be so.[2] In demonstration we start from true premisses and arrive with necessity at a true conclusion: in other words, we have proof. In dialectical argument, on the other hand, the premisses are not known to be true, and there is no necessity that the conclusion be true. If there is an approach to truth through dialectic, it must be more indirect.

We may distinguish three types of discourse in which proof is sought and demanded. In pure mathematics we seek to prove abstract *a priori* truths, in metaphysics we seek to prove very general propositions about the structure of the world, and in everyday argument, especially political or forensic argument, we look for proofs of contingent propositions. Of these three only mathematics answers obviously to Aristotle's description of demonstrative argument, and mathematics provides the larger number of his illustrations of demonstration. Since it is likely that the first logical inquiries were stimulated by reflection on such reasoning, we shall consider this first.

2. *Geometrical Demonstration*

It seems probable that the notion of demonstration attracted attention first in connexion with geometry. It is well known that

[1] *De Soph. Elench.* 2 (165^b1).

[2] *An. Post.* i. 19 (81^b18).

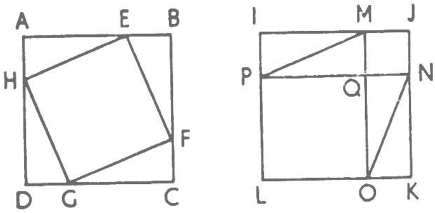
the Egyptians had discovered some truths of geometry empirically, e.g. a formula for the volume of a truncated pyramid, and the name 'geometry', which originally meant the same as 'land measurement', shows how the study was considered when it was introduced to Greece. The great achievement of the Greeks was to replace this empirical study by a demonstrative *a priori* science. Some stories give Thales (640–546 B.C.) the credit for *proving* the first theorem in geometry,[1] but the systematic study of the science seems to have begun in the Pythagorean school.

Pythagoras is said to have been born at Samos some time in the first half of the sixth century B.C. and to have emigrated to Croton, a Greek city in southern Italy, where he founded an ascetic order and taught the doctrine of metempsychosis. Here we have the origin of the notion of philosophy as a way of life. It is possible, indeed, that the word 'philosophy', meaning originally love of wisdom, was coined in the Pythagorean school to describe the way which the Master had shown when he called himself φιλόσοφος. Here also we have the beginning of intellectualism, the doctrine that the most important faculty of man is his intellect and that truths which can be learnt only by the use of the intellect are in some way more noble and fundamental than those learnt by observation. We may regret the evils of a *priori* metaphysics which were brought into the world by this doctrine, but it is only fair to say that it gained influence because the discovery of *a priori* knowledge naturally excites the admiration of intelligent men. For a while the order of Pythagoras was dominant in Croton, but there was presently a reaction, and Pythagoras went to live at Metapontum, where he died at the end of the sixth or the beginning of the fifth century. It has been conjectured that the final downfall of the order as a political force came about 450 B.C., and that it was this collapse which brought Pythagoreans to the mainland of Hellas. If we may take Plato's *Phaedo* as evidence, there were Pythagoreans living at Thebes when Socrates died in 399 B.C., but they were men who had dropped a good deal of the religious teaching of their founder, and in particular his theory of the transmigration of souls, in order to concentrate attention on the scientific side of the tradition.

Let us now consider what is involved in the customary presentation of elementary geometry as a deductive science. First of all, certain propositions of the science must be taken as true without demonstration; secondly, all the other propositions of the science must be derived from these; and, thirdly, the derivation

[1] Proclus, *In Primum Euclidis Elementorum Librum Commentarii*, ed. Friedlein, p. 65.

must be made without any reliance on geometrical assertions other than those taken as primitive, i.e. it must be *formal* or independent of the special subject matter discussed in geometry. From our point of view the third is the most important requirement: elaboration of a deductive system involves consideration of the relation of logical consequence or entailment. Historically geometry was the first body of knowledge to be presented in this way, and ever since Greek times it has been regarded as the paradigm of deductive system-building. Hence, for example, the title of Spinoza's work, *Ethica more geometrico demonstrata*. It would be wrong, however, to suggest that all this was clear to Pythagoras and his immediate followers. On the contrary, we must suppose that for the first Greek geometers any procedure was admissible which helped them to 'see' the truth of a theorem. They probably used methods like those of some modern teachers who find Euclid's work too academic; but they had the excuse that they were still seeking the light, not sinning against it. This point can be illustrated by consideration of the theorem about the square on the hypotenuse of a right-angled triangle which is always attributed to Pythagoras and was presumably well known in his school. The proof we find in Euclid is rather complicated, in that it requires a number of lemmata, or preliminary theorems. These other theorems may have been known to the early Pythagoreans, but it is hardly to be supposed that they discovered things in the order in which they are presented by Euclid, and it seems likely that the first 'demonstration' of Pythagoras' theorem consisted in the construction of a figure from which the theorem could be 'read off'. Modern editors of Euclid have suggested such a figure, or rather pair of figures:[1]



In the first of these, four equal right-angled triangles are arranged in such a way that their hypotenuses enclose an area: in the second, four triangles of the same dimensions are arranged to form two rectangles with the sides OQ and PQ at right angles. It then seems obvious that the area EFGH in the first figure is the square on the hypotenuse and equal to the sum of the areas PQOL and

[1] T. L. Heath, *The Thirteen Books of Euclid's Elements*, i, p. 354.

MJNQ of the second figure, which are the squares on the other two sides of one of the right-angled triangles. But for a rigorous proof it would be necessary to show that the three areas just mentioned are indeed squares. By the beginning of the third century B.C. when Euclid wrote, the ideal of demonstration had become clear to geometers. No one can read Euclid's *Elements* without realizing that his aim was to put all his special geometrical assumptions at the beginning and to construct chains of demonstration in which the theorems followed from axioms by purely formal necessity. It is true that in the proof of the first proposition of his first book Euclid assumed that two equal circles each of which has its centre on the circumference of the other must intersect in two places, although he had laid down no explicit postulate from which this follows; but it seems clear that if anyone had brought this defect to his notice he would have tried to remove it by setting down a new postulate.

Unfortunately we have no complete work of geometry earlier than Euclid's *Elements*, and we cannot trace in detail the process by which the Greeks became aware of the requirements of demonstration; but we know that there were books of elements, i.e. deductive treatises, before Euclid's. Scraps of early proofs are preserved in the works of Plato and Aristotle,[1] and material from a history of geometry by Aristotle's pupil Eudemus is to be found in Proclus' commentary on Euclid. It is therefore safe to say that the ideal of a deductive system was known in the Pythagorean school and in the Platonic Academy, which continued some of its traditions. But there was probably a good deal of confusion in the minds of many who read the earliest books of elements. Aristotle tells us that some people said there could be no demonstration and others that demonstration could be circular.[2] In order to understand why queer views of this kind were current, we must realize that the earliest books of elements probably differed in their choice of axioms, since there may be many different ways of presenting geometrical propositions in a deductive system. If this was so, propositions which were derivative in one system would be primitive in another, and the project of demonstration might easily become suspect to the half-initiated.

Now if reflection of the kind we call logical began in this context, what parts of logic, as we know it, should we expect to find stressed in the earliest exposition In the first place we should expect to find special attention paid to general propositions, that is to say, propositions about *kinds* of things. For in geometry we are not concerned with individuals. We may sometimes talk of

[1] T. L. Heath, *Mathematics in Aristotle.*

[2] *An. Post.* i. 3 (72^b5–18).

'the line AB' as though we were referring to a particular line, but it is always understood by geometers that this is just a way of speaking about all lines that satisfy a certain condition, e.g. that of being hypotenuse in a right-angled triangle. Secondly, among universal propositions (i.e. general propositions about *all* of a kind) we should expect to find special attention paid to those which are necessarily true. For when we do geometry in the Greek way, we must distinguish between universal propositions which must be true from the nature of the case and those which just happen to be true (e.g. that each book of the *Iliad* contains less than a thousand lines), and we suppose that the universal propositions of geometry are all of the first kind. It is not likely, of course, that the Greeks were able to formulate the distinction clearly as soon as they began to do geometry; as we shall see, it cost Aristotle some effort to reach this position. But a sure instinct guided them to pay special attention to those propositions which are in fact necessarily true. Thirdly, among universal propositions which are necessarily true we should expect definitions to receive special (but not exclusive) attention. A reader who is familiar with modern logic may perhaps deny that definitions are necessarily true propositions, and suggest that they are merely records of our determination to use certain abbreviations when we find it convenient to do so. To the Greeks, however, it did not seem that definitions were mere conventions. There is a great deal of muddle in the doctrine of real definition which started at this time, but it is easy to understand the Greek attitude if we remember that before the Greeks began to do demonstrative geometry words such as 'circle' had meaning only as standing for certain perceptual patterns. When a Greek said 'A circle is the locus of points equidistant from a given point', he was not introducing the word 'circle' for the first time, but rather giving it new connexions; and to himself he seemed to be expounding a truth of great importance about circles. Fourthly, we should expect to find great interest in the subsumption of specific varieties under general rules, since this seems to be the most common pattern of argument in geometry. Now all these features are to be found, as we shall see, in the logic of Aristotle, and some of them already in Plato's work or earlier. Aristotle tells us, for example, that Archytas, a Pythagorean mathematician who influenced Plato, had views about the proper form of definitions.[1] It is, therefore, reasonable to suppose that one trend in Greek logic was determined in large part by reflection on the problems of presenting geometry as a deductive system.

[1] *Metaphysica, H,* 2 (1043a21).

3. *Dialectic and Metaphysical Argument*

The character of Greek logic cannot be explained wholly in terms of demonstration (ἀπόδειξις). As we have already seen, Aristotle in the first account of syllogistic considered that his study covered also dialectical arguments. The word 'dialectic' had a number of different shades of meaning even in the early stages of philosophy, and it is of particular interest to us as the first technical term to be used for the subject we now call logic. Aristotle's word 'analytics' refers to his treatises rather than to their subject-matter, and 'logic' itself does not appear with its modern sense until the commentaries of Alexander of Aphrodisias, who wrote in the third century A.D.

In its earliest sense the word 'dialectic' is the name for the method of argument which is characteristic of metaphysics. It is derived from the verb διαλέγεσθαι, which means 'discuss', and, as we have already seen, Aristotle thinks of a dialectical premiss as one chosen by a disputant in an argument.[1] Plato's dialogues give numerous illustrations of the method of argument intended. In the *Theaetetus*, for example, Theaetetus lays down the thesis that knowledge is perception, and from this premiss Socrates draws conclusions which eventually force Theaetetus to abandon it.[2] The same illustration serves also to bring out the more precise meaning which 'dialectic' has for Plato in the dialogues of his middle period. There it is the examination of propositions called 'hypotheses' by drawing consequences from them. If a consequence is unacceptable, the hypothesis from which it is derived must be rejected. It is clear that, in general, this procedure can lead only to negative results; for the argument will proceed in accordance with the logical schema, 'If P then Q; but not-Q; therefore not-P'. This is the standard argument-pattern of refutation (ἔλεγχος) and it was probably suggested to Plato not only by the practice of Socrates in refuting the uncritically held opinions of his contemporaries, but also by the use of the *reductio ad impossibile* argument in metaphysics by Zeno of Elea. In his *Parmenides* Plato makes Zeno claim to have written a book in which he defends the monism of Parmenides by drawing out the absurd consequences of the supposition that there is plurality.[3] It was perhaps to this remark that Aristotle referred when he said, as reported by both Diogenes Laertius[4] and Sextus Empiricus,[5] that Zeno was the inventor of dialectic.

[1] *An. Pr.* i. 1 ($24^{b}1$).
[2] *Theaetetus*, 151E ff.
[3] *Parmenides*, 128D.
[4] *Vitae*, viii. 57 and ix. 25.
[5] *Adversus Mathematicos*, vii. 7.

In any case what Aristotle attributed to Zeno was presumably the discovery of the use of the *reductio ad impossibile* in metaphysics, and it is possible that this was suggested to Zeno himself by its use in Pythagorean mathematics. For the Pythagoreans are supposed to have discovered the incommensurability of the diagonal with the side of a square (in modern terminology, the irrationality of $\sqrt{2}$), and the pròof of this proposition, preserved as an interpolation in our text of Euclid, has the form of a leading-away to the impossible (ἀπαγωγὴ εἰς τὸ ἀδύνατον).[1] When Aristotle has occasion to mention such reasoning, he cites this theorem as though it were the standard example.[2] In the proof it is first supposed for the sake of argument that $\sqrt{2}$ is rational, i.e. that there are two integers, say m and n, which are mutually prime and such that $m/n = \sqrt{2}$ or $m^2 = 2n^2$. From this it follows that m^2 must be even and with it m, since a square number cannot have any prime factor which is not also a factor of the number of which it is the square. But if m is even, n must be odd according to our initial supposition that they are mutually prime. Assuming that $m = 2k$, we can infer that $2n^2 = 4k^2$ or $n^2 = 2k^2$; and from this it can be shown by a repetition of the reasoning used above that n must be even. Our hypothesis, therefore, entails incompatible consequences, and so it must be false.

Once established by Zeno as a method of reasoning in philosophy, dialectic had a long history. Among the philosophers who are sometimes called Minor Socratics it was practised by Euclides of Megara. Evidently he stood close to Socratès, for Plato and some other Athenian friends went to stay with him in Megara immediately after the death of Socrates. But it is said that he was also a follower of Parmenides and Zeno. The members of his school were called dialecticians, and we read that Euclides himself 'attacked demonstrations not by the premisses but by the conclusion', which presumably means that he tried to refute his opponents by drawing absurd consequences from their conclusions. Apparently he found all this consistent with his admiration for Socrates; for he even tried to identify the Good of Socrates with the One of Parmenides.[3] But this is not surprising if Socrates himself had adopted the method of Zeno for his own purposes. It is difficult to reach any certainties about the teaching of the historical Socrates, but those passages of Plato's works which because of their dramatic quality seem most reliable as evidence in this connexion suggest that he was not merely a lover of philosophical conversation but one who practised a definite

[1] *Elements*, x. 117, relegated to an appendix in Heiberg's edition.
[2] *An. Pr.* i. 23 (41^a26). [3] Diogenes Laertius, ii. 106–7; Cicero, *Academica*, ii. 129.