

CIRCUIT ANALYSIS FOR ENGINEERS

Continuous and
Discrete Time Systems

Dwight F. Mix
Neil M. Schmitt

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Dwight F. Mix
Neil M. Schmitt
University of Arkansas

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PREFACE

Circuit analysis forms the foundation of nearly all curricula in Electrical Engineering education. This text is intended to assist the professor in providing the student with both a broad and a deep understanding of this topic. Students enter engineering programs with a wide range of mathematical capabilities. This text is written so that students can take the first course in circuit analysis while they are taking their first course in calculus. In the early portions of the text any necessary calculus or differential equation concepts needed are introduced as part of the text material. In the latter portions of the text we have assumed that the student has completed a course in calculus and the mathematical concepts presented are considerably more sophisticated.

There is a letter on record from one fifteenth-century Italian nobleman to another advising the recipient to send his son to a certain German university "because they teach long division there." Considerable progress in our educational system has been made since then, and this progress will continue in the future. Discrete-time signal processing was first introduced into the graduate curriculum of many universities some 10 to 20 years ago. Since that time this topic has gradually filtered into the undergraduate curriculum. The advent of very sophisticated but inexpensive digital hardware has now dictated that this topic be a required component of the undergraduate curriculum. In this text we introduce this topic in the beginning circuits sequence and make it a natural part of beginning circuit analysis.

It is our contention that unfamiliar topics are neither inherently difficult nor inherently easy to understand. The manner in which they are presented, coupled with the student's background, primarily determines the level of difficulty experienced in comprehending new ideas. With this in mind, we have employed the following tactics to augment the learning process.

1. **Tell the student what he is expected to learn.** Each section of the text begins with a set of objectives that establishes the learning goals for the student for that section. We assume that these will be augmented or modified as desired by the professor.
2. **Help the student determine whether she has grasped the concepts.** Each section concludes with a set of drill problems that measures how well the student has met the learning objectives. The problems at the end of the chapter

are keyed to the sections in the chapter to further enhance the evaluation process. The end-of-chapter problems are more difficult than the drill problems and some are open-ended or extend the material presented in the chapter.

3. **Present the most basic concepts first and then build on these.** Essentially all concepts and terminology used in the text are introduced in the text as they are encountered rather than relying on previous experiences of the student in physics or similar courses. The meaning of circuit analysis is explained and illustrated as much as possible through the use of the resistive circuit. The addition of other circuit elements then leads to expansion of these concepts to include analysis for any deterministic input signal and analysis in the frequency domain.
4. **Use examples frequently.** For any student encountering a subject for the first time, illustration of principles and concepts by means of carefully chosen examples is important. You will find that examples are used generously as part of the text and they are easily identified within the text material.

We have included two other topics often omitted in a beginning circuits text: linearity and the convolution operation. Linearity is assumed in all beginning texts, but the exploration of this and the time invariance concept allows its use in deriving the transfer function and convolution operation. Under certain conditions (controllability and observability), there are three equivalent methods of finding the response of linear, time-invariant circuits: namely, convolution, transforms, and differential or difference equations. We give complete treatment to all three methods. Convolution has attained additional importance because of its ease of use in digital computer analysis of circuits. Also, more material on transforms than is found in the average two-semester circuits text is included.

The concept of state and the state-variable approach are not introduced. There are several reasons for this.

It is confusing to the beginner to learn two closely related procedures at the same time. This is well documented in a famous article by D. R. Entwistle and W. H. Huggins, "Interference in the Learning of Circuit Theory," (*Proc. IEEE*, July 1963, 986-990). We felt it would be especially confusing to introduce the closely related but distinct concepts of zero-state, zero-input response along with the traditional homogeneous (transient) and particular (steady-state) responses. The state-variable concepts are usually introduced in higher-level courses.

Approximately three-fourths of the text is devoted to traditional topics (continuous-time circuits) and one-fourth is devoted to discrete-time analysis. The new material is integrated with traditional topics as much as possible. For example, discrete convolution is presented in one section, followed by continuous-time convolution in the next section. Whenever a principle applies to both continuous and discrete systems, examples of both are given. Since discrete-time systems are so prevalent in industry, it is our belief that discrete-time topics will eventually occupy a major portion of the typical electrical engineering undergraduate curriculum. Since this text is among the first to introduce this topic in the beginning course, however, we have chosen to be cautious rather than bold. Also, there is some evidence that continuous-time signal processing will make a

comeback because of the time limitations of discrete signal processing. It may take seconds to find the two-dimensional Fourier transform of a large array. This same processing can be done optically in microseconds.

It has been our intent to write this book for the students. We hope that they find the style less formal than that of many texts, the historical insights enlightening and amusing, and the order and method of presentation helpful in assimilating the myriad concepts and principles contained within these pages. Likewise, we hope this text is of value in assisting the professor in the educational process.

Each author acknowledges that all errors or omissions are the fault of the other author. Seriously, we would like to acknowledge the helpful review of an earlier version of this text by Professor Edwin C. Jones at Iowa State University. None of the faults in this text should be attributed to him.

Dwight F. Mix
Neil M. Schmitt

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CHAPTER

1

FUNDAMENTAL DEFINITIONS AND LAWS

1.1 Circuit Analysis: What Is It?

You are about to embark on a journey through the world of circuit analysis. Naturally you must be curious about this topic and want to know why it is so fundamental to the engineering profession. A primary function of an engineer is to apply fundamental laws of science to problems facing the human race in order to effect a solution or at least to minimize the effect of the problem. A significant portion of these laws embrace the area of electricity. We all know that the atom is the basic building block of things that exist in our universe. Atoms are made up in part of charged particles (electrons and protons). The movement of these charged particles is what we commonly call electricity.

An electrical circuit is a grouping of certain devices in order to control the flow of electrons in such a manner as to achieve a desired outcome. The display of a picture on a television screen is the result of controlled movement of electrons by electrical circuits. The uncontrolled flow of electrons, such as in a bolt of lightning, can have devastating effects.

One function of the engineer is to develop devices to be used in electrical circuits, but that is a subject for other texts. A second important function is circuit design—the interconnection of devices with the intent of achieving some goal. Another function is to analyze a group of devices that have been interconnected in some way to determine their effect on the flow of electrons. This function is called circuit analysis, and is the topic of this book. Circuit analysis and circuit design are obviously interrelated; indeed, analysis exists primarily to support the design task.

There are many ways to analyze electric circuits, and the choice of the best method of analysis is made by the engineer based on knowledge and experience. It is the objective of this text to provide you with a “bag of circuit analysis tools” from which you may choose the most appropriate tool for the task you face. It is also the purpose of this text to provide you with experience in using these tools to solve circuit analysis problems.

We will begin by explaining the system of units to be used in the book and by defining basic elements of electricity. Two of the most fundamental laws of circuit analysis will also be introduced. Finally, conventions to be followed in the text will be agreed upon. It is our sincere desire that the subject matter introduced through this text will launch you into a rewarding and satisfying lifelong career as an engineer.

1.2 System for Describing Circuit Values

LEARNING OBJECTIVES

After completing this section you should be able to do the following:

1. Express measures of units in scientific notation.
2. Convert values to the international system of units.

In this book we are interested in conversing with (and in training you to converse with) technically oriented individuals. Consequently we use a system of units that has been adopted by all the professional engineering societies in the United States and by most professional technical societies in the world. It is called the International System of Units and is abbreviated "SI."

There are six basic elements involved in this system of units. The five that are applicable to circuit analysis provide measures of time, weight, distance, temperature, and current. They are respectively: second (s), kilogram (kg), meter (m), degree Kelvin ($^{\circ}\text{K}$), and ampere (A). The sixth is luminous intensity measured in candela (cd). The accepted abbreviations are shown in parentheses.

Each unit has been accurately defined so that it is both permanent and reproducible. For instance the meter is defined as:

$1.65076373 \times 10^6 \times$ the wavelength of radiation of the orange line of krypton 86

We encounter both very large and very small numbers with respect to the standard units of measure in our study of electric circuits. In order to easily accommodate these numbers we will use a combination of scientific notation (with which we assume the reader is familiar) and standard prefixes. Table 1.1 gives prefixes commonly used, along with their symbol and meaning. Hence, the following statements are equivalent:

1,280,000 seconds

1.28×10^6 seconds

1.28 megaseconds

1.28 Ms

TABLE 1.1
COMMONLY USED PREFIXES

Prefix	Symbol	Meaning
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
deci	d	10^{-1}
kilo	k	10^3
mega	M	10^6
giga	G	10^9
tera	T	10^{12}

Calculations involving large or small numbers are very handily performed using scientific notation. In fact, most electronic calculators have the capability to use this method to accept input and to display results.

Additional units of measure related to the SI standard will be introduced as they are encountered in the text.

LEARNING EVALUATIONS

1. Express each of the following terms in three additional ways.
 - a. 0.00168 meters
 - b. 3.577×10^{-2} s
 - c. 14 G °K
 - d. 0.877 A
2. Convert the following values to SI values.
 - a. 281 ft-lb
 - b. 40 °F
 - c. 6 in.
 - d. 422 s

1.3 Charge and Current

LEARNING OBJECTIVES

After completing this section you should be able to do the following:

1. Find the charge in coulombs of elementary particles.
2. Derive current $i(t)$ from charge $q(t)$.

Early investigators in electricity had few devices to work with, either to produce electric phenomena or to measure electric quantities. They rubbed amber with cat's fur or glass with silk in order to produce electricity. A number of other substances were found to be capable of being "electrified," including gems and rock crystal. These substances were called "electrics."

There were a number of substances, including metal, that could not be electrified, and hence were called "nonelectrics." Today we call these substances conductors, and the electrics are called insulators.

Electricity was considered somehow to be a fluid, for it could be demonstrated that if a glass rod was electrified, and if another electric such as cork was attached to the rod, then it too would become electrified. This was demonstrated in 1729 by an English electrician, Stephen Grey (1696–1736). Thus the fluid spread throughout the configuration. If a nonelectric (conductor) was connected between the electric and ground, the electric fluid would flow rapidly to ground.

It was recognized that there were two types of electricity. When glass was rubbed and the charge transferred to two cork balls, they repelled each other. The same thing happened when the charge from resin was used. But if one cork ball was charged with glass while the other was charged with resin they attracted each other. The glass induced charge was called "vitreous electricity" (from a Latin word for glass), while the resin induced charge was called "resinous electricity."

Benjamin Franklin (1706–1790) was one of the pioneers in this young science. In the 1740s he conducted experiments that led him to speculate that every

substance contained electric fluid, but that a deficiency or excess of this fluid could be induced by rubbing electrica. It was known that when both glass and resin were rubbed they attracted each other, hence, one must contain an excess of fluid while the other had a deficiency. Franklin called vitreous electricity positive, and resinous electricity negative. This is the source of our present convention of calling electrons "negative" and protons "positive," for it is now known that vitreous electricity has a deficiency of electrons, while resinous electricity has a surplus of electrons.

Franklin intended for current to be the flow of excess fluid from positive regions to negative regions, but he had no way of knowing which was which. He was forced to guess, and he guessed wrong, for electric current is primarily the flow of electrons in conductors. But he could hardly have known, for it was not for another century and a half that electricity came to be associated with subatomic particles. We still use his convention today for several reasons. First, no harm is done (despite the horror of laymen who first discover this error). Second, current flow in many devices (especially semiconductors) is considered to be the flow of positive charges. Third, and most important, is long standing convention. The scientific community has persisted in using Franklin's convention, and they are not about to change now.

Current is the flow of charge. Measurements show that negatively charged electrons drift through a metal conductor under the influence of an electric field, but the effect of this charge movement travels at the speed of light down a conductor. Thus we get the picture of charge moving in unison down the length of a conductor, much like soldiers marching in single file. The last one moves almost simultaneously with the first one, the slight delay being due to the finite velocity of light.

The fundamental unit of charge is called the coulomb (C) and 1 C of negative charge has been internationally defined as the net charge of 6.24×10^{18} electrons. The symbol for charge is q and the charge of one electron then is

$$q = \frac{-1}{6.24 \times 10^{18}} \text{ C} = -1.602 \times 10^{-19} \text{ C} \quad (1.3.1)$$

Likewise, the charge of one proton is $+1.602 \times 10^{-19} \text{ C}$. The charge on an object can be a function of time and would be shown as $q(t)$.

Charge, however, is not of primary interest in circuit analysis. Rather, it is the movement of charge that demands our attention. Through control of the movement of charge we are able to transmit (move from one point to another) energy and information. The movement of charge is called current and is measured in terms of the ampere, after Andre Ampere, the French mathematician who published a treatise explaining the concept of current in 1820. The symbol for current is i and the abbreviation for the ampere (commonly called amp) is A. We normally are interested only in those situations where the electrons' movement is confined, such as in a copper wire. The ampere is defined as follows:

Definition 1.3.1. One ampere of current at a given point is the movement of one coulomb of charge through that point in exactly one second.

Let us return to our copper wire and choose a point on the wire. If an impartial official counts the number of electrons moving past that point in the same direction for a 1 s interval and gets an answer of 6.24×10^{18} , then 1 A of current has passed through that point.

Several questions should arise in your mind at this point. What if the charge were positive instead of negative? What if the charge were going in the other direction? An understanding of these concepts form the basis of circuit theory and is worthy of your efforts to conquer the subject matter. The answers to the two questions are not universally accepted, but the majority of engineering professionals assume the following convention:

Positive current results from the movement of positive charge. The direction of the positive current flow is the direction of the movement of the positive charge.

This seems simple enough but is deceptively difficult to apply because the movement of charge is usually caused by the movement of electrons (negative charges). Consider the wire in Fig. 1.1. If we observe that there is a current of 1 A flowing from point *A* to point *B*, this can be caused by any of three situations:

1. A positive charge moving from *A* to *B* at the rate of 1 C/s.
2. A negative charge moving from *B* to *A* at the rate of 1 C/s.
3. A movement of positive charge from *A* to *B* and negative charge from *B* to *A* in such a manner that the *net* rate of charge moving from *A* to *B* is 1 C/s.

It is also equivalent to state that a current of $-i$ is flowing from *B* to *A*.

Mathematically we see that current is the time rate of change of charge

$$i = \frac{\Delta q}{\Delta t} \quad (1.3.2)$$

Consider the plot of charge as a function of time given in Fig. 1.2. To calculate the current for the interval $0 \leq t \leq 2$ we simply calculate $\Delta q / \Delta t$ and note that the slope is constant over the time interval. Hence,

$$i(t) = \frac{(-1) - (2)}{2} = -1.5 \text{ A} \quad 0 < t < 2$$

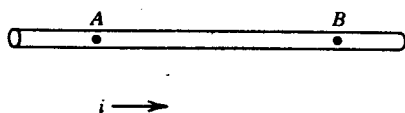


Figure 1.1. Direction of current.

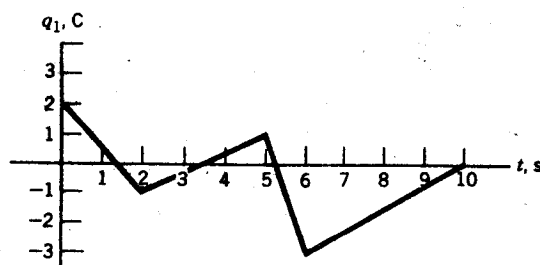


Figure 1.2. Charge as a function of time.

In the same manner,

$$i(t) = \frac{1 - (-1)}{3} = 0.67 \text{ A} \quad 2 \leq t \leq 5$$

$$i(t) = \frac{(-3) - (1)}{1} = -4 \text{ A} \quad 5 \leq t \leq 6$$

$$i(t) = \frac{0 - (-3)}{4} = 0.75 \text{ A} \quad 6 \leq t \leq 10$$

We can plot $i(t)$ as shown in Fig. 1.3.

It is emphasized that to specify a current correctly we must specify a magnitude and a direction. Usually in circuit analysis we will not know beforehand the magnitude or direction of currents in our circuit. We will assume a direction of positive current flow and indicate this direction in our circuit by an arrow. If later calculations show the value of current to be negative we know that we assumed the direction of positive current flow in the direction opposite to the actual positive current flow. Return to Fig. 1.1. If we find that $i = -5 \text{ A}$, then we say -5 A flows from A to B or $+5 \text{ A}$ flows from B to A .

LEARNING EVALUATIONS

- Find the charge associated with
 - 10^{12} protons.
 - 10^{14} electrons and 10^{12} protons.
- Find and plot $i(t)$ if $q(t)$ is given by the graph in Fig. 1.4.

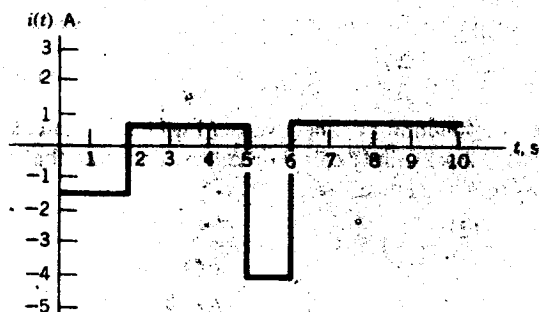


Figure 1.3. Current versus time.

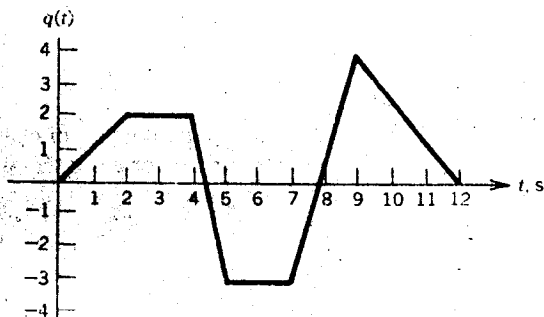


Figure 1.4

1.4 Kirchhoff's Current Law

LEARNING OBJECTIVES

After completing this section you should be able to do the following:

- State Kirchhoff's current law.
- Use Kirchhoff's current law to find the current entering or leaving a node.