ELECTROMAGNETICS

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WILEY EASTERN LIMITED
NEW DELHI BANGALORE BOMBAY CALCUTTA

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40/8 Ballygunge Circular Road, Calcutta 700019

This book has been subsidized by the Government of India,
through the National Book Trust, India,
for the benefit of the students

ISBN 0 85226 506 9

Rs. 21.50

Published by Mohinder Singh Sejwal for Wiley Eastern Limited,
4835/24 Ansari Road, Daryaganj, New Delhi 110002 and
printed by Abhay Rastogi at Prabhat Press, 20/1 Nauchandi Grounds,
Meerut 250002. Printed in India.

Preface

Electromagnetics has become a part of the foundations of Physics and as such is an essential ingredient of the Physics curriculum at different levels. The present book is intended as a text-book in electromagnetics for the B.Sc. and M.Sc. students in Physics.

The method of approach is dictated by the desire to meet the needs of the students. Hence, the primary concern of the book is to enable the students to obtain a satisfactory intuitive grasp of the subject and to make them realize that the subject is relevant and useful. The basic ideas are developed along familiar lines.

Electromagnetics is essentially mathematical in character. Without mathematics as an aid to thought the development of electromagnetism would have been almost impossible. However, no attempt has been made in this book, to achieve mathematical rigour because it would be difficult to be much more rigorous without causing the student to lose sight of the pragmatic physical content. No more mathematical background is required of the student than the customary undergraduate courses in Calculus and Vector analysis. More advanced techniques are outlined as they arise.

Problems at the end of the chapters provide details for which there is no room in the body of the text and call for additional applications. They also tend to test the students' understanding of the concepts discussed in the chapter.

The references at the end indicate the author's indebtedness to the ideas of others, but the list is, by no means, exhaustive.

B.B. LAUD

A Guide to Symbols

Symbol	*Explanation*
α	Polarizability of the atom
A_m	Molecular refractivity
$\mathbf{A}$	Vector potential
$\mathcal{A}$	Four vector potential
$\mathbf{B}$	Magnetic flux density
χ	Electric susceptibility
χ_m	Magnetic susceptibility
c	Velocity of light
C	Capacitance
δ	Skin depth
δ_{nm}	Kronecker delta
$\delta(r)$	Dirac delta function
∇	$\hat{\mathbf{e}}_x \frac{\partial}{\partial x} + \hat{\mathbf{e}}_y \frac{\partial}{\partial y} + \hat{\mathbf{e}}_z \frac{\partial}{\partial z}$
$\mathbf{D}$	Electric displacement
ϵ	Permittivity
ϵ_r	Relative permittivity, dielectric constant
$\mathcal{E}$	E.M.F.
e	Electronic charge
$\hat{\mathbf{e}}_r$	Unit vector in the direction r
$\mathbf{E}$	Electric field strength
ϕ	Electrostatic potential
Φ	Magnetic flux
ϕ_m	Magnetic scalar potential
$\mathbf{F}$	Force
$F_{\mu\nu}$	Electromagnetic field tensor
γ	Damping coefficient
$\mathcal{G}(x, x')$	Green's function
$\mathbf{H}$	Magnetic field strength
I	Current
$\mathbf{J}$	Current density
J_ν	Bessel's function
$\mathcal{J}$	Four-vector current-density
$\mathbf{k}$	Propagation vector

*λ	Charge per unit length, wavelength
λ_g	Gravitational constant
$\mathcal{L}$	Differential operator
*L	Lagrangian; Self-inductance
*μ	Magnetic moment; permeability
m	Magnetic dipole moment
M	Mutual inductance
n	Refractive index
$\mathbf{N}$	Poynting vector
*Ω	Solid angle; ohm
p	Dipole moment;
$\mathbf{P}$	Macroscopic polarization density
$\mathcal{P}$	Magnetic pressure
P_l	Legendre polynomial
P_l^m	Associated Legendre polynomial
q, Q	Charge
ρ	Charge density
*R	Reflection coefficient; resistance
*σ	Conductivity; scattering cross-section
S_l	Surface harmonic
*τ	Relaxation time; torque
T	Transmission coefficient
U	Energy
$\mathcal{U}$	Four-vector velocity
V	Potential
V	Volume
ω	Angular frequency
W	Energy
Z_0	Impedance

*These symbols are used to represent different quantities. The context generally makes clear which quantity is under discussion.

Contents

Chapter 1

Force, Field and Energy in Electorstatics

The world around us is made up of atoms which consist of positive and negative charges. The dominant force between atomic particles, therefore, is electrostatic. An understanding of a few basic laws of forces of nature can go a long way towards guiding us through the mysteries of science. One such law—the electrostatic law of force—was provided by Coulomb in 1785. Atomic reactions can be explained with great precision by Coulomb's Law.

1.1 Coulomb's Law

Coulomb showed experimentally that in free space oppositely charged bodies attract each other, while similarly charged bodies repel with a force that varies directly as the magnitude of each charge and inversely as the square of the distance between them, the force being directed along the line joining the charges.

Let us describe these observations of Coulomb in a mathematical form. We will use vector notation throughout this book. This has some advantages:

(i) the arbitrariness associated with the choice of the coordinate systems disappears and the physical content becomes more clear, and

(ii) the equations of electrodynamics become more concise and vivid if written in vector notation.

If '1' and '2' are two particles carrying charges q_1 and q_2 respectively and separated by a distance r_{12} in vacuum (Fig. 1.1), then the electric force exerted by the particle '1' on the particle '2' as given by Coulomb's law is

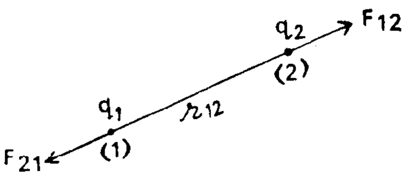

Fig. 1.1

$$F_{12} \propto \frac{q_1 q_2}{r_{12}^2}$$

i.e.

$$F_{12} = K\frac{q_1 q_2}{r_{12}^2} = -F_{21} \tag{1.1}$$

where K is the constant of proportionality and F_{21} is the force exerted by the particle '2' on the particle '1'.

In vector notation this can be written as

$$\mathbf{F}_{12} = K \frac{q_1 q_2}{|\mathbf{r}_{12}|^2} \hat{\mathbf{e}}_r = K \frac{q_1 q_2}{|\mathbf{r}_{12}|^3} \mathbf{r}_{12} \tag{1.2}$$

where $\hat{\mathbf{e}}_r = \frac{\mathbf{r}_{12}}{|\mathbf{r}_{12}|}$ is the unit vector along r_{12}.

The sign of the charges decides whether the force is attractive or repulsive. If the charges are similar (i.e. both positive or both negative), $\mathbf{F}_{12}$ is positive and represents the force of repulsion; while if one is positive and the other negative, $\mathbf{F}_{12}$ is negative and is the force of attraction.

We have two alternatives for choosing the value of K and the units of charge: either

(i) K is arbitrarily given some fixed numerical value and Eq. (1.2) used to determine the units of charge; or

(ii) the unit of charge is taken as some arbitrary value and the constant K determined experimentally.

In the Gaussian system (CGS) distances are measured in centimetres, mass in grams, time in seconds, force in dynes and the electrical units are defined by assuming the constant of proportionality K to be unity (first alternative). That is,

$$\mathbf{F}_{12} = \frac{q_1 q_2}{|\mathbf{r}_{12}|^3} \mathbf{r}_{12} \text{ (dynes)} \tag{1.3}$$

Thus, the force between two unit charges separated by a distance of one centimetre is one dyne. The unit of charge, therefore, is that charge which experiences a force of 1 dyne when placed 1 cm from an identical charge. The unit of charge, thus, defined, is called the *statcoulomb* or electrostatic unit (esu).

In SI system (*Système Internationale*) of units, the distances are measured in metres, mass in kilograms, time in seconds, force in newtons and the charge in coulombs (second alternative). The constant K is then equal to $\frac{1}{4\pi\epsilon_0}$ and the Coulomb's law is written as

$$\mathbf{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\mathbf{r}_{12}|^3} \mathbf{r}_{12} \quad \text{(newtons)} \tag{1.4}$$

Thus, the force between two particles each carrying a charge of one coulomb and separated by a distance of one metre, is one *newton*.

The factor 4π has been introduced in order to simplify the form of some important relations occurring in the electromagnetic theory. In developing the various formulae, we will have often to deal with spherical shapes and hence a factor 4π is bound to appear. It will be advantageous, therefore, to use a constant containing 4π explicitly. The constant ϵ_0 has the value

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ coulomb}^2 \text{ newton}^{-1} \text{ metre}^{-2} \tag{1.5}$$

You will see later that this is the so-called *permittivity of free space*.

Which of these two systems shall we use? One would think it best to use CGS system as in this system $K=1$ and the relation connecting the force with the charges is relatively simple. However, if we adopt this system, the electric current will have to be measured in clumsy units of $\frac{1}{3\times10^9}$ amperes. Thus, although we thought we have chosen the units judiciously so that the constant is banished from Eq. (1.1), it appears in a different guise in another place. Any of these systems, therefore, is as good as the other. However, as the ordinary measuring instruments are calibrated in SI units, we shall use SI system in this book.

Since Coulomb's law is based primarily on experiments, one may ask whether the law is exactly that of inverse square? That is, if the force is proportional to $1/r^n$, is n exactly equal to 2? Cavendish showed that $n=2\pm0.02$. Plimpton and Lawton (1936) found that n differs from 2 by not more than one part in 10^9. More recently Lamb and Rutherford (1947) also confirmed from their measurements of the energy levels of the hydrogen atom that the exponent in Coulomb's law is correct to one part in 10^9 at distances of the order of 10^{-10} metres. Evidence from nuclear physics has shown that the electrostatic forces vary approximately according to the inverse square law even at distances of the order of 10^{-15} metres. Considering the areas of our interest, we may use the inverse square law with complete confidence. It must be mentioned here that the logical construction of a consistent theory of electromagnetism that is developed in this book is based on the laws such as the one due to Coulomb, which are the results of experiments and as such are probably approximate. However, these are very good approximations and lead to the correct results.

Notice the similarity between Coulomb's law and the Newton's law of gravitation

$$F=\lambda_g\frac{m_1m_2}{r^2}$$

where λ_g is the gravitational constant.

It must be mentioned here that although the charge on the electron is extremely small, electrostatic forces are immensely strong. A comparison between the force of attraction between two electrons due to gravitation and the force of repulsion due to electronic charges, will give an idea about the relative magnitudes of these forces.

$$\frac{\text{Electrostatic repulsion}}{\text{Gravitational attraction}}=\frac{1}{4\pi\epsilon_0}\cdot\frac{e^2}{r^2}\cdot\frac{r^2}{\lambda_g m^2}$$

$$=\frac{e^2}{4\pi\epsilon_0\lambda_g m^2}=9\times10^9\times\frac{(1.60\times10^{-19})^2}{(6.67\times10^{-11})\times(9.1\times10^{-31})^2}$$

$$=4.17\times10^{42}\qquad\left(\because\ \frac{1}{4\pi\epsilon_0}=9\times10^9\right)$$

(How is it, then, that we normally do not notice electrostatic forces, which are so powerful?)

EXAMPLE 1.1. In Rutherford's scattering experiment gold nuclei were bombarded with α-particles energetic enough to approach within 2×10^{-14} metres of the nucleus. Find the electrostatic force of repulsion experienced by the α-particles.

The charge on the gold nucleus is '79 e' and that on an α-particle is '2 e' where 'e' is the charge on an electron.

$$\therefore F = \frac{1}{4\pi\epsilon_0} \times \frac{2 \times 79 \times e^2}{(2 \times 10^{-14})^2}$$
$$= 91.2 \text{ newtons.}$$

1.2 Principle of Superposition

If there are more than two particles present with charges, say, q_1, $q_2, \ldots$, the total force on any one particle is the vector sum of forces it experiences due to all other particles separately. This is known as the *Principle of superposition.* For example, in Fig. 1.2, in which there are three charges, force on q_3 is given by

$$\mathbf{F} = \mathbf{F}_{13} + \mathbf{F}_{23} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_3}{|\mathbf{r}_{13}|^3}\mathbf{r}_{13} + \frac{1}{4\pi\epsilon_0}\frac{q_2 q_3}{|\mathbf{r}_{23}|^3}\mathbf{r}_{23}$$

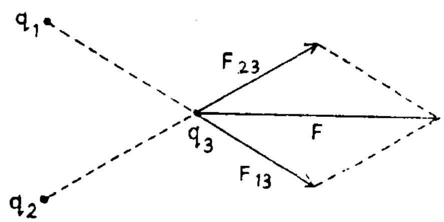

Fig. 1.2

The force acting on a charge q_j due to a number of other charges present is

$$\mathbf{F}_j = \frac{1}{4\pi\epsilon_0}\sum_{i \neq j}\frac{q_i q_j}{|\mathbf{r}_{ij}|^3}\mathbf{r}_{ij} \tag{1.6}$$

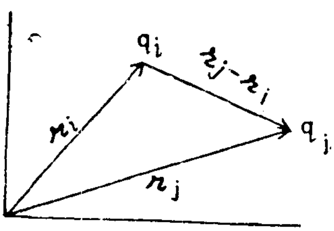

Fig 1.3

Equation (1.6) is often stated in terms of the position vectors. If $\mathbf{r}_i$ and $\mathbf{r}_j$ (Fig. 1.3) are the vectors giving the location of the charges q_i, q_j respectively, Eq. (1.6) can be written as

$$\mathbf{F}_j = \frac{1}{4\pi\epsilon_0}\sum_{i \neq j}\frac{q_i q_j}{|\mathbf{r}_j - \mathbf{r}_i|^3}(\mathbf{r}_j - \mathbf{r}_i) \tag{1.7}$$

The principle of superposition has facilitated considerably the mathematical handling of the theory. The principle fails in nuclear interactions and this is one of the reasons why the nuclear theory is somewhat more troublesome than the theory of atomic interactions.

1.3 Electric Field

In the treatment of physical problems, the concept of a 'field' is found to be of great utility. There are as many kinds of fields as there are types of mathematical or physical quantities that can be represented at a point. One finds, for instance, scalar, vector and tensor fields in the ordinary three dimensional space of classical physics and four dimensional vector fields in the four dimensional space-time domain of relativity theory. In the study of classical electromagnetics we shall be concerned with scalar and vector fields in three dimensions.

The space in which electrostatic forces act is called electrostatic field. This, however, is merely a qualitative description of the field. How do we define it quantitatively? Consider a system of charges distributed in space (Fig. 1.4).

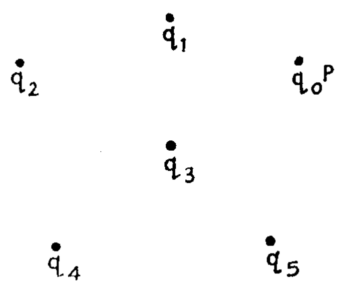

Fig. 1.4

What is the field due to these charges at a point P? To answer this question we put a charge q_o at P. We assume the charge q_o to be so small that it does not disturb the properties of the field, i.e. it exerts negligible force on the other charges. We call it a test charge. The force acting on this charge is

$$\mathbf{F}_0 = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_0 q_i}{|\mathbf{r}_0 - \mathbf{r}_i|^3} (\mathbf{r}_0 - \mathbf{r}_i) \tag{1.8}$$

where $\mathbf{r}_0$ is the location vector of the point P, and $\mathbf{r}_i$ the vectors giving the location of the other charges.

The force per unit charge experienced by the test particle at the point of interest is

$$\frac{\mathbf{F}_0}{q_0} = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{|\mathbf{r}_0 - \mathbf{r}_i|^3} (\mathbf{r}_0 - \mathbf{r}_i)$$

We have assumed that q_0 does not disturb the properties of the field. This, however, is not possible in practice. We, therefore, assume q_0 to be vanishingly small and define *electric field* as

$$\mathbf{E}(\mathbf{r}_0) = \lim_{q_0 \to 0} \frac{\mathbf{F}_0}{q_0} = \frac{1}{4\pi\epsilon_0} \sum \frac{q_i}{|\mathbf{r}_0 - \mathbf{r}_i|^3} (\mathbf{r}_0 - \mathbf{r}_i) \tag{1.9}$$

The strength of this field $|\mathbf{E}(\mathbf{r})|$ is called the *electric field intensity*.

From the definition of the electric field we see that if a charge q is placed at a point at which the field is **E**, the force acting on the charge is

$$\mathbf{F} = q\,\mathbf{E} \tag{1.10}$$

The electric field **E**(r) is a function of position and is itself a vector. An electric field, therefore, is a vector field. Examine carefully the Eq. (1.9) for the field **E**(r). It shows that the field at a point, due to the distribution of several charges, is the vector sum of the fields due to all the charges except the charge, if any, at the point under consideration. For the calculation of the electric field, we do not consider any charge to be at the point except the test charge. Otherwise, the contribution to the field at the point due to the charge present there would be infinite because of the singularity ($r = 0$) in the inverse square law and the theory would be useless. The concept such as that of a 'point' charge is meaningful if one accepts that measurements, in practice, are never made closer to such a charge than the distance of the order of atomic radii. Note that even a single electron has a finite size. However, it is often convenient to regard a small region of charged particles as a 'point charge'.

If the charge is not confined to a point, but is distributed over a region of space, it is possible to consider it as a continuous quantity and talk about a charge density or charge per unit volume. The charge density ρ is defined as

$$\rho = \lim_{V \to 0}\left(\frac{Q}{V}\right) \tag{1.11}$$

Consider, for example, the charge distribution within a hydrogen atom. We know that the electrons are not stationary point charges and their positions cannot be sharply defined. It is convenient and quite appropriate to consider the charge on the electron to be smeared out in a cloud around the nucleus (Fig. 1.5). If the charge density at a point specified by the position vector **r** is $\rho(\mathbf{r})$, the charge contained in a small volume element $d\tau$ at **r** is $\rho(\mathbf{r})\,d\tau$ and the total charge in the atom is given by

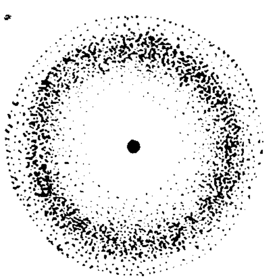

Fig. 1.5

$$\int \rho(\mathbf{r})\,d\tau = -e \tag{1.12}$$

The charge density also is a function of position, but it is a scalar quantity and its field is a scalar field.

In the case of continuous charge distribution the electric field is given by

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\mathbf{r})\,(\mathbf{r}-\mathbf{r}_i)}{|\mathbf{r}-\mathbf{r}_i|^3}\,d\tau \tag{1.13}$$

When the charge is distributed over surface, we talk of surface charge density or charge per unit area $\sigma(\mathbf{r})$. In this case