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RANALD V. GILES



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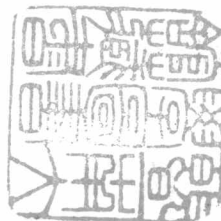
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SCHAUM'S OUTLINE OF
THEORY AND PROBLEMS
of
FLUID MECHANICS
and HYDRAULICS

SI (METRIC) EDITION



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SCHAUM'S OUTLINE SERIES

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Preface to SI Edition

The decision to produce an SI version of the second edition of *Fluid Mechanics and Hydraulics* was prompted by the increasing use of SI units in teaching and industry. A few textbooks have been produced in SI units, but there exists a demand for a book of worked examples.

SI units are still in a transition stage and common usage continues to modify the system. Large amounts of data are readily available only in foot-pound units and will be for some time to come. We have therefore included some conversion factors particularly for pseudo-dimensionless numbers such as specific speed.

This edition is essentially an adaptation of the second edition and all the examples have been retained and, where necessary, converted. In some cases the conversions have been rounded to convenient numbers, but in many cases accuracy or usage has required less convenient numbers. We believe that this method has considerable merit since practising engineers deal with numerical values as they arise and these are seldom round numbers. While our conversions have been made with a high accuracy, the answers to the unworked problems are quoted to slide rule accuracy only. This approach has been adopted since it is important that an engineer should gain some appreciation of the actual and necessary accuracy of his working and should not be deluded by the pretentious accuracy of his calculator. The tables have been converted to conform with the text, but, in addition, some useful foot-pound units have to be retained.

We wish to express our gratitude to the staff of McGraw-Hill (UK) for their helpful cooperation.

D. J. POLLARD

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Guildford
August 1976

Symbols and Abbreviations

The following tabulation lists the letter symbols used in this book. Because the alphabet is limited, it is impossible to avoid using the same letter to represent more than one concept. Since each symbol is defined when it is first used, no confusion should result.

a	acceleration in m/s^2 , area in m^2	I	moment of inertia in m^4
A	area in m^2	I_{xy}	product of inertia in m^4
b	weir length in m, width of water surface in m, bed width of open channel in m	k	ratio of specific heats, isentropic (adiabatic) exponent, von Karman constant
c	coefficient of discharge, celerity of pressure wave in m/s (acoustic velocity)	K	discharge factors for trapezoidal channels, lost head factor for enlargements, any constant
c_c	coefficient of contraction	K_c	lost head factor for contractions
c_v	coefficient of velocity	l	mixing length in m
C	coefficient (Chezy), constant of integration	L	length in m
CG	center of gravity	L_E	equivalent length in m
C_p	center of pressure, power coefficient for propellers	m	roughness factor in Bazin formula, weir factor for dams
C_D	coefficient of drag	\dot{m}	mass flow rate
C_F	thrust coefficient for propellers	M	mass in Kg, molecular mass
C_L	coefficient of lift	n	roughness coefficient, exponent, roughness factor in Kutter's and Manning's formulas
C_T	torque coefficient for propellers		
C_1	Hazen-Williams coefficient	N	rotational speed in rev/min
cfs	cubic feet per second	N_s	specific speed
d, D	diameter in m	N_u	unit speed
D_1	unit diameter in m	N_F	Froude number
e	efficiency	N_M	Mach number
E	bulk modulus of elasticity in Pa or N/m^2 , specific energy in Nm/N or J/N	N_W	Weber number
f	friction factor (Darcy) for pipe flow	p	pressure, wetted perimeter in m
F	force in N, thrust in N	p'	pressure
g	gravitational acceleration in m/s^2 = 9.81 m/s^2	P	force in N, power in kW
gpm	gallons per minute	P_u	unit power
h	head in m, height or depth in m, pressure head in m	psf	lb/ft^2
H	total head (energy) in m or J/N	psia	lb/in^2 , absolute
H_L, h_L	lost head in m (sometimes LH)	psig	lb/in^2 , gage
hp	horsepower = 0.746 kW	q	unit flow in $\text{m}^3/\text{s}/\text{unit width}$
		Q	volume rate of flow in m^3/s
		Q_u	unit discharge
		r	any radius in m

r_o	radius of pipe in m	v_s	specific volume = $1/\rho = \text{m}^3/\text{kg}$
R	gas constant, hydraulic radius in m	v_*	shear velocity in $\text{m/s} = \sqrt{\tau/\rho}$
R_E	Reynolds Number	V	average velocity in m/s (or as defined)
rl dn	relative density	V_c	critical velocity in m/s
S	slope of hydraulic grade line, slope of energy line	w	weight in N/m^3
S_o	slope of channel bed	W	weight in N, weight flow in $\text{N/s} = \rho g Q$
sp gr	specific gravity	x	distance in m
t	time in s, thickness, viscosity in Saybolt sec	y	depth in m, distance in m
T	temperature, torque in Nm, time in s	y_c	critical depth in m
u	peripheral velocity of rotating element in m/s	y_N	normal depth in m
u, v, w	components of velocity in X, Y and Z directions	Y	expansion factors for compressible flow
v	volume in m^3 , local velocity in m/s , relative velocity in hydraulic machines in m/s	z	elevation (head) in m
		Z	height of weir crest above channel bottom, in m
α (alpha)	angle, kinetic-energy correction factor		
β (beta)	angle, momentum correction factor		
δ (delta)	boundary layer thickness in m		
Δ (delta)	flow correction term		
ϵ (epsilon)	surface roughness in m		
η (eta)	eddy viscosity		
θ (theta)	any angle		
μ (mu)	absolute viscosity in Pa s (or poises)		
ν (nu)	kinematic viscosity $\text{m}^2/\text{s} = \mu/\rho$		
π (pi)	dimensionless parameter		
ρ (rho)	density kg/m^3		
σ (sigma)	surface tension in N/m , intensity of tensile stress in N/m^2 (or Pa)		
τ (tau)	shear stress in N/m^2 (or Pa)		
ϕ (phi)	speed factor, velocity potential, ratio		
ψ (psi)	stream function		
ω (omega)	angular velocity in rad/s		

Useful Conversion Factors

1 cubic foot	= 7.48 U.S. gallons = 28.32 litres = 0.02832 m^3
1 U.S. gallon	= 8.338 pounds of water at 60°F = $3.785 \times 10^{-3} \text{ m}^3$
1 cubic foot per second	= 0.646 million gallons per day = 448.8 gallons per minute = $0.02832 \text{ m}^3/\text{s}$
1 pound-second per square foot (μ)	= 478.7 poises = 47.87 Pa s (or kg/ms)
1 square foot per second (ν)	= 929 square centimeters per second = $0.0929 \text{ m}^2/\text{s}$
1 horsepower	= 550 foot-pounds per second = 0.746 kilowatts
30 inches of mercury	= 10.3 m of water = 14.7 pounds per square inch = 101 353 Pa
1 bar	= 10^5 Pa
1 poise	= 10^{-1} Pa s
1 stokes	= $10^{-4} \text{ m}^2/\text{s}$

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Chapter One

Properties of Fluids

FLUID MECHANICS and HYDRAULICS

Fluid mechanics and hydraulics represent that branch of applied mechanics dealing with the behavior of fluids at rest and in motion. In the development of the principles of fluid mechanics, some fluid properties play principal roles, others only minor roles or no roles at all. In fluid statics, weight is the important property, whereas in fluid flow, density and viscosity are predominant properties. Where appreciable compressibility occurs, principles of thermodynamics must be considered. Vapor pressure becomes important when negative pressures (gage) are involved, and surface tension affects static and flow conditions in small passages.

DEFINITION of a FLUID

Fluids are substances which are capable of flowing and which conform to the shape of containing vessels. When in equilibrium, fluids cannot sustain tangential or shear forces. All fluids have some degree of compressibility and offer little resistance to change of form.

Fluids may be divided into liquids and gases. The chief differences between liquids and gases are (a) liquids are practically incompressible whereas gases are compressible and often must be so treated and (b) liquids occupy definite volumes and have free surfaces whereas a given mass of gas expands until it occupies all portions of any containing vessel.

S.I. UNITS

Three selected reference dimensions (fundamental dimensions) are mass, length and time. In this book the corresponding fundamental units used will be the kilogram (kg) of mass, the metre (m) of length and the second (s) of time. All other units may be derived from these. The unit of force derived from these units is the newton (N). Thus unit volume is the m^3 , unit acceleration is the m/s^2 , unit work is the Nm called the joule (J), and unit pressure is the N/m^2 called the pascal (P). Should data be given in other units, they must be converted to S.I. Units before applying them to the solution of problems.

The unit for force in this system, the newton, is derived from the units of mass and acceleration. From Newton's second law,

$$\text{force in newtons} = \text{mass in kilograms} \times \text{acceleration in } \text{m/s}^2 \quad (1)$$

or

a force of 1 newton accelerates a mass of 1 kilogram at the rate 1 m/s^2 .

MASS DENSITY OF A SUBSTANCE (ρ)

The density of a substance is the mass of unit volume of the substance. For liquids the density may be taken as constant for practical changes of pressure. The density of water is 1000 kg/m^3 at 4°C . See Appendix, Tables 1C and 2 for additional values.

The density of gases may be calculated using the *equation of state* for the gas.

or
$$\frac{pv_s}{T} = R \text{ (Boyle's and Charles' laws)} \quad (2)$$

where p_3 is absolute pressure in pascals, specific volume v_s per unit mass m^3/kg , temperature T is the absolute temperature in degrees Kelvin ($273 + \text{degrees Celsius}$) and R is the gas constant in J/kg K . Since $\rho = 1/v_s$ the above equation may be written

$$\rho = \frac{p}{R T} \quad (3)$$

On occasions particularly in dealing with liquids the product ρg is used, where g is the gravitational acceleration 9.81 m/s^2 nominally. Formerly this product was called *specific weight* and given the symbol w . In S.I. units the prefix *specific* must be used solely to describe properties per unit mass and the term specific weight is no longer used.

RELATIVE DENSITY of a BODY (rl dn) [Formerly Specific Gravity]

The relative density of a body is that pure number which denotes the ratio of the mass of a body to the mass of an equal volume of a substance taken as a standard. Solids and liquids are referred to water (at 4°C) as standard, while gases are often referred to air free of CO_2 or hydrogen (at 0°C and 1 atmosphere = $1.013 \times 10^5 \text{ Pa}$ pressure) as standard. For example,

$$\begin{aligned} \text{relative density of a substance} &= \frac{\text{mass of the substance}}{\text{mass of equal volume water}} \\ &= \frac{\text{density of substance}}{\text{density of water}} \end{aligned} \quad (4)$$

Thus if the relative density of a given oil is 0.750, its density is $0.750 (1000 \text{ kg/m}^3) = 750 \text{ kg/m}^3$.

The relative density of water is 1.00 and of mercury is 13.57. The relative density of a substance is the same in any system of measures. See Appendix, Table 2.

VISCOSITY of a FLUID

The viscosity of a fluid is that property which determines the amount of its resistance to a shearing force. Viscosity is due primarily to interaction between fluid molecules.

Referring to Fig. 1-1, consider two large, parallel plates at a small distance y apart, the space between the plates being filled with a fluid. Consider the upper plate acted on by a constant force F and hence moving at a constant velocity U .

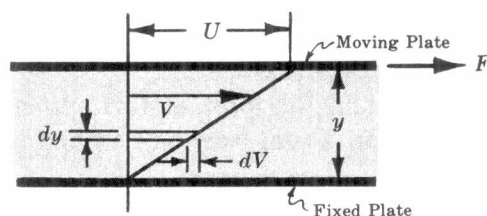


Fig. 1-1

The fluid in contact with the upper plate will adhere to it and will move at velocity U , and the fluid in contact with the fixed plate will have velocity zero. If distance y and velocity U are not too great, the velocity variation (gradient) will be a straight line. Experiments have shown that force F varies with the area of the plate, with velocity U , and inversely with distance y . Since by similar triangles, $U/y = dV/dy$, we have

$$F \propto \frac{AU}{y} = A \frac{dV}{dy} \quad \text{or} \quad \frac{F}{A} = \tau \propto \frac{dV}{dy}$$

where $\tau = F/A =$ shear stress. If a proportionality constant μ (mu), called the *absolute (dynamic) viscosity*, is introduced,

$$\tau = \mu \frac{dV}{dy} \quad \text{or} \quad \mu = \frac{\tau}{dV/dy} \quad (5)$$

The units of μ are Pa s, since $\frac{\text{Pa}}{(\text{m/s})/\text{m}} = \text{Pa s}$. Fluids which follow the relation of equation (5) are called *Newtonian fluids* (see Problem 9).

Another viscosity coefficient, the *kinematic coefficient of viscosity*, is defined as

$$\text{Kinematic coefficient } \nu \text{ (nu)} = \frac{\text{absolute viscosity } \mu}{\text{mass density } \rho}$$

$$\text{or} \quad \nu = \frac{\mu}{\rho} \quad (6)$$

The units of ν are $\frac{\text{m}^2}{\text{s}}$, since $\frac{\text{Pa s}}{\text{kg/m}^3} = \frac{\text{kg/ms}}{\text{kg/m}^3} = \frac{\text{m}^2}{\text{s}}$.

Viscosities are reported in handbooks as poises and stokes (cgs units) and on occasion as Saybolt seconds, from viscosimeter measurements. Conversions to S.I. units are illustrated in Problems 6–8. A few values of viscosities are given in Tables 1 and 2 of the Appendix.

Viscosities of liquids decrease with temperature increases but are not affected appreciably by pressure changes. The absolute viscosity of gases increases with increase in temperature but is not appreciably changed due to pressure. Since the density of gases changes with pressure changes (temperature constant), the kinematic viscosity varies inversely as the pressure. However, from the equation above, $\mu = \rho \nu$.

VAPOR PRESSURE

When evaporation takes place within an enclosed space, the partial pressure created by the vapor molecules is called vapor pressure. Vapor pressures depend upon temperature and increase with it. See Table 1C for values for water.

SURFACE TENSION

A molecule in the interior of a liquid is under attractive forces in all directions, and the vector sum of these forces is zero. But a molecule at the surface of a liquid is acted on by a net inward cohesive force which is perpendicular to the surface. Hence it requires work to move molecules to the surface against this opposing force, and surface molecules have more energy than interior ones.

The surface tension of a liquid is the work that must be done to bring enough molecules from inside the liquid to the surface to form one new unit area of that surface (Nm/m^2). The work is numerically equal to the tangential contractile force acting across a hypothetical line of unit length on the surface (Nm).

In most problems of introductory fluid mechanics, surface tension is not of particular importance. Table 1C gives values of surface tension σ (sigma) for water in contact with air.

CAPILLARITY

The rise or fall of a liquid in a capillary tube (or in some equivalent circumstance, such as in porous media) is caused by surface tension and depends on the relative magnitudes of the cohesion of the liquid and the adhesion of the liquid to the walls of the containing vessel. Liquids rise in tubes they wet (adhesion $>$ cohesion) and fall in tubes they do not wet (cohesion $>$ adhesion). Capillarity is important when using tubes smaller than about 10 mm in diameter.

FLUID PRESSURE

Fluid pressure is transmitted with equal intensity in all directions and acts normal to any plane. In the same horizontal plane the pressure intensities in a liquid are equal. Measurements of unit pressures are accomplished by using various forms of gages. Unless otherwise stated, gage or relative pressures will be used throughout this book. Gage pressures represent values above or below atmospheric pressure.

UNIT PRESSURE or PRESSURE is expressed as force divided by area. In general,

$$p \text{ (N/m}^2 \text{ or Pa)} = \frac{dP \text{ (N)}}{dA \text{ (m}^2\text{)}}$$

For conditions where force P is uniformly distributed over an area, we have

$$p \text{ (Pa)} = \frac{P \text{ (N)}}{A \text{ (m}^2\text{)}} \quad \text{and} \quad p' \text{ (bar)} = \frac{P \text{ (N)}}{A \text{ (m}^2\text{)}} \times 10^{-5}$$

DIFFERENCE in PRESSURE

Difference in pressure between any two points at different levels in a liquid is given by

$$p_2 - p_1 = \rho g(h_2 - h_1) \text{ in Pa} \quad (7)$$

where ρg = unit weight of the liquid (N/m^3) and $h_2 - h_1$ = difference in elevation (m).

If point 1 is in the free surface of the liquid and h is positive downward, the above equation becomes

$$p = \rho gh \text{ (in Pa), a gage pressure} \quad (8)$$

To obtain the bar pressure unit, we use

$$\text{gage pressure } p' = \frac{p}{10^5} = \frac{\rho gh}{10^5} \text{ (in bar)} \quad (9)$$

These equations are applicable as long as ρ is constant (or varies so slightly with h as to cause no significant error in the result).

PRESSURE VARIATIONS in a COMPRESSIBLE FLUID

Pressure variations in a compressible fluid are usually very small because of the small unit weights and the small differences of elevation being considered in hydraulic calculations. Where such differences must be recognized for small changes in elevation dh , the law of pressure variation may be written

$$dp = -\rho g dh \quad (10)$$

The negative sign indicates that the pressure decreases as the altitude increases, with h positive upward. For applications, see Problems 29–31.

PRESSURE HEAD h

Pressure head h represents the height of a column of homogeneous fluid that will produce a given intensity of pressure. Then

$$h \text{ (m of fluid)} = \frac{p \text{ (Pa)}}{\rho g \text{ (N/m}^3\text{)}} \quad (11)$$

BULK MODULUS of ELASTICITY (E)

The bulk modulus of elasticity (E) expresses the compressibility of a fluid. It is the ratio of the change in unit pressure to the corresponding volume change per unit of volume.

$$E = \frac{dp'}{-dv/v} = \frac{\text{Pa}}{\text{m}^3/\text{m}^3} = \text{Pa (or N/m}^2\text{)} \quad (12)$$

COMPRESSION of GASES

Compression of gases may occur according to various laws of thermodynamics. For the same mass of gas subjected to two different conditions,

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} = MR \quad \text{and} \quad \frac{p_1}{\rho_1 T_1} = \frac{p_2}{\rho_2 T_2} = R \quad (13)$$

where p = absolute pressure in Pa, v = volume in m^3 , M = mass in kg, ρ = density in kg/m^3 , R = gas constant in J/kg K, T = absolute temperature in degrees Kelvin ($273 + ^\circ\text{C}$).

FOR ISOTHERMAL CONDITIONS (constant temperature) the above expression (13) becomes

$$p_1 v_1 = p_2 v_2 \quad \text{and} \quad \frac{\rho_1}{\rho_2} = \frac{p_1}{p_2} = \text{constant} \quad (14)$$

Also, Bulk Modulus $E = p$ (in Pa) (15)

FOR REVERSIBLE ADIABATIC or ISENTROPIC CONDITIONS (no heat exchanged) the above expressions become

$$p_1 v_1^k = p_2 v_2^k \quad \text{and} \quad \left(\frac{\rho_1}{\rho_2}\right)^k = \frac{p_1}{p_2} = \text{constant} \quad (16)$$

Also
$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k} \quad (17)$$

and Bulk Modulus $E = kp$ (in Pa) (18)

where k is the ratio of the specific heat at constant pressure to the specific heat at constant volume. It is known as the isentropic exponent.

Table 1A in the Appendix lists some typical values of R and k . For many gases, R times molecular weight is about 8314.

PRESSURE DISTURBANCES

Pressure disturbances imposed on a fluid move in waves. These pressure waves move at a velocity equal to that of sound through the fluid. The velocity, or celerity, in m/s is expressed as

$$c = \sqrt{E/\rho} \quad (19)$$

where E must be in Pa. For gases, this acoustic velocity is

$$c = \sqrt{kp/\rho} = \sqrt{kRT} \quad (20)$$

Solved Problems

1. Calculate the density ρ and specific volume v_s of methane at 40°C and an absolute pressure of 8.3 bar.

Solution:

From Table 1A in the Appendix, $R = 96.3 \times 5.38 = 518$

$$\text{Density } \rho = \frac{p}{RT} = \frac{8.3 \times 10^5}{518(273 + 40)} = 5.1 \text{ kg/m}^3$$

$$\text{Specific volume } v_s = \frac{1}{\rho} = \frac{1}{5.1} = 0.196 \text{ m}^3/\text{kg}$$

2. If 5.6 m³ of oil weighs 46 800 N, calculate its density ρ and relative density.

Solution:

$$\text{Weight of Unit Volume} = \rho g = \frac{46\,800}{5.6} = 8360 \text{ N/m}^3$$

$$\text{Density } \rho = \frac{\rho g}{g} = \frac{8360}{9.81} = 852 \text{ kg/m}^3$$

$$\text{Relative Density} = \frac{\rho_{\text{oil}}}{\rho_{\text{water}}} = \frac{852}{1000} = 0.852$$

3. At 32°C and 2 bar absolute the specific volume v_s of a certain gas was 0.74 m³/kg. Determine the gas constant R and the density ρ .

Solution:

$$\text{Since } \rho = \frac{p}{RT}, \text{ then } R = \frac{p}{\rho T} = \frac{pv_s}{T} = \frac{(2 \times 10^5)(0.74)}{(273 + 32)} = 485.2$$

$$\text{Density } \rho = \frac{1}{v_s} = \frac{1}{0.74} = 1.35 \text{ kg/m}^3$$

4. (a) Find the change in volume of 1.00 m^3 of water at 26.7°C when subjected to a pressure increase of 20 bar. (b) From the following test data determine the bulk modulus of elasticity of water: at 35 bar the volume was 1.000 m^3 and at 240 bar the volume was 0.990 m^3 .

Solution:

(a) From Table 1C in the Appendix, E at 26.7°C is $2.24 \times 10^9 \text{ Pa}$. Using formula (12),

$$dv = -\frac{v dp'}{E} = -\frac{1.00 \times 20 \times 10^5}{2.24 \times 10^9} = -0.00089 \text{ m}^3$$

(b) The definition associated with formula (12) indicates that *corresponding* changes in pressure and volume must be considered. Here an increase in pressure corresponds to a decrease in volume.

$$E = -\frac{dp'}{dv/v} = -\frac{(240 - 35)10^5}{(0.990 - 1.000)/1.000} = 2.05 \times 10^9 \text{ Pa} = 2.05 \text{ GPa}$$

5. A cylinder contains 0.35 m^3 of air at 50°C and 2.76 bar absolute. The air is compressed to 0.071 m^3 . (a) Assuming isothermal conditions, what is the pressure at the new volume and what is the bulk modulus of elasticity? (b) Assuming isentropic conditions, what is the final pressure and temperature and what is the bulk modulus of elasticity?

Solution:

(a) For isothermal conditions,

$$p_1 v_1 = p_2 v_2$$

$$\text{Then } (2.76 \times 10^5)0.35 = p_2' \times 10^5)0.071 \text{ and } p_2' = 13.6 \text{ bar}$$

The bulk modulus $E = p' = 13.6 \text{ bar}$.

(b) For isentropic conditions, $p_1 v_1^k = p_2 v_2^k$ and Table 1A in the Appendix gives $k = 1.40$.

$$\text{Then } (2.76 \times 10^5)(0.35)^{1.40} = (p_2' \times 10^5)(0.071)^{1.40} \text{ and } p_2' = 25.8 \text{ bar}$$

The final temperature is obtained by using equation (17)

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k}, \frac{T_2}{(273 + 50)} = \left(\frac{25.8}{2.76}\right)^{0.40/1.40}, T_2 = 612 \text{ Kelvin} = 339^\circ\text{C}$$

The bulk modulus $E = kp' = 1.40 \times 25.8 \times 10^5 = 3.61 \text{ MPa}$.

6. From the International Critical Tables, the viscosity of water at 20°C is 0.010 08 poises. Compute (a) the absolute viscosity in Pa s units. (b) If the relative density at 20°C is 0.998, compute the value of the kinematic viscosity in m^2/s units.

Solution:

The poise is measured in dyne sec/cm². Since $1 \text{ dyne} = 1 \text{ g cm/s}^2 = 10^{-5} \text{ N}$, we obtain

$$1 \text{ poise} = \frac{10^{-5} \text{ N s}}{(10^{-2})^2 \text{ m}^2} = 10^{-1} \text{ Pa s}$$

$$(a) \mu \text{ in Pa s} = 0.01008/10 = 1.008 \times 10^{-3} \text{ Pa s}$$

$$(b) \nu \text{ in m}^2/\text{s} = \frac{\mu}{\rho} = \frac{1.008 \times 10^{-3}}{0.998 \times 1000} = 1.01 \times 10^{-6} \text{ m}^2/\text{s}$$

7. Convert 15.14 poises to kinematic viscosity in m^2/s units if the liquid has relative density 0.964.

Solution:

The steps illustrated in Problem 6 may be taken or an additional factor may be established for water from

$$\frac{1}{10} \times \frac{1}{1000} = 10^{-4} = 0.001078. \text{ Hence } \nu \text{ in m}^2/\text{s} = \frac{15.14 \times 10^{-4}}{\text{rd dn} = 0.964} = 1.57 \times 10^{-3}$$

8. Convert a viscosity of 510 Saybolt seconds at 60°F to kinematic viscosity ν in m^2/s units.

Solution:

Two sets of formulas are given to establish this conversion when the Saybolt Universal Viscosimeter is used:

$$(a) \text{ for } t \leq 100, \mu \text{ in poises} = (0.002\,26t - 1.95/t) \times \text{rl dn}$$

$$\text{for } t > 100, \mu \text{ in poises} = (0.002\,20t - 1.35/t) \times \text{rl dn}$$

$$(b) \text{ for } t \leq 100, \nu \text{ in stokes} = (0.002\,26t - 1.95/t)$$

$$\text{for } t > 100, \nu \text{ in stokes} = (0.002\,20t - 1.35/t)$$

where t = Saybolt second units. To convert stokes (cm^2/s) to m^2/s units, divide by $(100)^2$ or 10^4 .

$$\text{Using group (b), and since } t > 100, \nu = \left(0.002\,20 \times 510 - \frac{1.35}{510}\right) \times 10^{-4} = 11.19 \times 10^{-3} \text{ m}^2/\text{s}.$$

9. Discuss the shear characteristics of the fluids for which the curves have been drawn in Fig. 1-2.

Solution:

(a) The Newtonian fluids behave according to the law $\tau = \mu(dV/dy)$, or the shear stress is proportional to the velocity gradient or rate of shearing strain. Thus for these fluids the plotting of shear stress against velocity gradient is a straight line passing through the origin. The slope of the line determines the viscosity.

(b) For the "ideal" fluid, the resistance to shearing deformation is zero, and hence the plotting coincides with the x-axis. While no ideal fluids exist, in certain analyses the assumption of an ideal fluid is useful and justified.

(c) For the "ideal" or elastic solid, no deformation will occur under any loading condition, and the plotting coincides with the y-axis. Real solids have some deformation and, within the proportional limit (Hooke's law), the plotting is a straight line which is almost vertical.

(d) Non-Newtonian fluids deform in such a way that shear stress is not proportional to rate of shearing deformation, except perhaps at very low shear stresses. The deformation of these fluids might be classified as plastic.

(e) The "ideal" plastic material could sustain a certain amount of shearing stress without deformation, and thereafter it would deform in proportion to the shearing stress.

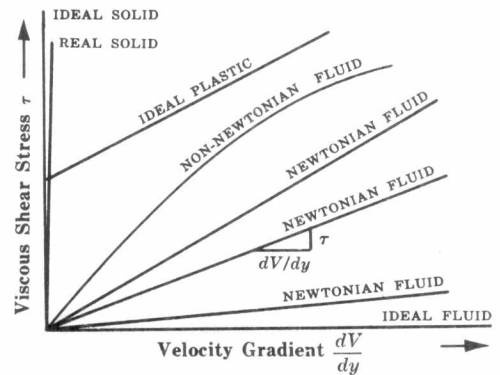


Fig. 1-2

10. Refer to Fig. 1-3. A fluid has absolute viscosity 0.048 Pa s and relative density 0.913. Calculate the velocity gradient and the intensity of shear stress at the boundary and at points 25 mm, 50 mm, and 75 mm from the boundary, assuming (a) a straight line velocity distribution and (b) a parabolic velocity distribution. The parabola in the sketch has its vertex at A. Origin is at B.

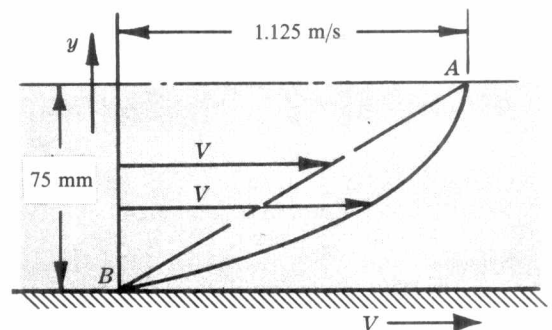


Fig. 1-3