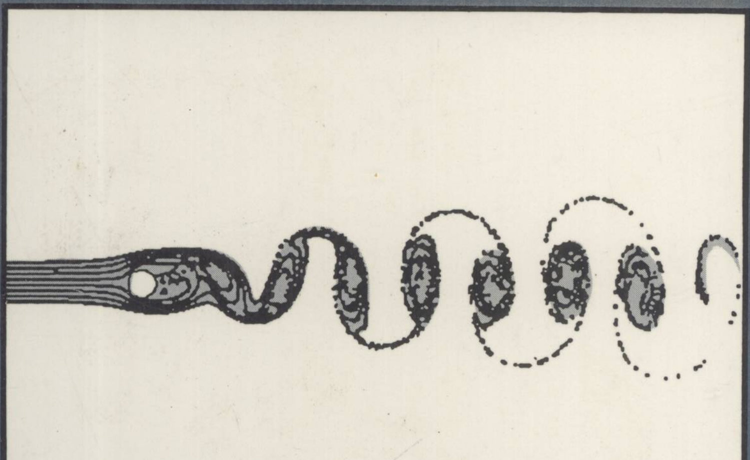
Flow Analysis Using a PC

H. Ninomiya and K. Onishi



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Base SI units used in the text:

Quantity name	Unit name	Unit symbol
length	meter	m
mass	kilogram	kg
time	second	s
thermodynamic temperature	kelvin	K
	(Supplementary unit)	
plane angle	radian	rad

Derived SI units from the base units:

Physical quantity	SI unit (symbol)	Definition for unit
force	newton (N)	$1N = 1kg \times 1m/s^2,\ kg\,m/s^2$
pressure	pascal (Pa)	$1Pa = 1N/m^2,\ kg\,/m\,s^2$
energy	joule (J)	$1J = 1Nm,\ kg\,m^2/s^2$
power	watt (W)	$1W = 1J/s,\ kg\,m^2/s^3$
	(Customary units)	
pressure	standard atmosphere (atm)	$1atm = 101325Pa$
Celsius temperature	degree Celsius (^oC)	$\theta(^oC) = T - 273.15(K)$
energy	calorie (cal)	$1cal\,(15^oC) = 4.1855J$

PREFACE

This book is written expressly as a first course in numerical fluid mechanics. We have carefully selected fundamental and important flows of practical interest from various fluid flows and formulated them in the framework of the finite element method. Comparatively low cost personal computers such as the IBM-PC, NEC-PC, or more promising recent EWS (Engineering Work Stations) are assumed to be used. BASIC programs are provided on a diskette which accompanies the text. The aim of this book is to present a short introductory course on simple numerical modelling of fluid flows, using the elementary finite element method.

Flows of various degrees of complexity occur naturally. In this book, we shall mainly consider two-dimensional flows. We start our course with potential flows, which seem to be the simplest. The reader will be led easily through some typical examples in numerical computation of potential flow problems to the notion of flow velocity, velocity potential, streamfunction, and boundary conditions. The governing equation is the Laplace equation. As a starting point for transient analysis, we next consider unsteady heat conduction in solids. The corresponding governing equation is a parabolic equation. We then move to consideration of incompressible viscous fluid. The governing equations here are Navier-Stokes equations, which are expressed in terms of streamfunction and vorticity. The reader will learn related initial and boundary conditions. The discussion is extended to thermal convection due to buoyancy effects which include natural convection, forced convection, density-dependent convective diffusion, and the analysis of tidal currents in shallow sea water.

We confine ourselves to an elementary finite element method for the computational method. The minimum requisite of the finite element method is discussed in each chapter. We use the Ritz-Galerkin method as an introduction to the finite element method. Triangular 3-node linear elements are used exclusively for the two-dimensional analysis. These types of elements may not be the most efficient, but are the most familiar to beginners. This simple tool is valuable for solving the problems considered in this book. One should notice that some problems can be successfully formulated using only higher order finite elements in the flow analysis. Some special techniques will eventually lead to useful applications of the finite element method in computational fluid dynamics. However, detailed discussions are beyond the scope of this book. Interested readers are recommended to consult other more advanced references.

The best way to understand numerical modelling is to practise on available computers. Some sophisticated modelling concepts will reveal themselves through computation. Through practice, one can learn optimal mesh generation, appropriate boundary conditions, suitable time step size, *etc.* For this purpose, some numerical examples are included throughout the text as well as a series of exercises at the end of each chapter. As is often said, to write down mathematical expressions on a paper sheet is easy, but to obtain numerical solutions in practice is difficult. Experience of methods for numerical computation can be obtained through this work which should be very valuable.

A diskette of computer programs coded in BASIC is supplied with the text. They will run on standard PC (Personal Computers) and EWS. Perhaps it is worthwhile to note here that a 'computer program' is not synonymous with 'software'. 'Software' includes not only the programs but the technique of implementation. Unfortunately there are many people with much experience in programming but with no knowledge of how to input data, how to run the program, or even what was obtained from their output files. The users usually need more information than the programmer anticipated. To avoid such pitfalls, we offer in this book not only programs corresponding to respective flow problems, but also programs for automatic mesh generation, plotting of the mesh, renumbering of nodes as preprocessor, as well as to provide contour and vector plots of the numerical results as postprocessor.

The authors of this book are a physicist and an applied mathematician. Some useful programs they developed during their individual research activities were collected and edited in the form of a book. It was published in Japanese under the same title, and is the basis of this book. However, much new material and discussion has been added. All programs have been made compatible with IBM-PC. The revision is so extensive that we felt that we have virtually written a new book. Valuable advice and criticism from a wide variety of Japanese readers are cordially acknowledged and some of them were incorporated into this new edition.

Ninomiya and I wish to express our indebtedness to many people. We are most grateful to Mr Kenji Hayashi of Japan Weather Association for making the first Japanese edition of the book possible. We are equally grateful to Dr Carlos A. Brebbia of Computational Mechanics Institute in Southampton, England for his continuing encouragement. We also would like to express our appreciation to Dr Sebastian Koh for helpful suggestions and a careful reading of a part of the manuscript. Many thanks are due to Mrs Kinko Kobayashi, Mrs Yoko Ohura and my students for the preparation of the manuscript.

Kazuei Onishi

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Chapter 1

BASIC FLOW PROPERTIES

To find numerical solutions of fluid flow problems, one requires basic equations governing the fluid flow. These equations are derived from laws of physics. In addition to the basic equations, one requires a knowledge of boundary conditions as well as physical properties of the fluid. In this chapter, we shall present some preliminaries for viscosity and thermal property of the fluid, which may strongly affect the flow behavior. We shall also present the basic flow equations and associated boundary conditions for two-dimensional flows in rectangular coordinate system.

1.1 Fundamentals in Flow Analysis

The three states of matter can be classified into a solid, liquid and gas. Liquid and gas are commonly called *fluids*. The main distinction between a liquid and a gas lies in their rate of change in the density. The density of gas changes more readily than that of liquid. However, they can be treated in the same way without taking into account the change of density, provided that the speed of flow is low as compared with the speed of sound propagating in the fluid. The fluid is called *incompressible* if the change of the density is negligible.

1.1.1 Streamlines and streamfunction

Suppose that ink is injected into a gently moving fluid. We can observe a streak of ink, as shown by the bold curve in Figure 1.1. The curve thus obtained is called a *streakline*. In general, the ink at the point B travelled there, not along the streakline but along a different curve, such as the one shown by the dotted line. This curve is called a *particle trajectory*.

We can consider yet another curve, which presents the flow pattern at the instant the ink reaches the point B. This curve is defined in such a way that the

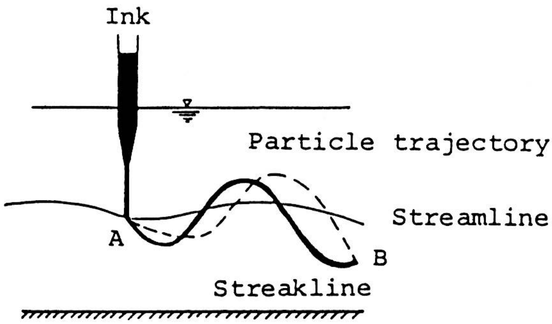

Figure 1.1: Movement of ink.

tangent to the curve and the flow velocity at the instant have the same direction at every point on the curve. The curve thus defined is called a *streamline.*

These curves coincide, only when the flow pattern does not change with time. In this case, the flow is called a *steady flow.* If the flow pattern changes with time, then it is called an *unsteady* or *transient flow.*

We denote by u and v the x- and y- components of the flow velocity (m/s), respectively. Since the flow vector with the components u, v at every point on the streamline has the same direction as the tangent vector at the same point, we have

$$\frac{dx}{u} = \frac{dy}{v} . \tag{1.1}$$

A planar curve can be generally expressed by means of a function in two variables. We consider the function $\psi(x, y)$ such that the relation: $\psi(x, y)$ = const., represents the streamlines. When ψ is a smooth function, the total derivative along the line must be equal to zero. Namely,

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy = 0 . \tag{1.2}$$

The function ψ is called a *streamfunction.* As a possible consequence, it follows from (1.1) and (1.2) that

$$u = \frac{\partial \psi}{\partial y} \quad , \quad v = -\frac{\partial \psi}{\partial x} . \tag{1.3}$$

These relations reveal an important fact that the partial derivatives of ψ form a vector which, after rotating 90° clockwise, coincides with the velocity vector.

1.1.2 Viscosity and vorticity

Let us consider forces acting in a fluid that is stationary in a vessel. The *gravity force* is acting in the isothermal fluid. Since the fluid is stationary, a counterforce must be acting in the fluid. This force is called *pressure* which will be denoted by

$p\,(Pa)$. Strictly speaking, the pressure is not a force. It is a force acting on any surface in the normal direction. The force per unit area (N/m^2) is called ***stress.*** Therefore, the pressure is a normal stress. Figure 1.2 illustrates the equilibrium of forces. Here we consider an imaginary cube in the fluid. Pressures acting on vertical surfaces balance out. The difference between the pressure acting on top and bottom surfaces exerts an upward force, known as ***buoyancy force***, which balances with the gravity force. As the result, the fluid remains stationary.

The moving viscous fluid encounters an internal frictional force in the direction of motion due to the ***viscosity.*** This force is expressed as ***shearing stress***, which acts on the unit area of a surface in the tangential direction. Consider a fluid at rest between two parallel plates. If one of the plates begins to move at constant velocity as shown in Figure 1.3, a motion is induced in the fluid. Through this process, the momentum is transported by the fluid due to its viscosity.

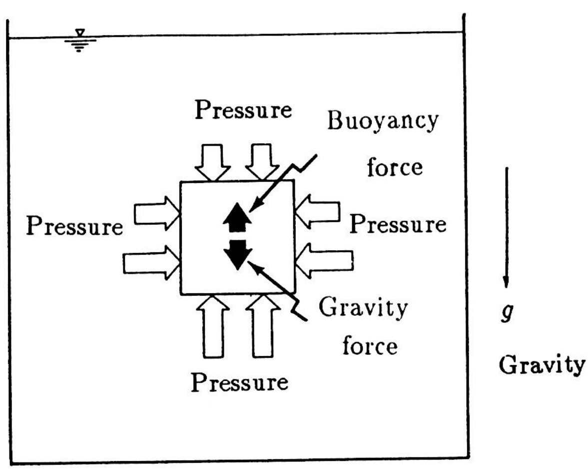

Figure 1.2: Forces acting in fluid at rest.

For most types of fluid, the shearing stress $\tau\,(Pa)$ is proportional to the negative of the spatial variation of the flow velocity du/dy, which is known as the ***rate of strain.*** Such fluid is called a *Newtonian fluid.* We have thus

$$\tau = -\,\mu \frac{du}{dy}\,, \tag{1.4}$$

where the proportionality constant μ is called the ***viscosity coefficient*** $(Pa \cdot s)$. The quotient of viscosity divided by the density $\rho\,(kg/m^3)$ of the fluid is called the ***kinematic viscosity*** (m^2/s), and it is written as

$$\nu = \mu/\rho\,. \tag{1.5}$$