TIME SERIES ANALYSIS:

SMOOTHING BY STAGES

MAVERICK

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PREFACE

THIS book is concerned with trends or smoothing lines, and with cycles. In difficulty it is designed for the student or statistical clerk who has had but little training in statistics. He should have fitted straight line trends by least squares, and have calculated seasonal indexes and the normal line; he should be familiar with the use of the normal line as a base of reference in studying business cycles; he should have plotted time series, trends, and moving averages on quadrille paper and on semi-logarithmic paper; and he should be ready to form conclusions as to the relative advantages, as trends, of the straight line and sundry curves.

BUT in this Preface the work must be defended; consequently appeal is made here to more highly qualified readers.

THE process of time series analysis through "smoothing by stages" owes much to Ragnar Frisch, whose ideas underlie the whole. A minor difference from Frisch is in the emphasis upon moving averages rather than upon points of inflection (see the first reference, below, to Econometrica; in that article, the writer followed Frisch closely, even to the extent of relying upon points of inflection). Other important predecessors are Simon S. Kuznets and C. A. R. Wardwell.

THE method of smoothing by stages has been described or used by the writer in several publications: "Time Series, their Analysis by Successive Smoothings", Econometrica, I, 2, July 1933, pp. 238-246. "Cycles in Real Estate Activity", Journal of Land and Public Utility Economics, VIII, 2, May 1932, pp. 191-199. "Cycles in Real Estate Activity, Los Angeles County", ibid., IX, 1, Feb. 1933, pp. 52-56."Real Estate Activity in California", ibid., X, 3, Aug. 1934, pp. 291-295. Economic and Social Statistics, University of California Press, 1941.

THE method was used and commended by Elizabeth Waterman Gilboy in"Time Series and the Derivation of Demand and Supply Curves: A Study of Coffee and Tea 1850-1930", Quarterly Journal of Economics, XLVIII, August 1934, pp. 667-685.

THE procedure has been improved and reduced to routine, so that now it seems appropriate to present a full report, with illustrations, in the hope that statisticians may find it useful.

A JUNIOR statistician who follows this method of analyzing time series can arrive at serviceable smoothing lines, of which that of the highest order will approximate the underlying or secular trend. The method gives him tools to accomplish a cycle analysis which formerly lay wholly beyond his powers. The smoothing lines and cycles give him material for a rich description of each time series, so that comparison between series can be made similarly extensive. This information enables him to make a good mechanical forecast; to be sure, a mechanical forecast is inadequate, and a really adequate forecast must always lie beyond the powers of anyone not master both of the technical processes of statistics and of the field studied; but the mechanical forecast is at least an excellent beginning, upon which someone more expert may make modifications to allow for expected forces and tendencies.

THE method of smoothing by stages, when employed by an accomplished statistician, saves no time in determining the trend line, for there is an unavoidable amount of detail in the process. But the highest order smoothing line probably furnishes a closer fitting trend than he can secure by a total algebraic process. And he may rest assured that for him too, as well as for the novice, the method gives new powers in the fields of cycle analysis, the comparison of time series, and forecasting.

MANY have undertaken cycle analysis, using other methods of segregating the cyclical movements from the non-recurrent components of the series. A leader among such statisticians is Simon S. Kuznets; extensive reference is made to one of his books in Chapter V herein, and to an article in this Preface. See also in Wesley C. Mitchell, Business Cycles, the chapter on Statistics. Analyses by these men and by others have shown the significance of this phase of time series study. It is hoped that statisticians may find that the method of smoothing by stages improves the tools for the study.

CONSIDER now a classification of types of trends, and place among the categories the smoothing lines secured under the method of "smoothing by stages". The chief reference will be to the searching theoretical article by Simon S. Kuznets, "On the Analysis of Time Series". (1)

TRENDS fall into two broad classes: empirical, and mathematically fitted by some total process. An empirical trend grows out of the data in its own vicinity in time, by an inductive process. It fits close to the plotted points representing the data. It has no preconceived form, and when it has been located, it usually defies description by a mathematical equation. Examples of empirical trends are moving averages and lines drawn free-hand. As will be seen, the smoothing lines secured through the method of "smoothing by stages" combine the properties of these two sub-classes. Most critics have granted that the moving average is satisfactorily objective (the subjective element lying principally in the choice of the length or period of the average), but some contend that the free-hand trend has been so subjective as to call for complete rejection. In the process of smoothing by stages, there is some departure from the moving average in the direction of a free-hand curve, but the process is protected by several objective criteria.

KUZNETS, in the article just referred to, "On the Analysis of Time Series", questions whether an empirical trend can contain enough internal evidence of the persistence of form through successive periods, to warrant a forecast. In response, it may be pointed out that the moving average shows the actual local central tendency of the variable through each cycle,with a faithfulness to current conditions that cannot be approached by a trend line fitted by a total mathematical procedure. Consequently, any persistence of form in the moving average furnishes a much better basis for a forecast than does a similar apparent persistence of form in a total mathematical curve. To be sure, persistence of form can be shown more reliably by another mathematical procedure: if the period be broken into parts and a trend fitted separately to each short part, the series of trends so secured will give a satisfactory basis to judge persistence of trend form; but this procedure is quite different from fitting one trend to the totality of the data.

THE second general class of trends comprises those that are mathematically fitted - usually by a total process; some description of this second type of trend has already been offered, for contrast, in discussing the first type. One and sometimes two decisions involving subjective judgment are

(1) Journal of the American Statistical Assn. *XXIII, 1928, pp. 398-410.*

required: (a) what type of curve to fit; and sometimes, (b) a critical date or other parameter, such as the date of the point of inflection of a logistic curve. Trends that have been mathematically fitted are sufficiently objective to satisfy the requirements of economic statistics.

THE commoner types of curves employed in the mathematical process are: (1) the straight line, (2) the parabola or second degree polynomial, (3) the cubic or third degree polynomial, sometimes called a third degree parabola, (4) the simple exponential curve, which appears as a straight line on semi-logarithmic paper, and (5) the logistic and Gompertz curves, which are characterized by an S-shape. Other forms have been considered: higher degree polynomials are not practical; the arc-tangent might be added in class 5; and recurring trigonometric functions like the sine and cosine might form a sixth class; they have been of interest to statisticians who think of the long tidal movements.

ONE may distinguish - though it is of theoretical interest only - between two types of mathematically fitted curves: on the one hand, curves that really essay an explanation of the changes in the variable under study --- with parameters that correspond to real phenomena. Such curves have not been discovered for economic time series. On the other hand, there are curves which serve as smoothing devices. These differ in the amount of explanation they seem to offer of the phenomena under study, and in the "reasonableness" of their shape. Kuznets, in the book to be examined in Chapter V, offers the S-shaped logistic curve as the "proper" trend form for industrial growth, etc., although he is unable to give physical meaning to the parameters in the equation.

BUT no one has yet come forward with a general type of curve to fit price series. In a period of stable money (or if correction were made for the changing general value of money), a straight line or a logistic might fit. This problem will be referred to again, below.

AN important theoretical issue between empirical trends and those fitted by a total mathematical process, is with respect to the assumption of homogeneity of the forces affecting the value of the variable. "If . . we have forecasting done from a single line of trend, from a description that is . . . . historically limited, the assumption is that the forces that have been determining such movement in the past will continue to do so in the future - will repeat themselves. The basis of expectation here is not at all the statistical analysis, but information from a different source,which enables the forecaster to assert that the period for which the trend line was fitted was homogeneous, that is, under a preponderant influence of one and the same known set of forces, which is expected to repeat its influence in the future". (1)

TO illustrate how radical and unrealistic is the assumption of underlying homogeneity of the affecting forces, even through the observed period - - without extrapolating into the future - - - let us consider several time series. Suppose one were studying the method of lighting in American homes since 1800 (or the financial outlay upon that lighting,or the total candle power). Heterogeneity is striking, for the homes have been lighted by oil lamps, candles, kerosene, gas, and several types of electric lamps.

IF a price series were under examination, not only would there be involved problems of supply (discoveries, exhaustion of resources, etc.) and demand in the single industry (in which homogeneity might not be impossible), but also in the supplying industries, the rival industries, and the industries which use the product of this one as their raw material. An invention, or

(1) Kuznets, "On the Analysis of Time Series", p. 400.

a change in import or excise taxes, in any of these fields, would change the underlying forces. And, most fickle of all, the changing purchasing power of money makes for heterogeneity over time, in any price series.

IT would be unwise to fit a smooth mathematical curve to the number of votes cast in American presidential elections. There have been changes from property qualification to universal white manhood suffrage, to the freeing of the slaves, and to woman suffrage; territorial growth from thirteen states to forty-eight; the Civil War as an affecting episode;changes in the flow of immigration and of the westward movement of population; and the subjects voted upon have also changed, as for example in the matter of the direct election of senators.

FOR the business of the Port of San Francisco, homogeneity cannot be predicated, for the record runs through Spanish, Mexican, and American sovereignty, the gold strike of 1848, the Civil War, the completion of the transcontinental railroad in 1869, and of other lines, the Spanish-American War and the resulting development of far eastern trade, the earthquake and fire of 1906, the first World War, the opening of the Panama Canal in 1915, and the Second World War - with the emergence of air traffic.

QUITE naturally, the assumption of homogeneity of the affecting forces often strikes the operator himself as untenable; see, in Chapter V, how Kuznets has broken into two fragments the trend that he fitted to the series on Erie Canal freights. And, even when the fit of a mathematical curve is not so bad as to demand such fragmentation, it may, nevertheless, be worse than the fit of a well-adapted empirical curve of the same gentleness (long radius) of curvature.

EVEN in the case of fitting an empirical trend or smoothing line, it may sometimes prove advisable to regard some of the data as so completely different from the rest that the empirical line should be made discontinuous. An excellent recent example of such treatment is to be found in Norman J. Silberling, Dynamics of Business, (1) page 154. Silberling, in dealing with price series, regards the inflationary episodes of wartime as belonging in another "statistical universe" from prices during peacetime. Consequently, he discontinues his smoothing line through those inflationary periods.

THE present writer faced another problem in studying the various business series of San Francisco as they were affected by the earthquake and fire of 1906. I had plotted monthly data. In the first smoothing line, SL A, I allowed a saltatory displacement to stand, at the time of the catastrophe. In the second smoothing line, SL B, which cut through the short business cycle, I again permitted a saltatory displacement, though along a somewhat sloped line, rather than abruptly vertical. As for the third smoothing line, SL M, which cut through the major cycle, I felt it advisable to draw the curve in a continuous manner rather than to permit again a saltatory displacement. But I recognized the subjective nature of the decision; possibly another saltatory decline should have been admitted, and the continuous line reserved for the next stage of smoothing, designed to remove the long wave.

AN important publication in the field of the present book is Macaulay: The Smoothing of Time Series. Macaulay provides a number of weighted formulas for moving averages, designed for series in which the length or period of the cycle is reasonably uniform. Some of Macaulay's formulas have the advantage of giving greater weight to the middle values and less to

(1) McGraw-Hill, 1943.

the ends. This brings the moving average to the full desirable departure in the convex direction (departure from the less satisfactory position which an unweighted average would occupy), in periods when there is marked curvature in the moving average. But that same purpose is accomplished herein by the reiteration of the moving average (see correction for curvature, Chapter II), with an advantage over Macaulay's formulas, of adaptation to changing lengths of the cycles.

C. A. R. WARDWELL devised the "moving cyclical average", which makes possible an objective check upon the smoothing process for series with changing cycle lengths. His moving cyclical average is here accepted as the principal objective check in the smoothing process. Some ingenious statistician may find a way to combine Wardwell's contribution of the variable length moving average with Macaulay's heavy weighting of the central values, and so make unnecessary a separate calculation to correct for curvature. But that separate calculation is not laborious, and gives excellent results.

THE method of smoothing by stages is <u>systematic</u>; each stage of smoothing is similar to the next. The only difference among the stages is in one detail. In smoothing out a daily, weekly, or annual cycle, a simple moving average with <u>fixed and uniform</u> length is employed, because each successive cycle has the same length. But in smoothing out the short business cycle, the major cycle, or the "long wave" - - and also in smoothing out the monthly cycle (the months consisting of 31 days, 28, 31, 30, etc.) - - Wardwell's moving cyclical average of <u>changing</u> length is used. But this minor adaptation of method does not impair the truly unified and systematic nature of the procedure.

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. . . "the analysis became necessary since recurrent changes had to be separated from the non-recurrent ones, and . . . the recurrences of different amplitude and duration had to be distinguished from one another." Kuznets (1)

CHAPTER I.

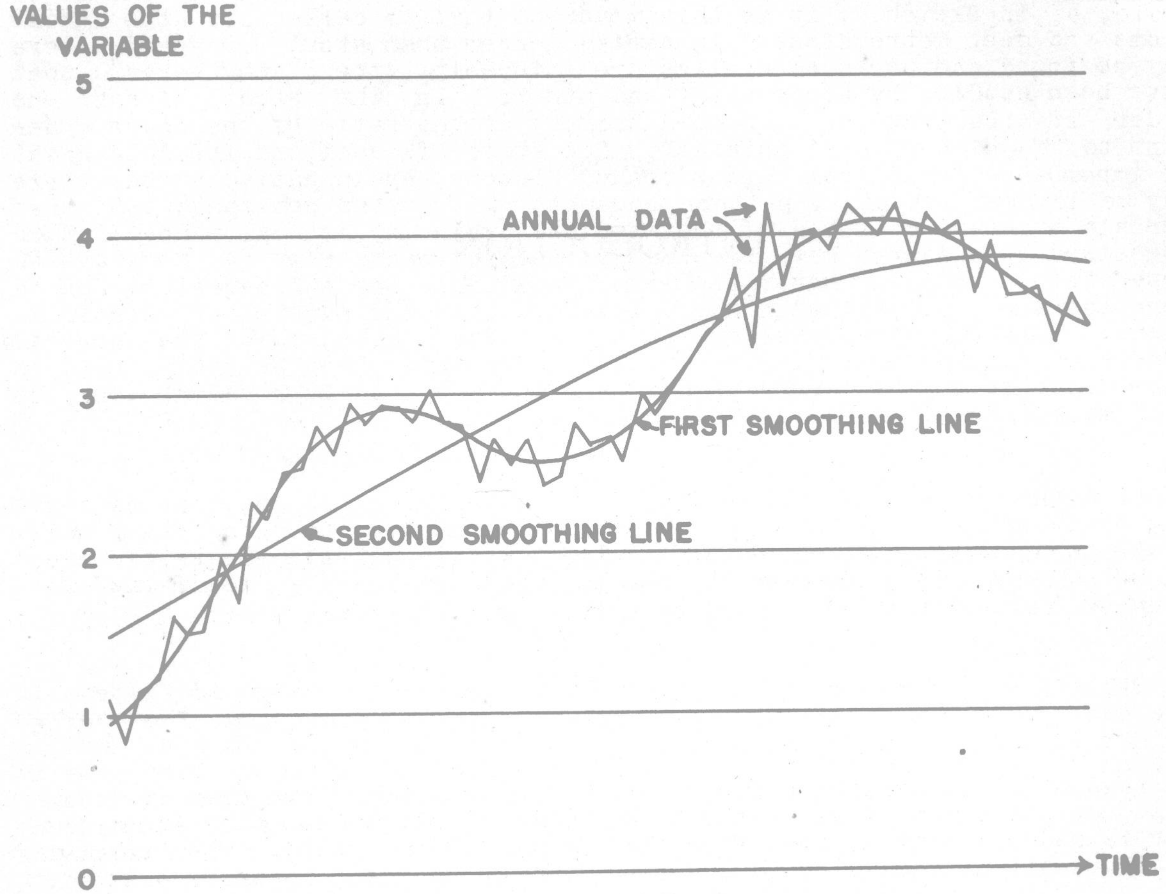
INTRODUCTION

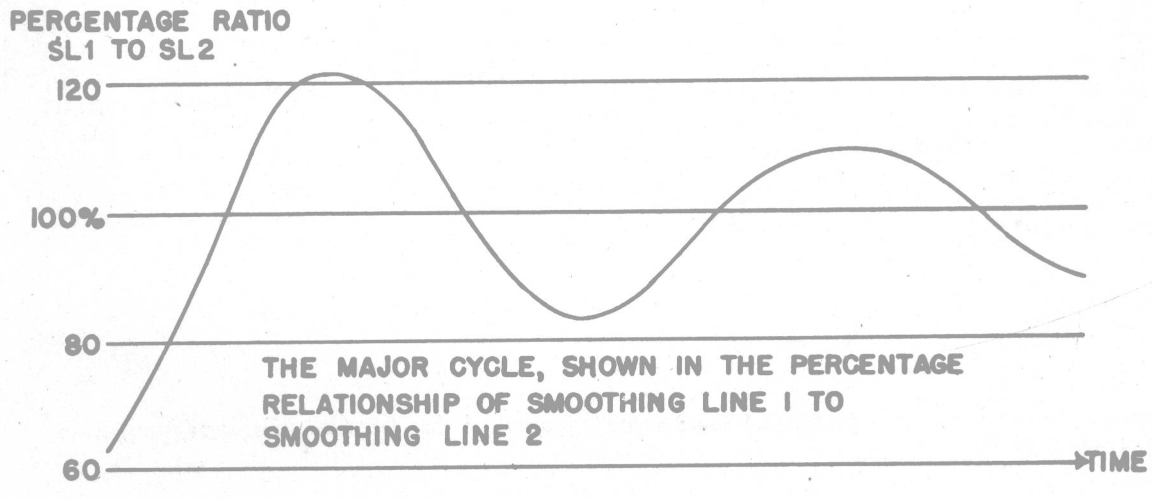
THE method of analyzing time series which we shall call "smoothing by stages", is primarily graphical. One arranges the data in two forms, as numerical values in a table, and as a time polygon upon a chart.

HE draws a "first smoothing line", which cuts through and "removes" the cycle of shortest period (the word "removes" means that the smoothing line is made completely free from that short-period cycle). Then through the fluctuations remaining in the first smoothing line, he draws a second smoothing line, which removes the cycle or fluctuation of next shortest period; etc., until in the final smoothing line there remain no recurrent movements. If the values of the time series have been given at monthly or quarterly intervals, his first task is to remove the seasonal fluctuation - which in its full historical record may be called the annual cycle - by the application of Smoothing Line A (so-called because it removes the annual cycle; abbreviated as SL A). Then SL A, in its turn, is smoothed by a second order smoothing line, SL B, which cuts through and thereby "eliminates" the short business cycle, and which derives its name from the initial letter of that cycle. But if the data are in annual form (instead of monthly or quarterly), the first task is to cut through the fluctuations of the short business cycle by drawing SL B. In this book, except for the brief illustration in this Introduction, all the examples have annual data, so that SL B is the first smoothing line obtained. SL B, in turn, is smoothed by the application of SL M, which eliminates the major cycle, a fluctuation ten to thirty years in length. The highest order smoothing line obtained (usually it is SL M) serves as an approximation to the underlying "secular" trend, the trend through the centuries.

(1) *Simon S. Kuznets, "On the Analysis of Time Series",* Journal of the American Statistical Association, *XXIII, 1928, page 399.*

SKETCH a, SMOOTHING LINES AND CYCLE





IN the relationship between any pair of successive smoothing lines, is found the history of one order of cyclical movement. The relationship between the original monthly or quarterly data and SL A gives the cyclical and irregular movements of the shortest order, termed the seasonal movement or the annual cycle. The relationship of the first order smoothing line, SL A, to the second order line, SL B, gives the history of the cyclical-irregular movement commonly called the short business cycle. In case the original figures are annual, instead of monthly or quarterly, the short business cycle is revealed in the relationship between the annual data and SL B. The relationship of SL B to SL M gives the major cycle, as in Sketch a; it is this movement that is marked by the great booms and deep depressions. In series longer than about 80 years, there may be found another such cyclical relationship, the "long waves" that have been studied by Kondratieff and others. In the study of some one order of fluctuation, the full history of the ratio of the lower order line to the higher is of interest. But it is of equal,and possibly greater importance, that from this extended history, by inductive steps, there may be derived a typical pattern,somewhat uniform and constant; and that certain standard measures of that typical cycle may be calculated. When the standard measures have been calculated for each order of fluctuation separately, excellent material becomes available for a forecast - for a more thorough forecast than has yet lain within the power of statisticians. This forecast makes use of the standard measures of the several orders of fluctuation; it is realistic, extensive, and helpful; It is hoped that it may be generally accepted as a decided improvement on the customary forward extension of the normal line.

TABLE A and Chart 1 will show that there already exist ways of segregating or isolating the annual or seasonal fluctuation from the other movements in a time series. This table and chart do not illustrate technically the method of "smoothing by stages" but merely serve as an introduction to it.

IN Table A and on Chart 1, a twelve month moving average is fitted; in the case of this particular series, the curve connecting the average points is found to be smooth; consequently it will serve, without modification, as an acceptable approximation to SL A, and as a good base of reference for the study of the annual cycle. Under the method of smoothing by stages, the moving average curve might be more carefully smoothed; but, practically speaking, that further refinement is not often necessary unless the twelve month moving average curve is quite irregular. In Chapter II, the recommendation will be made that monthly data be consolidated into quarterly figures before undertaking the smoothing process.That consolidation makes it even more unlikely that irregularities will disturb the smooth flow of the four quarter or "annual" moving average.

Table A. SAN FRANCISCO REAL ESTATE ACTIVITY, 1920 TO 1929

The annual cycle, as shown in the ratio of actual value to moving average.

Source: San Francisco Real Estate Circular, Thomas Magee and Sons.

Month	Number of Deeds	Moving Average, 12 months length: not recent-ered	Moving Average, 12 months length: recent-ered	Percentage Ratio, Actual to Moving Average	Trend (fitted by least squares)
X	Y	ma_{12}	ma_{12r}	$\frac{100\ Y}{ma_{12r}}$	T
1920 Jan	838				1034
Feb	759				1035
Mar	969				1036
Apr	771				1037
May	812				1038
June	676				1039
July	706	767	764	92	1040
Aug	645	761	758	85	1040
Sept	766	756	754	101	1041
Oct	791	753	758	104	1042
Nov	768	763	765	100	1043
Dec	702	767	772	91	1044
1921 Jan	764	778	784	97	1045
Feb	702	789	791	89	1046
Mar	928	793	791	117	1046
Apr	873	789	790	110	1047
May	883	790	796	110	1048
June	803	803	807	100	1049

Month	Deeds	Moving Average: not recent-ered	Moving Average: recent-ered	Ratio	Trend
X	Y	ma_{12}	ma_{12r}	$\frac{100\ Y}{ma_{12r}}$	T
1921 July	848	811	822	103	1050
Aug	685	834	844	81	1051
Sept	720	854	866	83	1052
Oct	901	878	891	101	1052
Nov	824	904	910	91	1053
Dec	805	915	921	87	1054
1922 Jan	1030	927	932	111	1055
Feb	945	937	953	99	1056
Mar	1215	969	975	129	1057
Apr	1195	981	994	120	1057
May	1006	1008	1021	99	1058
June	952	1034	1038	92	1059
July	947	1043	1051	90	1060
Aug	971	1059	1070	91	1061
Sept	986	1082	1092	90	1062
Oct	1218	1101	1115	109	1062
Nov	1135	1129	1144	99	1063
Dec	919	1160	1176	87	1064

Table A (continued) San Francisco Real Estate Activity

Month	Deeds	Moving Average: not recentered	Moving Average: recentered	Ratio	Trend
X	Y	ma_{12}	ma_{12r}	$\frac{100\ Y}{ma_{12r}}$	T
1923 Jan	1217	1191	1192	102	1065
Feb	1121	1192	1199	94	1066
Mar	1544	1206	1208	127	1067
Apr	1527	1211	1219	125	1068
May	1388	1227	1232	112	1068
June	1119	1238	1242	90	1069
July	1161	1245	1249	93	1070
Aug	1135	1253	1266	89	1071
Sept	1047	1278	1278	82	1072
Oct	1412	1277	1282	110	1073
Nov	1251	1288	1292	97	1074
Dec	1018	1295	1298	78	1074
1924 Jan	1321	1302	1304	101	1075
Feb	1416	1306	1298	111	1076
Mar	1521	1290	1296	110	1077
Apr	1666	1302	1301	128	1078
May	1468	1300	1297	113	1079
June	1210	1294	1299	93	1080
July	1207	1304	1303	93	1080
Aug	1046	1302	1304	80	1081
Sept	1093	1306	1310	83	1082

Month	Deeds	Moving Average: not recentered	Moving Average: recentered	Ratio	Trend
X	Y	ma_{12}	ma_{12r}	$\frac{100\ Y}{ma_{12r}}$	T
1924 Oct	1379	1313	1318	105	1083
Nov	1182	1324	1326	89	1084
Dec	1141	1328	1335	85	1085
1925 Jan	1301	1342	1356	95	1086
Feb	1453	1371	1381	105	1086
Mar	1608	1391	1410	113	1087
Apr	1801	1429	1454	124	1088
May	1529	1478	1484	103	1089
June	1366	1490	1507	91	1090
July	1552	1524	1533	101	1091
Aug	1383	1542	1539	89	1092
Sept	1461	1536	1540	95	1092
Oct	1841	1543	1548	119	1093
Nov	1541	1552	1552	99	1094
Dec	1446	1552	1550	93	1095
1926 Jan	1529	1547	1547	98	1096
Feb	1381	1547	1530	90	1097
Mar	1690	1513	1500	113	1098
Apr	1908	1488	1477	128	1098
May	1526	1466	1448	105	1099
June	1275	1429	1416	90	1100

Table A. (concluded) San Francisco Real Estate Activity

Month	Deeds	Moving Average not recent-ered	Moving Average recent-ered	Ratio	Trend
X	Y	ma_{12}	ma_{12r}	$\frac{100\ Y}{ma_{12r}}$	T
1926		1402			
July	1172		1382	85	1101
		1362			
Aug	1083		1355	80	1102
		1348			
Sept	1204		1336	90	1103
		1323			
Oct	1388		1306	106	1104
		1290			
Nov	1217		1275	95	1104
		1260			
Dec	1170		1243	94	1105
1927		1226			
Jan	1164		1215	95	1106
		1204			
Feb	1084		1198	90	1107
		1192			
Mar	1394		1192	117	1108
		1191			
Apr	1448		1180	113	1109
		1169			
May	1117		1150	97	1110
		1132			
June	1009		1117	90	1110
		1102			
July	1023		1091	94	1111
		1080			
Aug	1074		1070	100	1112
		1059			
Sept	936		1054	89	1113
		1049			
Oct	947		1034	92	1114
		1020			
Nov	864		1001	86	1115
		982			
Dec	900		979	92	1116
1928		976			
Jan	918		968	94	1116
		959			
Feb	955		950	100	1117
		940			
Mar	1050		934	112	1118

Month	Deeds	Moving Average not recent-ered	Moving Average recent-ered	Ratio	Trend
X	Y	ma_{12}	ma_{12r}	$\frac{100\ Y}{ma_{12r}}$	T
1928		927			
Apr	991		927	106	1119
		927			
May	1050		927	113	1120
		927			
June	908		916	98	1121
		915			
July	913		910	100	1122
		905			
Aug	843		900	94	1122
		894			
Sept	790		894	88	1123
		882			
Oct	952		888	100	1124
		872			
Nov	858		877	97	1125
		854			
Dec	752		851	88	1126
1929		848			
Jan	799		842	95	1127
		837			
Feb	819		833	98	1128
		829			
Mar	905		824	109	1128
		820			
Apr	880		814	108	1129
		808			
May	827		802	103	1130
		795			
June	841		790	107	1131
		785			
July	776				1132
Aug	751				1133
Sept	675				1133
Oct	812				1134
Nov	707				1135
Dec	624				1136

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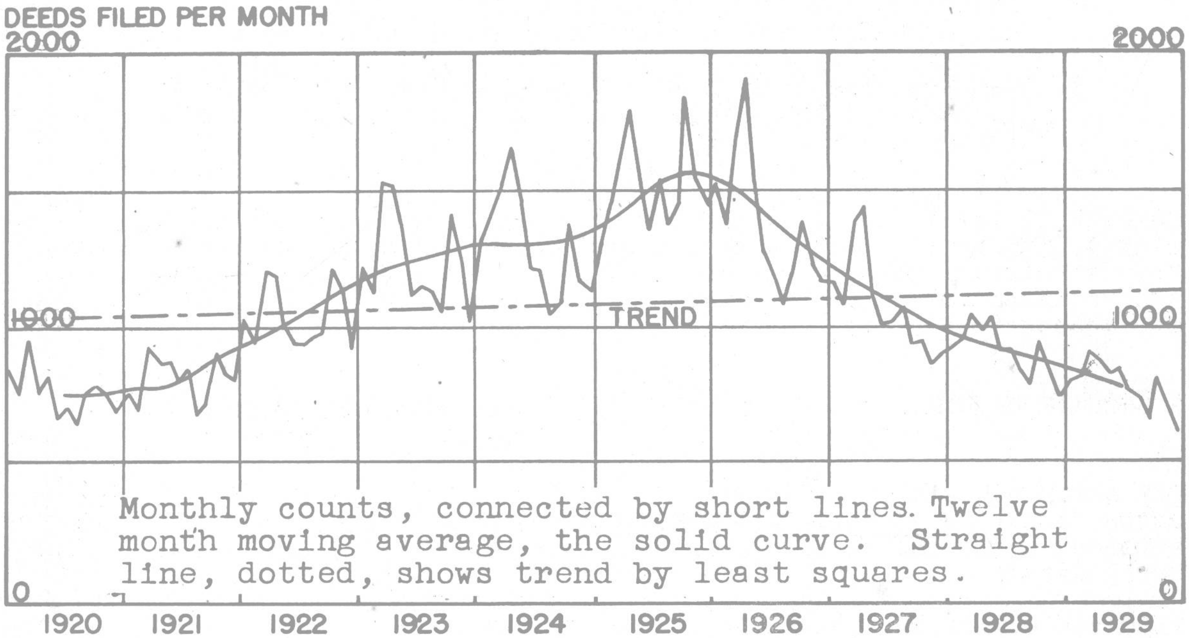
CHART I.
SAN FRANCISCO REAL ESTATE ACTIVITY, 1920 TO 1929

(a) APPLICATION OF MOVING AVERAGE AND TREND



(b) THE ANNUAL CYCLE