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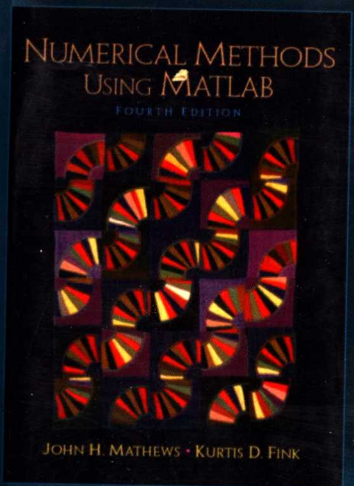
数值方法（MATLAB版）

（第四版） 英文版

Numerical Methods Using MATLAB, Fourth Edition



[美] John H. Mathews
Kurtis D. Fink 著

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内容简介

本书介绍了数值方法的理论及实用知识，并讲述了如何利用MATLAB软件实现各种数值算法，以便为读者今后的学习打下坚实的数值分析与科学计算基础。本书内容丰富，教师可以根据不同的学习对象和学习目的选择相应的章节，形成理论与实践相结合的学习策略。书中的每个概念均以实例说明，同时还包含大量的习题，范围涉及多个不同领域。通过这些实例进一步说明数值方法的实际应用。本书的突出特点是强调利用MATLAB进行数值方法的程序设计，可提高读者的实践能力并加深对数值方法理论的理解；同时它的覆盖范围广，包含数据方法的众多研究领域，可以满足不同专业和不同层次学生的需求。

本书概念清晰、逻辑性强，可作为大专院校计算机、工程和应用数学专业的教材和参考书。

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出 版 说 明

21世纪初的5至10年是我国国民经济和社会发展的重要时期，也是信息产业快速发展的关键时期。在我国加入WTO后的今天，培养一支适应国际化竞争的一流IT人才队伍是我国高等教育的重要任务之一。信息科学和技术方面人才的优劣与多寡，是我国面对国际竞争时成败的关键因素。

当前，正值我国高等教育特别是信息科学领域的教育调整、变革的重大时期，为使我国教育体制与国际化接轨，有条件的高等院校正在为某些信息学科和技术课程使用国外优秀教材和优秀原版教材，以使我国在计算机教学上尽快赶上国际先进水平。

电子工业出版社秉承多年来引进国外优秀图书的经验，翻译出版了“国外计算机科学教材系列”丛书，这套教材覆盖学科范围广、领域宽、层次多，既有本科专业课程教材，也有研究生课程教材，以适应不同院系、不同专业、不同层次的师生对教材的需求，广大师生可自由选择和自由组合使用。这些教材涉及的学科方向包括网络与通信、操作系统、计算机组织与结构、算法与数据结构、数据库与信息处理、编程语言、图形图像与多媒体、软件工程等。同时，我们也适当引进了一些优秀英文原版教材，本着翻译版本和英文原版并重的原则，对重点图书既提供英文原版又提供相应的翻译版本。

在图书选题上，我们大都选择国外著名出版公司出版的高校教材，如Pearson Education培生教育出版集团、麦格劳－希尔教育出版集团、麻省理工学院出版社、剑桥大学出版社等。撰写教材的许多作者都是蜚声世界的教授、学者，如道格拉斯·科默(Douglas E. Comer)、威廉·斯托林斯(William Stallings)、哈维·戴特尔(Harvey M. Deitel)、尤利斯·布莱克(Uyless Black)等。

为确保教材的选题质量和翻译质量，我们约请了清华大学、北京大学、北京航空航天大学、复旦大学、上海交通大学、南京大学、浙江大学、哈尔滨工业大学、华中科技大学、西安交通大学、国防科学技术大学、解放军理工大学等著名高校的教授和骨干教师参与了本系列教材的选题、翻译和审校工作。他们中既有讲授同类教材的骨干教师、博士，也有积累了几十年教学经验的老教授和博士生导师。

在该系列教材的选题、翻译和编辑加工过程中，为提高教材质量，我们做了大量细致的工作，包括对所选教材进行全面论证；选择编辑时力求达到专业对口；对排版、印制质量进行严格把关。对于英文教材中出现的错误，我们通过与作者联络和网上下载勘误表等方式，逐一进行了修订。

此外，我们还将与国外著名出版公司合作，提供一些教材的教学支持资料，希望能为授课老师提供帮助。今后，我们将继续加强与各高校教师的密切联系，为广大师生引进更多的国外优秀教材和参考书，为我国计算机科学教学体系与国际教学体系的接轨做出努力。

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Preface

This book provides a fundamental introduction to numerical analysis suitable for undergraduate students in mathematics, computer science, physical sciences, and engineering. It is assumed that the reader is familiar with calculus and has taken a structured programming course. The text has enough material fitted modularly for either a single-term course or a year sequence. In short, the book contains enough material so that instructors will be able to select topics appropriate to their needs.

Students of various backgrounds should find numerical methods quite interesting and useful, and this is kept in mind throughout the book. Thus, there is a wide variety of examples and problems that help to sharpen one's skill in both the theory and practice of numerical analysis. Computer calculations are presented in the form of tables and graphs whenever possible so that the resulting numerical approximations are easier to visualize and interpret. MATLAB programs are the vehicle for presenting the underlying numerical algorithms.

Emphasis is placed on understanding why numerical methods work and their limitations. This is challenging and involves a balance between theory, error analysis, and readability. An error analysis for each method is presented in a fashion that is appropriate for the method at hand, yet does not turn off the reader. A mathematical derivation for each method is given that uses elementary results and builds the student's understanding of calculus. Computer assignments using MATLAB give students an opportunity to practice their skills at scientific programming.

Shorter numerical exercises can be carried out with a pocket calculator/computer, and the longer ones can be done using MATLAB subroutines. It is left for the instructor to guide the students regarding the pedagogical use of numerical computations. Each instructor can make assignments that are appropriate to the available computing resources. Experimentation with the MATLAB subroutine libraries is encouraged. These materials can be used to assist students in the completion of the numerical analysis component of computer laboratory exercises.

In this edition a section on Bézier curves has been added to the end of the chapter on curve fitting. Additionally, the chapter on numerical optimization has been expanded to include an introduction to both direct and derivative based methods for optimizing functions of one or more variables. A listing of the MATLAB programs in this textbook is available upon request from the authors (*http://math.fullerton.edu/mathews/numerical.html*). An instructor's solution manual for the exercise sets is available from the publisher.

Previously, we took the attitude that any software program that students mastered would work fine. However, many students entering this course have yet to master a programming language (computer science students excepted). MATLAB has become the tool of nearly all engineers and applied mathematicians, and its newest versions have improved the programming aspects. So we think that students will have an easier and more productive time in this MATLAB version of our text.

Acknowledgments

We would like to express our gratitude to all the people whose efforts contributed to the various editions of this book. I (John Mathews) thank the students at California State University, Fullerton. I thank my colleagues Stephen Goode, Mathew Koshy, Edward Sabotka, Harris Schultz, and Soo Tang Tan for their support in the first edition; additionally, I thank Russell Egbert, William Gearhart, Ronald Miller, and Greg Pierce for their suggestions for the second edition. I also thank James Friel, Chairman of the Mathematics Department at CSUF, for his encouragement.

Reviewers who made useful recommendations for the first edition are Walter M. Patterson, III, Lander College; George B. Miller, Central Connecticut State University; Peter J. Gingo, The University of Akron; Michael A. Freedman, The University of Alaska, Fairbanks; and Kenneth P. Bube, University of California, Los Angeles. For the second edition, we thank Richard Bumby, Rutgers University; Robert L. Curry, U.S. Army; Bruce Edwards, University of Florida; and David R. Hill, Temple University.

For the third edition we wish to thank Tim Sauer, George Mason University; Gerald M. Pitstick, University of Oklahoma; Victor De Brunner, University of Oklahoma; George Trapp, West Virginia University; Tad Jarik, University of Alabama, Huntsville; Jeffrey S. Scroggs, North Carolina State University; Kurt Georg, Colorado State University; and James N. Craddock, Southern Illinois University at Carbondale.

Reviewers for the fourth edition were Kevin Kreider, University of Akron; Demetrio Labate, Washington University at St. Louis; Lee Johnson, Virginia Tech; and Azmy Ackleh, University of Louisiana at Lafayette. We are grateful to the reviewers for their time and recommendations.

Suggestions for improvements and additions to the book are always welcome and can be made by corresponding directly with the authors.

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Contents

1

Preliminaries

Consider the function $f(x) = \cos(x)$, its derivative $f'(x) = -\sin(x)$, and its antiderivative $F(x) = \sin(x) + C$. These formulas were studied in calculus. The former is used to determine the slope $m = f'(x_0)$ of the curve $y = f(x)$ at a point $(x_0, f(x_0))$, and the latter is used to compute the area under the curve for $a \le x \le b$.

The slope at the point $(\pi/2, 0)$ is $m = f'(\pi/2) = -1$ and can be used to find the tangent line at this point (see Figure 1.1(a)):

$$y_{\tan} = m\left(x - \frac{\pi}{2}\right) + 0 = f'\left(\frac{\pi}{2}\right)\left(x - \frac{\pi}{2}\right) = -x + \frac{\pi}{2}.$$

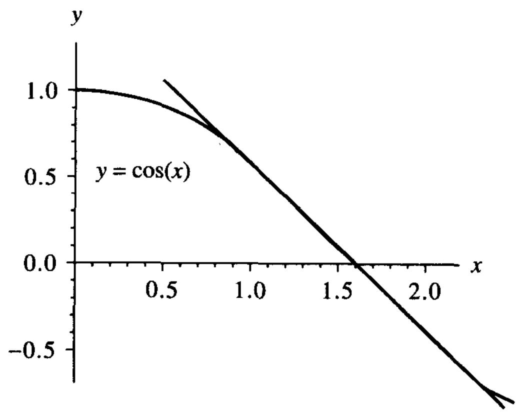

Figure 1.1 (a) The tangent line to the curve $y = \cos(x)$ at the point $(\pi/2, 0)$.

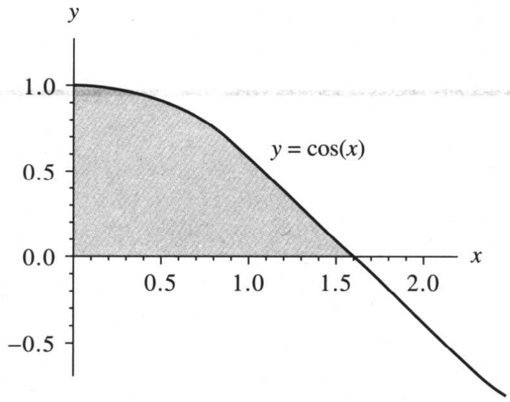

Figure 1.1 (b) The area under the curve $y = \cos(x)$ over the interval $[0, \pi/2]$.

The area under the curve for $0 \leq x \leq \pi/2$ is computed using an integral (see Figure 1.1(b)):

$$\text{area} = \int_0^{\pi/2} \cos(x)\,dx = F\left(\frac{\pi}{2}\right) - F(0) = \sin\left(\frac{\pi}{2}\right) - 0 = 1.$$

These are some of the results that we will need to use from calculus.

1.1 Review of Calculus

It is assumed that the reader is familiar with the notation and subject matter covered in the undergraduate calculus sequence. This should have included the topics of limits, continuity, differentiation, integration, sequences, and series. Throughout the book we refer to the following results.

Limits and Continuity

Definition 1.1. Assume that $f(x)$ is defined on an open interval containing $x = x_0$, except possibly at $x = x_0$ itself. Then f is said to have the ***limit*** L at $x = x_0$, and we write

(1) $$\lim_{x \to x_0} f(x) = L,$$

if given any $\epsilon > 0$ there exists a $\delta > 0$ such that $|f(x) - L| < \epsilon$ whenever $0 < |x - x_0| < \delta$. When the h-increment notation $x = x_0 + h$ is used, equation (1) becomes

(2) $$\lim_{h \to 0} f(x_0 + h) = L.$$ ▲

Definition 1.2. Assume that $f(x)$ is defined on an open interval containing $x = x_0$. Then f is said to be ***continuous at*** $\boldsymbol{x = x_0}$ if

(3) $$\lim_{x \to x_0} f(x) = f(x_0).$$

The function f is said to be continuous on a set S if it is continuous at each point $x \in S$. The notation $C^n(S)$ stands for the set of all functions f such that f and its first n derivatives are continuous on S. When S is an interval, say $[a, b]$, then the notation $C^n[a, b]$ is used. As an example, consider the function $f(x) = x^{4/3}$ on the interval $[-1, 1]$. Clearly, $f(x)$ and $f'(x) = (4/3)x^{1/3}$ are continuous on $[-1, 1]$, while $f''(x) = (4/9)x^{-2/3}$ is not continuous at $x = 0$. ▲

Definition 1.3. Suppose that $\{x_n\}_{n=1}^{\infty}$ is an infinite sequence. Then the sequence is said to have the *limit* L, and we write

(4) $$\lim_{n \to \infty} x_n = L,$$

if given any $\epsilon > 0$, there exists a positive integer $N = N(\epsilon)$ such that $n > N$ implies that $|x_n - L| < \epsilon$. ▲

When a sequence has a limit, we say that it is a ***convergent sequence***. Another commonly used notation is "$x_n \to L$ as $n \to \infty$." Equation (4) is equivalent to

(5) $$\lim_{n \to \infty} (x_n - L) = 0.$$

Thus we can view the sequence $\{\epsilon_n\}_{n=1}^{\infty} = \{x_n - L\}_{n=1}^{\infty}$ as an ***error sequence***. The following theorem relates the concepts of continuity and convergent sequence.

Theorem 1.1. Assume that $f(x)$ is defined on the set S and $x_0 \in S$. The following statements are equivalent:

(6)
(a) The function f is continuous at x_0.
(b) If $\lim_{n \to \infty} x_n = x_0$, then $\lim_{n \to \infty} f(x_n) = f(x_0)$.

Theorem 1.2 (Intermediate Value Theorem). Assume that $f \in C[a, b]$ and L is any number between $f(a)$ and $f(b)$. Then there exists a number c, with $c \in (a, b)$, such that $f(c) = L$.

Example 1.1. The function $f(x) = \cos(x - 1)$ is continuous over $[0, 1]$, and the constant $L = 0.8 \in (\cos(0), \cos(1))$. The solution to $f(x) = 0.8$ over $[0, 1]$ is $c_1 = 0.356499$. Similarly, $f(x)$ is continuous over $[1, 2.5]$, and $L = 0.8 \in (\cos(2.5), \cos(1))$. The solution to $f(x) = 0.8$ over $[1, 2.5]$ is $c_2 = 1.643502$. These two cases are shown in Figure 1.2. ■

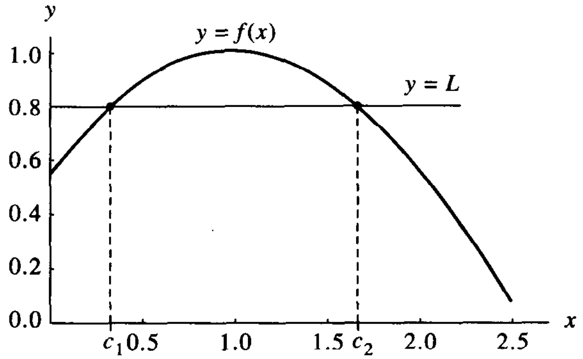

Figure 1.2 The intermediate value theorem applied to the function $f(x) = \cos(x - 1)$ over $[0, 1]$ and over the interval $[1, 2.5]$.

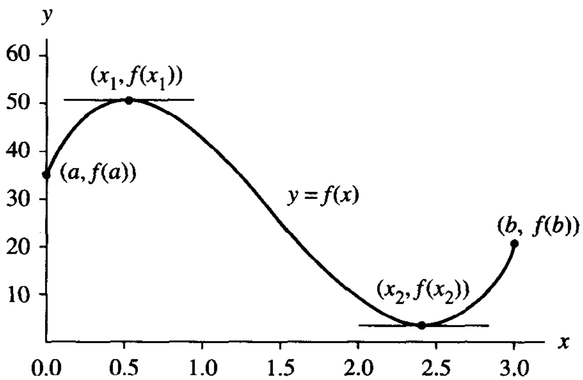

Figure 1.3 The extreme value theorem applied to the function $f(x) = 35 + 59.5x - 66.5x^2 + 15x^3$ over the interval $[0, 3]$.

Theorem 1.3 (Extreme Value Theorem for a Continuous Function). Assume that $f \in C[a, b]$. Then there exists a lower bound M_1, an upper bound M_2, and two numbers $x_1, x_2 \in [a, b]$ such that

$$M_1 = f(x_1) \le f(x) \le f(x_2) = M_2 \quad \text{whenever } x \in [a, b]. \tag{7}$$

We sometimes express this by writing

$$M_1 = f(x_1) = \min_{a \le x \le b} \{f(x)\} \quad \text{and} \quad M_2 = f(x_2) = \max_{a \le x \le b} \{f(x)\}. \tag{8}$$

Differentiable Functions

Definition 1.4. Assume that $f(x)$ is defined on an open interval containing x_0. Then f is said to be *differentiable* at x_0 if

$$\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} \tag{9}$$

exists. When this limit exists, it is denoted by $f'(x_0)$ and is called the ***derivative*** of f at x_0. An equivalent way to express this limit is to use the h-increment notation:

$$\lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h} = f'(x_0). \tag{10}$$

A function that has a derivative at each point in a set S is said to be ***differentiable*** on S. Note that the number $m = f'(x_0)$ is the slope of the tangent line to the graph of the function $y = f(x)$ at the point $(x_0, f(x_0))$. ▲

Theorem 1.4. If $f(x)$ is differentiable at $x = x_0$, then $f(x)$ is continuous at $x = x_0$.

It follows from Theorem 1.3 that if a function f is differentiable on a closed interval $[a, b]$, then its extreme values occur at the endpoints of the interval or at the critical points (solutions of $f'(x) = 0$) in the open interval (a, b).

Example 1.2. The function $f(x) = 15x^3 - 66.5x^2 + 59.5x + 35$ is differentiable on $[0, 3]$. The solutions to $f'(x) = 45x^2 - 123x + 59.5 = 0$ are $x_1 = 0.54955$ and $x_2 = 2.40601$. The maximum and minimum values of f on $[0, 3]$ are:

$$\min\{f(0), f(3), f(x_1), f(x_2)\} = \min\{35, 20, 50.10438, 2.11850\} = 2.11850$$

and

$$\max\{f(0), f(3), f(x_1), f(x_2)\} = \max\{35, 20, 50.10438, 2.11850\} = 50.10438$$

(see Figure 1.3). ■

Theorem 1.5 (Rolle's Theorem). Assume that $f \in C[a, b]$ and that $f'(x)$ exists for all $x \in (a, b)$. If $f(a) = f(b) = 0$, then there exists a number c, with $c \in (a, b)$, such that $f'(c) = 0$.

Theorem 1.6 (Mean Value Theorem). Assume that $f \in C[a, b]$ and that $f'(x)$ exists for all $x \in (a, b)$. Then there exists a number c, with $c \in (a, b)$, such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}. \tag{11}$$

Geometrically, the mean value theorem says that there is at least one number $c \in (a, b)$ such that the slope of the tangent line to the graph of $y = f(x)$ at the point $(c, f(c))$ equals the slope of the secant line through the points $(a, f(a))$ and $(b, f(b))$.

Example 1.3. The function $f(x) = \sin(x)$ is continuous on the closed interval $[0.1, 2.1]$ and differentiable on the open interval $(0.1, 2.1)$. Thus, by the mean value theorem, there is a number c such that

$$f'(c) = \frac{f(2.1) - f(0.1)}{2.1 - 0.1} = \frac{0.863209 - 0.099833}{2.1 - 0.1} = 0.381688.$$

The solution to $f'(c) = \cos(c) = 0.381688$ in the interval $(0.1, 2.1)$ is $c = 1.179174$. The graphs of $f(x)$, the secant line $y = 0.381688x + 0.099833$, and the tangent line $y = 0.381688x + 0.474215$ are shown in Figure 1.4. ■

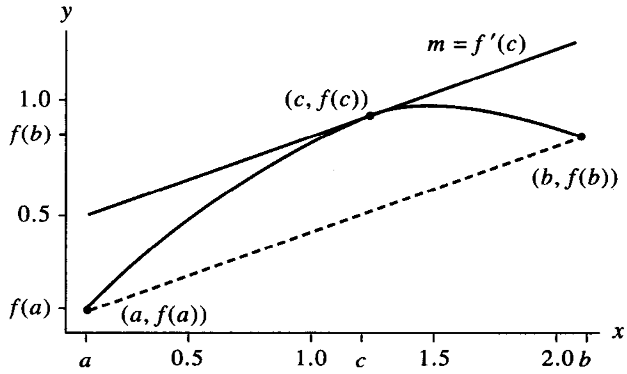

Figure 1.4 The mean value theorem applied to $f(x) = \sin(x)$ over the interval $[0.1, 2.1]$.

Theorem 1.7 (Generalized Rolle's Theorem). Assume that $f \in C[a, b]$ and that $f'(x), f''(x), \ldots, f^{(n)}(x)$ exist over (a, b) and $x_0, x_1, \ldots, x_n \in [a, b]$. If $f(x_j) = 0$ for $j = 0, 1, \ldots, n$, then there exists a number c, with $c \in (a, b)$, such that $f^{(n)}(c) = 0$.

Integrals

Theorem 1.8 (First Fundamental Theorem). If f is continuous over $[a, b]$ and F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a) \quad \text{where} \quad F'(x) = f(x). \tag{12}$$

Theorem 1.9 (Second Fundamental Theorem). If f is continuous over $[a, b]$ and $x \in (a, b)$, then

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x). \tag{13}$$

Example 1.4. The function $f(x) = \cos(x)$ satisfies the hypotheses of Theorem 1.9 over the interval $[0, \pi/2]$; thus by the chain rule

$$\frac{d}{dx}\int_0^{x^2} \cos(t)\,dt = \cos(x^2)(x^2)' = 2x\cos(x^2). \qquad \blacksquare$$

Theorem 1.10 (Mean Value Theorem for Integrals). Assume that $f \in C[a, b]$. Then there exists a number c, with $c \in (a, b)$, such that

$$\frac{1}{b-a}\int_a^b f(x)\,dx = f(c).$$

The value $f(c)$ is the average value of f over the interval $[a, b]$.