

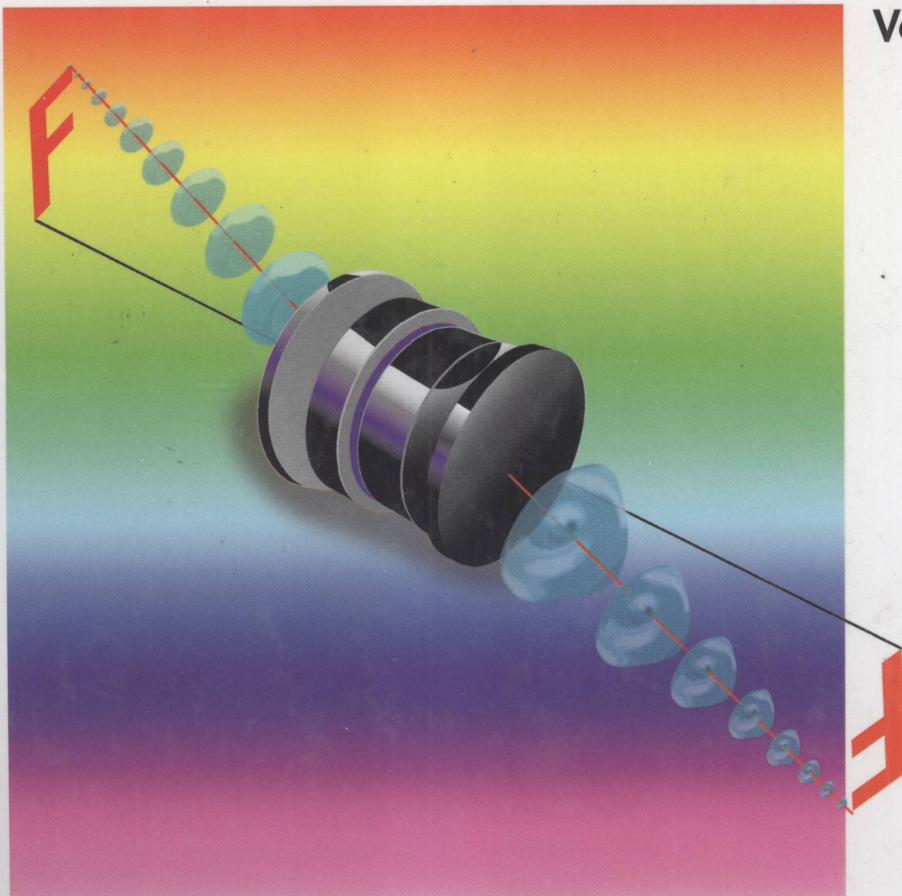
Edited by
Herbert Gross

 WILEY-VCH

Handbook of Optical Systems

H. Gross, H. Zügge, M. Peschka, F. Blechinger
Aberration Theory and Correction of Optical Systems

Volume 3



TH 740
H 236
v. 3

Handbook of Optical Systems

Edited by
Herbert Gross

Volume 3: Aberration Theory and Correction
of Optical Systems

Herbert Gross, Hannfried Zügge, Martin Peschka,
Fritz Blechinger



WILEY-VCH Verlag GmbH & Co. KGaA

Herbert Gross

Head of Optical Design Department
Carl Zeiss AG, Oberkochen, Germany
e-mail: Gross@zeiss.de

Hannfried Zügge

Oberkochen, Germany
e-mail: hannfried@zuegge.de

Martin Peschka

Department FT-OD
Carl Zeiss AG, Oberkochen, Germany
e-mail: m.peschka@zeiss.de

Fritz Blechinger

Leuze Elektronik, Esslingen, Germany
e-mail: fritz.blechinger@optenso.de

■ All books published by Wiley-VCH are carefully produced. Nevertheless, authors, editors, and publisher do not warrant the information contained in these books, including this book, to be free of errors. Readers are advised to keep in mind that statements, data, illustrations, procedural details or other items may inadvertently be inaccurate.

Library of Congress Card No.:
applied for

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

**Bibliographic information published by
Die Deutsche Bibliothek**

Die Deutsche Bibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data is available in the Internet at
<<http://dnb.ddb.de>>.

© 2007 WILEY-VCH Verlag GmbH & Co. KGaA,
Weinheim

All rights reserved (including those of translation into other languages). No part of this book may be reproduced in any form – nor transmitted or translated into machine language without written permission from the publishers. Registered names, trademarks, etc. used in this book, even when not specifically marked as such, are not to be considered unprotected by law.

Printed in the Federal Republic of Germany.

Printed on acid-free paper.

Cover Design 4t Matthes + Traut Werbeagentur
GmbH, Darmstadt

Typesetting Kühn & Weyh, Satz und Medien,
Freiburg

Printing Betz-Druck GmbH, Darmstadt

Bookbinding Litges & Dopf Buchbinderei GmbH,
Heppenheim

ISBN 978-3-527-40379-0 (Vol. 3)

ISBN 978-3-527-40382-0 (Set)

Handbook of Optical Systems

Edited by Herbert Gross

Volume 3:

Aberration Theory and Correction
of Optical Systems

Handbook of Optical Systems

Edited by
Herbert Gross

Volume 1: Fundamentals of Technical Optics

Volume 2: Physical Image Formation

Volume 3: Aberration Theory and Correction of Optical Systems

Volume 4: Survey of Optical Instruments

Volume 5: Metrology of Optical Components and Systems

Volume 6: Advanced Physical Optics



WILEY-VCH Verlag GmbH & Co. KGaA

Herbert Gross

Herbert Gross was born in 1955. He studied Physics at the University of Stuttgart and joined Carl Zeiss in 1982. Since then he has been working in the department of optical design. His special areas of interest are the development of simulation methods, optical design software and algorithms, the modelling of laser systems and simulation of problems in physical optics, and the tolerancing and the measurement of optical systems. Since 1995, he has been heading the central optical design department at Zeiss. He served as a lecturer at the University of Applied Sciences at Aalen and at the University of Lausanne, and gave seminars for the Photonics Net of Baden Württemberg as well as several company internal courses. In 1995, he received his PhD at the University of Stuttgart on a work on the modelling of laser beam propagation in the partial coherent region. He has published several papers and has given many talks at conferences.

Hannfried Zügge

Hannfried Zügge was born in 1940. He studied mathematics at the University of Marburg, receiving his PhD in 1970. He worked for Carl Zeiss for more than 33 years until his retirement in 2003. As a lens designer, he worked on photographic objectives, still and cine cameras, video optics, photogrammetric and reconnaissance lenses and instruments as well as on ophthalmic lenses. His experience includes design work and also practical aspects of tolerancing and manufacturing of optical systems. He has held many talks and seminars on optical design and has published several scientific articles. In addition to his work for Carl Zeiss, he has been an assistant lecturer on lens design and aberration theory at the University of Stuttgart since 1980.

Martin Peschka

Martin Peschka was born in 1971 and studied physics at the University of Tübingen. He received his PhD in 2000 at the Institute of Experimental Physics with a thesis on the conceptual design, construction and setup of a laser scanning microscope for cryogenic temperatures and high magnetic fields and worked there as a postdoc until 2002. Since 2002 he has been with Carl Zeiss in the department of optical design. His special areas of interest are the modelling of tolerances in optical system design and questions related to the practical realization of systems. He is developing software tools for the simulation of real systems and tolerancing.

Fritz Blechinger

Fritz Blechinger was born in 1954. He received his diploma in Feinwerktechnik in 1978 from the University of Applied Science in Munich. From 1978 until 1984 he worked at Optische Werke Rodenstock, where he developed medical systems, scanning systems, infrared optics, projection objectives and software solutions. Thereafter until 1995 he worked at MBB in Ottobrunn where he became the manager of the department of optical systems. His responsibilities included developments of digital cameras, telescopes, spectrometers, interferometer systems, test optics and several other systems. From 1996 he was head of the optical design group at Linos Photonics, Munich, where he was concerned with special objectives, infrared systems, printing objectives, photosystems, among others. Since 2006 he has been the head of the optical design department at Leuze Elektronik. He is the developer of the optical design software OpTaliX, and has held many talks at conferences and published several papers.

Preface

The first two volumes of this handbook series on optical systems covered the basics of technical and physical optics. These are a prerequisite for the understanding of the following parts of the handbook.

This third volume is the starting point of the second major part of the book series. It is concerned with aberration theory, the correction and improvement of optical systems, together with their optimization and tolerancing. These aspects are the central and most important parts in the daily work of an optical designer. It was one of our aims to provide the reader with the basic ideas and tools which are necessary in order to be able to calculate, analyse and develop optical systems themselves and to comprehend the examples within this book. Therefore, free access to an extended light version of the professional optical design software OpTaliX has been made available with this volume. In the appendix, this software is briefly described.

The book itself results from the cooperation of four authors: Hannfried Zügge wrote chapters 29 and 31, Martin Peschka contributed chapter 35 and Fritz Blechinger prepared the appendix. I would like to acknowledge these colleagues for their involvement, which included useful discussions, and also for their helpful cooperation.

Furthermore, several of our colleagues helped us to correct and improve the manuscript. We want to thank Bertram Aichtner, Hans-Joachim Frasch, Markus Seeßelberg, Christoph Menke and Michael Kempe for proof reading parts of the text and for many helpful hints, corrections and suggestions. We also thank Franz Merz and Dietmar Gängler for useful information and for their technical support in selecting the examples for chapter 35. Thanks are also due to Steffen Siegler for his helpful discussions.

Finally, we want to express our gratitude to Florian Bociort, from the University of Delft, for making figure 32-59 available to us and giving permission for its inclusion in the book.

Once more I want to thank my family for their continual understanding and patience during the progress of this work.

Essingen,
September 2006

Herbert Gross

Introduction

In this third volume of the book series, the reader first discovers an introduction to the theory of aberrations, which is the basic tool used for the description and understanding of optical systems. The basic geometrical and wave optical considerations used to define the aberrations are explained. The power series representation allows for the classification of the types of aberration and enables us to distinguish between the well known primary aberrations of spherical aberration such as astigmatism, coma, field curvature and distortion. The dispersion of the transparent optical materials leads to the definition of the chromatic aberrations, which are of a somewhat different nature, but are mostly similar in that they affect the quality of the system. Using the lowest expansion order, the Seidel terms can be determined. They allow for an analytical description and a quick estimation of the aberrations. Furthermore, they can help in the understanding of the basic dependencies of the aberrations on the system parameters. The representation of the aberrations in the Zernike polynomial basis is in common use. This calculus offers several advantages; for example, a direct relationship with interferometric measurements, or the orthogonality of the polynomials, which guarantees a robust computation and easy extension to higher orders. As a final consideration in the first chapter of the book, the important concept of aplanatic and isoplanatic systems are discussed, based on the sine condition.

In the second chapter, the large variety of possible descriptions of the image quality and representations is discussed in detail. The quality of an optical system must always be measured and described by criteria, which are appropriate to the application and the system type. As a consequence, there is no universal quality metric, which makes sense for all systems. This causes some problems in comparing systems on an absolute scale.

Beginning with the elementary geometrical aberration curves and spot diagrams, the reader is introduced into the possible representations and interpretations of aberration curves. The next section deals with the quality criteria based on the wavefront of an object point. These aberrations can be classified in very similar way to the geometrical aberrations. Some special criteria help us to find absolute quality measures. This is possible, since the wavelength of the light used is a universal scale which describes the effects of decreasing quality. If the physical image formation process according to volume 2 is considered, the quality of a system can be determined from its point spread function, and depends on its size and form. The Strehl

ratio is a measure which is used for diffraction-limited systems. An extension and generalization of this concept is used to describe coherent laser beams. The beam parameter product and the single M2 term are well known metrics which qualify a beam with a general intensity distribution.

If, again, image formation and the transfer of information are considered, the resolution and the contrast of an optical system have to be measured in a quantitative manner. This leads to the idea of the two-point resolution and, in a more general way, to the concept of the optical transfer function. Depending on the application criteria, the imaging quality of an edge or a line, the concentration of the energy distribution or some other criteria, can be considered. To complete this topic, finally more general aspects such as statistical aberrations, motion blur or parasitic light are considered.

In chapter 31, the dependence of the aberrations on system parameters such as radii, refractive indices and distances are discussed. These relationships help the optical designer to decide which kind of system changes would help to correct for special aberrations. The basic aberration types are discussed separately, while simple examples enable us to learn about these effects.

In the practical work, the correction of the aberrations of a given system is usually carried out with the help of numerical optimization software. Therefore, in chapter 32 the basic mathematical aspects of nonlinear numerical optimization are presented from a more practical point of view. Several locally working algorithms, together with the boundary conditions, are discussed in detail. Special problems in the application of these algorithms to the optical design tasks are described and illustrated with the help of some concrete examples.

Global optimization algorithms play an increasing role in optical design and are explained briefly in a more phenomenological way. Some special aspects of optimization in the optical design such as typical constraints or the role of the initial system, complete this chapter.

In practice, it is a complicated task to optimize a system from scratch. To demonstrate to the reader the application of the more theoretical considerations of the preceding chapters in a realistic problem, the optimization process of a real task is described in detail in chapter 33. As an example, a monochromatic microscopic objective lens of high numerical aperture is chosen. First, some general hints for the practical design work are summarized as a brief insight into the tool box of the designer. Then the most important features and properties of high numerical aperture microscopic objective lenses are described in order to introduce this special type of system to the reader. Several published patent designs are presented for illustration and to show the major concepts in the design and correction of microscopic lenses. In the central part of the chapter, the design process of a lens is described in detail starting from the very first steps. The successive steps are documented and explained to show the reader the way to find the solution. The final design is discussed and documented to indicate the performance and its limits.

Chapter 34 deals with some special and more unusual features, which can be used in optical design to obtain better solutions or additional functionalities. The use of aspheres is described and discussed. This surface type has been well known

for many decades and has been successfully applied. The properties of these components and their potential are illustrated by several examples. The second special component discussed is the gradient index lens. There is great potential in correcting systems with this lens type, but the problems in accurately realizing them have not been solved in a general way up to this point. A special type of gradient distribution yielding promising results involves the Gradium lenses, which are discussed in more detail. Lenses with a diffractive structure are discussed as a third special type of optical component. These lenses have rather unusual properties concerning their spectral behavior. The use of diffractive or hybrid refractive–diffractive lenses is nowadays controllable and is used in industrial solutions. The design principles for these components are described with many examples in order to show the reader their possible advantages. The major drawback of diffractive elements such as unwanted diffracting orders and the problems of a broad spectral application are described, and possible solutions for their avoidance are given. As a last example, for more unconventional designs, general three-dimensional systems are discussed. The unusual vectorial aberration theory of systems without symmetry is demonstrated and some examples illustrate the possible applications of this approach.

Chapter 35 contains the important issues of tolerancing and the assembly of optical systems. Beginning with an overview of tolerance standards, the centering tolerances together with centering errors in bonding processes and optical mounts are then dealt with in more detail. After some general cost aspects of tolerances the different types of aberrations introduced into an optical system by the most important tolerance types, are discussed. The idea of exploiting the connection between tolerance types and aberration types for the compensation of tolerance effects leads to the important topic of adjustment in the assembly and integration of real optical systems. The modeling of the adjustment of an optical system is discussed and the potential for adjustment is shown by the application of the model onto a microscopic objective lens. Next, the chapter treats sensitivity analysis and statistical simulations as basic tools for practical tolerancing in optical design and their application is again exemplified for the microscopic objective lens. Inverse tolerancing is briefly discussed and finally, some specific aspects of the tolerancing of prisms are considered.

In the appendix, the professional optical design software tool OpTaliX is introduced. The main features are explained and an overview of the possibilities and program parts is given.

In addition to the basic knowledge of technical optics, which is the content of volume 1, and the physical understanding of image formation as explained in volume 2, this volume 3 now provides insight into aberration theory, correction principles, the numerical framework of optimization in optical design and the practical aspects of tolerancing systems, which should enable the reader to successfully design and develop optical systems.

Contents of Volume 1

| | | |
|----|--|-----|
| 1 | Introduction | 1 |
| 2 | Paraxial Imaging | 5 |
| 3 | Interfaces | 61 |
| 4 | Materials | 111 |
| 5 | Raytracing | 173 |
| 6 | Radiometry | 229 |
| 7 | Light Sources | 269 |
| 8 | Sensor Technology and Signal Processing | 323 |
| 9 | Theory of Color Vision | 379 |
| 10 | Optical Systems | 425 |
| 11 | Aberrations | 485 |
| 12 | Wave Optics | 523 |
| 13 | Plano-optical Components | 569 |
| 14 | Gratings | 647 |
| 15 | Special Components | 693 |
| 16 | Optical Measurement and Testing Techniques | 759 |

Contents of Volume 2

| | | |
|----|--|-----|
| 17 | The Wave Equation | 1 |
| 18 | Scalar Diffraction | 41 |
| 19 | Interference and Coherence | 99 |
| 20 | The Geometrical Optical Description and Incoherent Imaging | 187 |
| 21 | The Abbe Theory of Imaging | 239 |
| 22 | Coherence Theory of Optical Imaging | 283 |
| 23 | Three-dimensional Imaging | 319 |
| 24 | Image Examples of Selected Objects | 355 |
| 25 | Special System Examples and Applications | 409 |
| 26 | Polarization | 465 |
| 27 | Vector Diffraction | 523 |
| 28 | Polarization and Optical Imaging | 589 |
| A1 | Mathematical Appendix | 627 |

Contents

Preface XIX

Introduction XXI

| | | |
|-----------|---|-----------|
| 29 | Aberrations | 1 |
| 29.1 | Introduction | 2 |
| 29.2 | Power Series Expansions | 8 |
| 29.3 | Chromatic Aberrations | 13 |
| 29.4 | Primary Aberrations | 16 |
| 29.4.1 | Aperture and Field Dependence | 16 |
| 29.4.2 | Symmetry and Periodicity Properties | 18 |
| 29.4.3 | Presentation of Aberrations and their Impact on Image Quality | 20 |
| 29.4.4 | Calculation of the Seidel Sums | 29 |
| 29.4.5 | Stop Shift Formulae | 36 |
| 29.4.6 | Several Aberration Expressions from the Seidel Sums | 38 |
| 29.4.7 | Thin Lens Aberrations | 41 |
| 29.5 | Pupil Aberrations | 45 |
| 29.6 | High-order Aberrations | 50 |
| 29.6.1 | Fifth-order Aberrations | 50 |
| 29.6.2 | Seventh and Higher-order Aberrations | 53 |
| 29.7 | Zernike Polynomials | 55 |
| 29.8 | Special Aberration Formulae | 56 |
| 29.8.1 | Sine Condition and the Offence against the Sine Condition | 57 |
| 29.8.2 | Herschel Condition | 60 |
| 29.8.3 | Aplanatism and Isoplanatism | 61 |
| 29.8.4 | Aldis Theorem | 61 |
| 29.8.5 | Spherical Aberration, a Surface Contribution Formula | 64 |
| 29.8.6 | Aplanatic Surface and Aplanatic Lens | 68 |
| 29.9 | Literature | 70 |
| 30 | Image Quality Criteria | 71 |
| 30.1 | Introduction | 74 |
| 30.2 | Geometrical Aberrations | 76 |

| | | |
|---------|--|-----|
| 30.2.1 | Transverse Aberrations | 76 |
| 30.2.2 | Spot Diagrams | 77 |
| 30.3 | Wave Aberrations | 80 |
| | Introduction | 80 |
| 30.3.1 | PV and RMS Value of the Wavefront | 81 |
| 30.3.2 | PV and RMS Values of Simple Aberrations | 83 |
| 30.3.3 | Influence of the Spatial Frequency | 85 |
| 30.4 | Strehl Ratio | 87 |
| 30.4.1 | Introduction | 87 |
| 30.4.2 | Simple Analytical Relations | 89 |
| 30.4.3 | Approximations of the Strehl Ratio | 91 |
| 30.5 | Special Criteria | 96 |
| 30.5.1 | Rayleigh Criterion | 96 |
| 30.5.2 | Marechal Criterion | 97 |
| 30.5.3 | 80% Strehl Criterion | 98 |
| 30.6 | Criteria for PSF and Intensity Distributions | 99 |
| 30.6.1 | Introduction | 99 |
| 30.6.2 | Apodization | 101 |
| 30.6.3 | Spatial Moments | 102 |
| 30.6.4 | Kurtosis Parameter | 104 |
| 30.6.5 | Beam Quality M^2 | 106 |
| 30.6.6 | Relation between M^2 and the Conventional Criteria | 109 |
| 30.6.7 | Spot or Beam Diameter | 111 |
| 30.7 | Point Resolution | 113 |
| 30.7.1 | Introduction | 113 |
| 30.7.2 | Incoherent Two-point Resolution | 116 |
| 30.7.3 | Coherent Two-point Resolution | 121 |
| 30.8 | Depth of Focus | 124 |
| 30.8.1 | Best Receiving Plane | 127 |
| 30.8.2 | Defocus Criterion of Fisher | 130 |
| 30.8.3 | Depth of Focus for Visual Detection | 131 |
| 30.9 | MTF Criteria | 132 |
| 30.9.1 | Introduction | 132 |
| 30.9.2 | Connection with other Criteria | 134 |
| 30.9.3 | MTF for Ideal and Defocused Systems | 135 |
| 30.9.4 | MTF for Aberrations | 138 |
| 30.9.5 | Sagittal and Tangential Structures | 140 |
| 30.9.6 | Polychromatic OTF | 141 |
| 30.9.7 | Geometrical Approximated Transfer Function GTF | 142 |
| 30.9.8 | Phase Transfer Function PTF | 144 |
| 30.9.9 | Argand Diagramm | 146 |
| 30.9.10 | Contrast Versus Resolution | 147 |
| 30.9.11 | Threshold Modulation | 151 |
| 30.9.12 | Hopkins Factor | 153 |
| 30.9.13 | Area Criteria of the MTF | 155 |

| | | |
|---------|--|-----|
| 30.9.14 | Coherent Transfer Function | 156 |
| 30.9.15 | Test Charts | 157 |
| 30.9.16 | Image Examples | 161 |
| 30.10 | Edge Criteria | 163 |
| 30.10.1 | Edge Width | 163 |
| 30.10.2 | Edge Steepness | 167 |
| 30.10.3 | Acutance and Edge Defect | 168 |
| 30.11 | Line Criteria | 168 |
| 30.11.1 | Resolution of Lines | 168 |
| 30.11.2 | LSF Criterion of Struve | 170 |
| 30.11.3 | Bossung Plots | 171 |
| 30.12 | Encircled Energy | 175 |
| 30.12.1 | Introduction | 175 |
| 30.12.2 | Energy Curve of the Airy Pattern | 177 |
| 30.12.3 | Ensquared Energy | 177 |
| 30.12.4 | Displaced Energy Criterion | 178 |
| 30.13 | Special Criteria | 179 |
| 30.13.1 | Relative Ceiling | 179 |
| 30.13.2 | Fidelity | 180 |
| 30.13.3 | Structural Content | 180 |
| 30.13.4 | Correlation | 181 |
| 30.13.5 | Relations and Comparison between the Criteria | 181 |
| 30.14 | Distortion | 182 |
| 30.15 | Color Aberrations | 187 |
| 30.15.1 | Transverse Color | 187 |
| 30.15.2 | Longitudinal Color | 188 |
| 30.16 | Transmission and Illumination | 191 |
| 30.16.1 | Illumination Fall-off | 191 |
| 30.16.2 | Special Illumination Profiles | 193 |
| 30.17 | Field Dependence of the Quality | 194 |
| 30.18 | Statistical Aberrations | 199 |
| 30.18.1 | Introduction | 199 |
| 30.18.2 | Statistical Surfaces | 199 |
| 30.18.3 | Statistical Wave Aberrations | 201 |
| 30.18.4 | Point Spread Function in the Presence of Statistical Aberrations | 202 |
| 30.18.5 | Transfer Function in the Presence of Statistical Aberrations | 204 |
| 30.18.6 | Atmospheric Perturbations | 205 |
| 30.19 | Special Aspects | 207 |
| 30.19.1 | Complete Chain of Image Formation | 207 |
| 30.19.2 | Discretization Problems | 207 |
| 30.19.3 | Motion Blur | 208 |
| 30.19.4 | Special Imaging Modes | 209 |
| 30.19.5 | Parasitic Light | 209 |
| 30.19.6 | Polarization | 209 |
| 30.20 | Literature | 211 |

| | | |
|-----------|---|------------|
| 31 | Correction of Aberrations | 215 |
| 31.1 | Strategies | 216 |
| 31.1.1 | Introduction | 216 |
| 31.1.2 | Lens Bending | 218 |
| 31.1.3 | Power Splitting | 219 |
| 31.1.4 | Power Combination | 219 |
| 31.1.5 | Distances | 220 |
| 31.1.6 | Stop Position | 220 |
| 31.1.7 | Refractive Index | 221 |
| 31.1.8 | Dispersion | 222 |
| 31.1.9 | Relative Partial Dispersion | 222 |
| 31.1.10 | GRIN, Gradient Index Material | 222 |
| 31.1.11 | Cemented Surface | 223 |
| 31.1.12 | Aplanatic Surface | 223 |
| 31.1.13 | Aspherical Surface | 223 |
| 31.1.14 | Mirror | 224 |
| 31.1.15 | Diffraction Surface | 224 |
| 31.1.16 | Symmetry Principle | 225 |
| 31.1.17 | Field Lens | 226 |
| 31.2 | Monochromatic Aberrations | 226 |
| 31.2.1 | Spherical Aberration | 226 |
| 31.2.2 | Coma | 242 |
| 31.2.3 | Astigmatism | 250 |
| 31.2.4 | Petzval Curvature | 252 |
| 31.2.5 | Distortion | 261 |
| 31.2.6 | High-order Aberrations | 265 |
| 31.3 | Chromatic Aberrations | 268 |
| 31.3.1 | Axial Color and Secondary Spectrum | 269 |
| 31.3.2 | Lateral Color | 280 |
| 31.3.3 | Spherochromatism | 283 |
| 31.4 | Coexistence of Aberrations | 285 |
| 31.5 | Literature | 289 |
| 32 | Principles of Optimization | 291 |
| 32.1 | Introduction | 293 |
| 32.2 | Numerics of Optimization | 295 |
| 32.2.1 | Notation | 295 |
| 32.2.2 | Linear Matrix Algebra | 297 |
| 32.2.3 | Local Expansion of the Error Function | 299 |
| 32.2.4 | The Control Function | 300 |
| 32.2.5 | One-dimensional Minimum Search | 302 |
| 32.2.6 | Significance of the Result | 304 |
| 32.2.7 | Termination of the Iteration | 305 |
| 32.2.8 | Efficiency of Variables and Weighting Factors | 305 |
| 32.2.9 | Performance of an Algorithm | 307 |

| | | |
|---------|---|-----|
| 32.2.10 | Numerical Calculation of Derivatives | 307 |
| 32.3 | Constraints | 309 |
| 32.3.1 | Introduction | 309 |
| 32.3.2 | Kuhn–Tucker Conditions | 312 |
| 32.3.3 | Penalty Function | 313 |
| 32.3.4 | Barrier Methods | 315 |
| 32.4 | Local Solution Methods | 316 |
| 32.4.1 | Introduction | 316 |
| 32.4.2 | Method of Steepest Descent | 317 |
| 32.4.3 | Method of Newton–Raphson without Constraints | 319 |
| 32.4.4 | Damped Least-squares Method without Constraints | 320 |
| 32.4.5 | Damped Least-squares Method with Constraints | 322 |
| 32.4.6 | Conjugate Gradient Method | 323 |
| 32.4.7 | Method of Davidon, Fletcher and Powell | 324 |
| 32.4.8 | Method of Levenberg–Marquardt | 325 |
| 32.4.9 | Orthogonalization of the System Matrix | 326 |
| 32.4.10 | Derivative-free Simplex Methods | 327 |
| 32.4.11 | Comparison of Algorithms | 329 |
| 32.5 | Global Optimization Methods | 330 |
| 32.5.1 | Introduction | 330 |
| 32.5.2 | Simulated Annealing | 332 |
| 32.5.3 | Genetic Optimization | 335 |
| 32.6 | Optimization of Optical Systems | 337 |
| 32.6.1 | Introduction | 337 |
| 32.6.2 | Example 1: Bending of a Thin Lens | 338 |
| 32.6.3 | Example 2: Achromatic Doublet | 339 |
| 32.6.4 | Parameters of Optical Systems | 347 |
| 32.6.5 | Constraints of Optical Systems | 347 |
| 32.6.6 | Merit Function | 349 |
| 32.6.7 | Special Aspects | 350 |
| 32.7 | Starting Systems in Lens Design | 353 |
| 32.7.1 | Introduction | 353 |
| 32.7.2 | Thin Lens Start System | 354 |
| 32.7.3 | Structural Approach according to Shafer | 355 |
| 32.8 | Controlling the Optimization Process | 356 |
| 32.8.1 | The Complete Design Process | 356 |
| 32.8.2 | Structural Changes in the System | 358 |
| 32.8.3 | Expert Systems | 359 |
| 32.8.4 | Global Optimization in Optical Design | 360 |
| 32.8.5 | The Saddle Point Method of Bociort | 362 |
| 32.8.6 | Isshikis Method of the Global Explorer | 365 |
| 32.8.7 | Adaptive Correction Method According to Glatzel | 368 |
| 32.9 | Literature | 369 |