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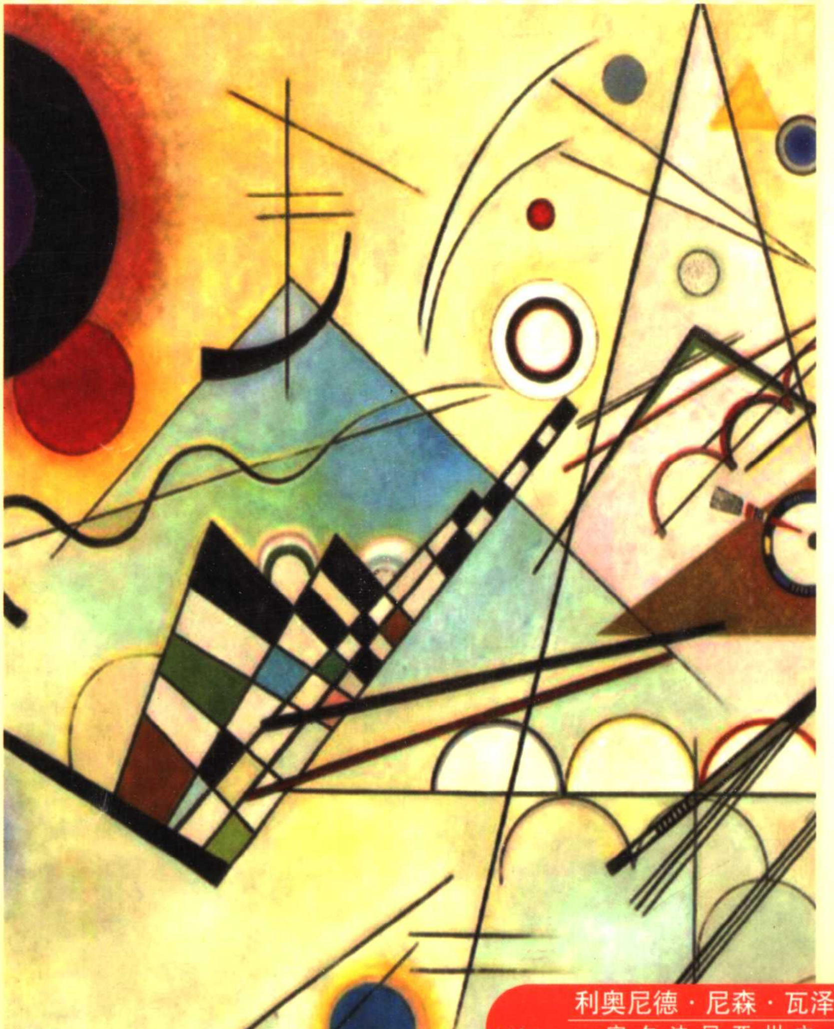
(英文版)



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(美) 利奥尼德·尼森·瓦泽斯坦 宾夕法尼亚州立大学
克里斯托弗·卡特利尔·伯恩 宾夕法尼亚州立大学 著

机械工业出版社
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前　言

为什么要写本书

这本教材是从一门高级的本科生数学课程演变而来的，课程的对象是具有不同数学基础和来自不同专业的学生，包括数学、计算机科学、统计学、工程科学、中等教育、精算学、计算机工程学、理学、工商学专业的学生。有些是按五年制理学和工商学整合型教学计划读工商管理硕士的学生，而有些是攻读博士学位的学生。

由于这门课程不需要诸如微积分、微分方程、抽象代数、拓扑学或数论等高深的数学理论，所以为那些只具有中等数学基础的学生学习一些实用而又重要的数学提供了一个机会。考虑到这一点，只要可能，本书就尽量避免使用诸如向量空间、行列式、梯矩阵、极限和导数等高深或复杂的数学概念。

许多学生之所以选修这门课程，是因为线性规划在商业和其他领域有广泛的使用。他们需要学习如何对实际问题建模，如何改写模型以便用具体的计算机软件求解，以及如何解释计算结果并应用于实际问题。一旦计算机不能得到任何计算结果或者结果没有意义，他们应该能够对问题进行调整或者另外选择一个合适的软件。

在线性规划方面有许多优秀的教科书，但其中大多数需要很强的数学基础，只适于数学专业使用，或者只有知识超前的学生能够阅读，而包含的材料又大大超出一个学期的课程。

真正具有挑战性的是同时让高层次的学生和初学者在同一个课堂学习！在宾夕法尼亚州立大学，虽然线性代数是学习线性规划的先修课程，但课堂上有些学生解线性方程组存在困难。另一方面，课堂上有些学生在数学或计算机科学方面却很强。

因此，我尝试不重蹈传统教材的老路，因为它们有点像民间故事中的金发姑娘拒绝食用的麦片粥一样——它们包含的题材要么太“冷”（内容过于平凡），要么太“热”（需要严格的数学基础）。前者会使许多学生感到厌烦；而后者会有一些内容使许多学生难于接受。

这本教材从入门开始介绍，只假定读者具有很少的数学基础。因此，我给读者提供了一个机会，在学习线性规划的同时，在看到单纯形方法之前，首先了解线性代数和逻辑学中的相关工具。有关逻辑的一节是线性规划的重要组成部分，虽然这一点经常被忽视。在整本书中，我介绍了大量的例子和应用，并要求学生尝试不同难度的习题。学生们很喜欢这种学习线性规划的方法，这可以从选修这门课程的学生数量以及他们在学期末的评价表中给出的评价得到证实。

计算机应用的普遍性并没有消除对计算技巧的需求，但增加了逻辑技巧的相对重要性。现在，如果你能通过手工计算得到圆周率的前100位数字，那只是出于好奇，而不能算是什么重要的结果，因为目前的计算机能够把圆周率的前10^{10}位数字算出来。但是，从逻辑上看，是否有可能把圆周率的第10^{100}位数字算出来呢？

如何使用本书

这本教材是按照三个层次写的。即使对于不了解线性代数和微积分的学生来说其中大部分

内容也可以读懂。对于程度更高的学生，本书给出了一些注释和习题。在书后的附录中，给出了线性规划和数学规划其他方面发展的一般思想，为进一步研究提供指导。附录中还给出了第1章至第8章中提到过的需要更强数学基础的一些主题的细节，并对高于典型的美国大学本科生水准的那部分学生提供经验和高深知识。

教材中给出了很多例子及其解答，所以我觉得没有必要给学生提供大量习题的答案。即便如此，在本书的最后，我还是给出部分习题的答案⊖，包括那些比较棘手的习题。习题的难度是不同的，但所有习题都可以用手工计算求解。我没有提供用计算机求解具有很大优势的习题。第1章第1节的习题除了可以检查对各种定义的理解外，还可以测试学生的数学基础。

致谢及参考资料

我的课堂讲稿经过了几年的演进，很多学生和阅卷评分者对讲稿的改进做出了贡献，他们指出其中的印刷错误和其他错误，并提出各种各样的问题。Prentice Hall出版社的审阅人和编辑也提了许多修改和改进意见。

我故意没有将本书与任何一个特定的软件联系起来，因为我相信学生学习了本书的材料后，当他们面对一个好的软件包时，能够聪明地应用这些知识。还有一个原因是，随着新软件包的出现以及计算机和操作系统的发展，任何一个特定的软件包都会很快过时。

但是，允许上课的学生使用他们喜欢的任何软硬件，即使在测验时也是如此。能够求解线性规划问题的软件包包括Mathemetica、Maple、Excel等。

在因特网上有很多有关线性规划的软件，有的可以免费下载，有的可以在线使用。因特网上也有很多关于线性规划的有用信息。这里我列举一些网址，不过要记住，网上的变化是很快的：

- http://carbon.cudenver.edu/hgreenbe/glossary/
  (数学规划词汇表)
- http://www.mathprog.org/
  (数学规划学会)
- http://iris.gmu.edu/asofer/siagopt.html
  (美国工业与应用数学学会最优化活动组)
- http://solon.cma.univie.ac.at/neum/glopt.html
  (全局最优化，维也纳)
- http://www.informs.org/Resources/
  (美国运筹学与管理科学学会)

在网络上以“线性规划”为关键词，可以搜索到很多网站。有关线性规划的书也很多，在2002年8月16日，从网站http://www.amazon.com上检索到的“线性规划”的书多达771本。

也有很多杂志发表线性规划和非线性规划方面的文章。在2002年8月16日，网站http://www.informs.org/Resources/上列出运筹学方面的36种纸介质的杂志和14种在线杂志。该网站还列出运筹学方面的35个学会。

利奥尼德·尼森·瓦泽斯坦
vstein @ math. psu. edu

⊖ 习题答案请参考华章网站（http://www.hzbook.com）。——编辑注

Contents

前言 iii

Chapter 1. Introduction

1. What Is Linear Programming? 1
2. Examples of Linear Programs 12
3. Graphical Method 25

Chapter 2. Background

4. Logic 34
5. Matrices 43
6. Systems of Linear Equations 52

Chapter 3. Tableaux and Pivoting

7. Standard and Canonical Forms for Linear Programs 67
8. Pivoting Tableaux 76
9. Standard Row Tableaux 89

Chapter 4. Simplex Method

10. Simplex Method, Phase 2 100
11. Simplex Method, Phase 1 112
12. Geometric Interpretation 121

Chapter 5. Duality

13. Dual Problems 133
14. Sensitivity Analysis and Parametric Programming 143
15. More on Duality 156

Chapter 6. Transportation Problems

16. Phase 1 166
17. Phase 2 180
18. Job Assignment Problem 192

Chapter 7. Matrix Games

19. What are Matrix Games? 199
20. Matrix Games and Linear Programming 210
21. Other Methods 220

Chapter 8. Linear Approximation

22. What is Linear Approximation? 229
23. Linear Approximation and Linear Programming 241
24. More Examples 251

Appendix. Guide to Mathematical Programming

A1. Mathematical Programming 257
A2. Univariate Programming 261
A3. Convex and Quadratic Programming 270
A4. Multivariate Programming 273
A5. Interior Methods 280
A6. Perturbation 286
A7. Goal Programming 287
A8. Linear Programming in Small Dimension 289
A9. Integer Programming 292
A10. Sorting, Order Statistics, and Saddle Points 295
A11. Other Topics and Recent Developments 298

Bibliography 301
Index 305

Chapter 1

Introduction

§1. What Is Linear Programming?

Perhaps the earliest examples of mathematical models for analyzing and optimizing the economy were provided almost 250 years ago by a French economist. In his *Tableau Économique,* written in 1758, François Quesnay (1694–1774) explained the interrelation of the roles of the landlord, peasant, and artisan in eighteenth-century France by considering several factors separately. For example, there are "The Economical Tableau considered relative to National Cash," and "The Economical Tableau considered in the Estimation of the Produce and Capital Stock of Every Kind of Riches."

The nineteenth-century French mathematician Jean-Baptiste-Joseph Fourier (1768–1830) had some knowledge of the subject of linear programming, as evidenced by his work in linear inequalities as early as 1826 (see §A.10 in the Appendix). He also suggested the simplex method for solving linear programs arising from linear approximation (see Chapter 8). In the late 1800s, the writings of the French economist L. Walras (1834–1910) demonstrated his use of linear programming. However, with a few other notable exceptions, such as Kantorovich's 1939 monograph *Mathematical Methods for Organization and Planning of Production,* there was comparatively little attention paid to linear programming preceding World War II.

The fortuitous synchronization of the advent of the computer and George B. Dantzig's reinvention of the simplex algorithm in 1947 contributed to the dizzyingly explosive development of linear programming with applications to economics, business, industrial engineering, actuarial sciences, operations research, and game theory. Progress in linear programming is noteworthy enough to be reported in the *New York Times.* In 1970 P. Samuelson (b. 1915) was awarded the Nobel Prize in Economics, and in 1975 L. Kantorovich (1912–1986) and T. C. Koopmans (1910–1985) received the

Nobel Prize in Economics for their work in linear programming. The subject of linear programming even made its way into Len Deighton's suspense spy story, *The Billion Dollar Brain,* published in 1966:

"I don't want to bore you," Harvey said, "but you should understand that these heaps of wire can practically think—linear programming—which means that instead of going through all alternatives they have a hunch which is the right one."

Optimization problems come in two flavors: maximization problems and minimization problems. In a maximization problem, we want to maximize a function over a set, and in a minimization problem, we want to minimize a function over a set,

In both cases, the function is real valued and it is called the *objective function.* The set is called the *feasible region* or the set of *feasible solutions.* To solve an optimization (maximization or minimization) problem means usually to find both the *optimal value* (maximal or minimal value, respectively) over the feasible region and an *optimal solution* or *optimizer* [i.e., how (where) to reach the optimal value, if it is possible]. It is not required unless otherwise instructed to find all optimal solutions. This is different from solving a system of linear equations, where a complete answer describes all solutions.

The optimal value is also known as *optimum* or *extremum.* Depending on the flavor, the terms *maximum* (max for short) and *minimum* (min for short) are also used. The set of all optimal solutions (maximizers or minimizers) is called the *optimality region.*

Now we consider a simple example.

Imagine that you are asked to solve the following optimization problem:

$$\begin{cases} \text{Maximize} & x \\ \text{subject to} & 2 \le x \le 3. \end{cases}$$

Clearly the goal is to find the largest value for x, given that this variable is limited as to the values it can assume. Since these limitations are explicitly stated as functions of the variable under consideration, called the *objective variable,* there is no difficulty in solving the problem; just take the maximum value. Thus, you can correctly conclude that the maximum value for x is 3, attained at $x = 3$.

However, it is more often the case that the range of values for the objective variable is given implicitly by placing limitations on another variable or other variables related with the objective variable. These variables are called *decision* or *control variables.* These variables are under our control: We are free to decide their values subject to given constraints. They are different from data that form an input for our optimization problem. The objective function is always a function of decision variables. Sometimes it has a name called the *objective variable.*

For instance, in a problem such as finding which rectangles of fixed perimeter encompass the largest area, the objective variable is "area," and the decision variables are l = length of the rectangle and w = width of the rectangle. In general, when the objective variable is given as a function of decision variables, we use the term *objective function* to describe the function we want to optimize. These limitations on the decision variables, however they might be described, are called the *constraints* or *restraints* of the problem.

Thus, a *mathematical program* is an optimization problem where the objective function is a function of real variables (decision variables) and the feasible region is given by conditions (constraints) on the variables. So a feasible solution is a set of values for all the decision variables satisfying all the constraints in the problem. Mathematical programs are addressed in *mathematical programming.*

What is *linear programming* then? Linear programming is the part of mathematical programming that studies optimization (extremal) problems having objective functions and constraints of particularly simple form. Mathematically, a *linear program* is an optimization problem of the following form: Maximize (or, sometimes, minimize) an *affine function* subject to a finite set of *linear constraints.* Contrary to modern perception, the word *programming* here does not refer to computer programming. In our context, which goes back to military planning, *programming* means something like "detailed planning."

Now we define the terms *affine function* and *linear constraint.* In this book, unless indicated otherwise, a *number* means a"real number" and a function means a "real-valued function."

Definition 1.1. A function f of variables $x_1, \ldots, x_n$ is called a *linear form* if it can be written as $c_1x_1 + \cdots + c_nx_n$, where the coefficients c_i are given real numbers (constants). A function f is called *affine* if it is the sum of a linear form and a constant. ■

Of course, it is not necessary to denote these variables as x_i, the coefficients as c_i, or the function as f. For example, $g(x, y) = 2x - a^2y$, where a is some fixed real number, is a linear form in two variables, which are denoted x and y instead of x_1 and x_2. Note that if a were a variable and y were a fixed nonzero number, then $f(x, a) = 2x - a^2y$ is not a linear form in x and a (see Problem 1.2). Here are three affine functions of two variables, x, y : $x - 4y - 3$, $y + 2$, $x + y$.

Problem 1.2. Show that the function $g(x, a) = 2x - a^2y$, where y is a fixed *nonzero* number, is not a linear form in x and a.

Solution. Suppose, to the contrary, that $g(x, a) = 2x - a^2y$ is a linear form in x and a; that is, $g(x, a) = 2x - a^2y = c_1x + c_2a$ with coefficients c_1, c_2 independent of x and a. Then $g(1, 0) = 2 = c_1$ and $g(0, 1) = -y = c_2$. Thus, $g(x, a) = 2x - a^2y = 2x - ya$, hence $a^2y = ya$ for all a. Taking $a = 2$, we see that $y = 0$. But, since y cannot equal zero by hypothesis, we have arrived at our hoped-for contradiction. ∎

The term *linear function* means "linear form" in some textbooks and "affine function" in others. The term *linear functional* in linear programming means "linear form."

Linear constraints come in three flavors, of type $=$, $\geq$, or $\leq$. The linear constraints of type $=$ are familiar linear equations, that is, the equalities of the form

an affine function = an affine function.

Most often, they come in the standard form

a linear form = a constant.

For illustration, $x = 2$, $x - y = 0$, $5y = 7$ are three linear equations for two variables x, y written in standard form, while $2 = x$, $x = y$, $3y + x + 3 = x - 2y - 4$ are the same equations written differently.

Two other types of linear constraints are inequalities of the form

an affine function ($\leq$ or $\geq$) an affine function.

Often they are written as

a linear form ($\leq$ or $\geq$) a constant.

Thus, a linear constraint consists of two affine functions (the left-hand side and the right-hand side) connected by one of three symbols: $=$, $\leq$, $\geq$. Strict linear inequalities such as $x > 0$ are not considered to be linear constraints.

Example 1.3

(i) $y = \sin 5$ is a linear constraint on the variable y.

(ii) $x \geq 0$ is a linear constraint on the variable x.

(iii) $2x + 3y \leq 7$ is a linear constraint on the variables x and y.

Note, however, that

(iv) $y + \sin x = 1$

is not a linear constraint on the variables x and y, since $\sin x$ is *not* a linear form in x.

Definition 1.4. A *linear program* (LP for short), or *linear programming problem,* is any optimization problem where we are required to maximize (or minimize) an affine function subject to a *finite* set of linear constraints. ■

For example, the following is a linear program:

$$\begin{cases} \text{minimize} & f(x_1, \ldots, x_n) = c_1x_1 + \cdots + c_nx_n + d, \\ \text{subject to} & \displaystyle\sum_{i=1}^{n} a_{ji}\, x_i \leq b_j \quad \text{for } j = 1, \ldots, m \\ & x_i \geq 0 \quad \text{for } i = 1, \ldots, n \end{cases} \tag{1.5}$$

where m, n are given natural numbers, $d, c_i, , b_j, a_{j,i}$ are constants, and x_i are decision (control) variables (unknowns). We call (1.5) a linear program in *canonical* form.

The finite set of constraints in Definition 1.4 can be empty. In other words, the number of constraints is allowed to be zero. If there are no constraints in an optimization problem, we talk about *unconstrained* optimization. Note that, unless otherwise instructed, we cannot ignore any of the given constraints in an optimization problem.

Recall that constant terms are not allowed in linear forms, but we allow constant terms in the objective functions of linear programs. Thus, according to our definitions, the function $x - 2y + 3$ of two variables x and y is not a linear form but it is an affine function, and it can be the objective function of a linear program. Some textbooks make different choices in definitions.

It is possible to have an optimization problem or even a linear program for which there are no feasible solutions (see Example 1.6). Such a problem is called *infeasible* or *inconsistent*. It is also possible for an optimization problem to have feasible solutions but no optimal solutions. For example, maximize x subject to $x < 1$. This explains why we do not allow these kind of constraints in linear programming.

An optimization problem is called *unbounded* if the objective function takes arbitrary large values in the case of the maximization problem and arbitrary small values in the case of the minimization problem (Example 1.7). We will see in Chapter 4 that any feasible linear program either has an optimal solution or is unbounded.

Note that there may be more than one optimal solution (or none at all, as in Example 1.6) among the feasible solutions (Example 1.8). However, the optimal (maximal or minimal) value of an optimization problem is unique (if it exists). Had we found two different values, one would be better, so the other would not be optimal.

Example 1.6. *An Infeasible LP*

$$\begin{cases} \text{Maximize} & 4x + 5y \\ \text{subject to} & 2x + y \leq 4 \\ & -2x - y \leq -5 \\ & x \geq 0, \quad y \geq 0. \end{cases}$$

Note that if x and y satisfy the constraint $2x + y \leq 4$, then, by multiplying by -1, we obtain $-2x - y \geq -4$. However, the second constraint demands that $-2x - y \leq -5$. Obviously, the two given constraints are mutually exclusive and therefore there are no feasible solutions. This linear program is *infeasible.* ■

Example 1.7. *An Unbounded LP*

$$\begin{cases} \text{Maximize} & x - 2y \\ \text{subject to} & -3x + 2y \leq -2 \\ & -6x - 5y \leq -1 \\ & x \geq 0, \quad y \geq 0. \end{cases}$$

This linear program does have feasible solutions (for example $x = 2/3$, $y = 0$), but none of them is optimal. For any real number M, there is a feasible solution x, y such that $x - 2y > M$. An example of such a feasible solution is $x = 2/3 + M$ and $y = 0$. In a sense, there are so many feasible solutions that none of them even gets close to being optimal. This linear program is *unbounded.* ■

Example 1.8 *A LP with Many Optimal Solutions*

$$\begin{cases} \text{Minimize} & x + y \\ \text{subject to} & x, y, z \geq 0. \end{cases}$$

In this problem with three variables x, y, z the optimal solutions are $x = y = 0$, $z \geq 0$ arbitrary nonnegative number. The optimal value is 0. ■

Example 1.9. *A LP with One Optimal Solution*

$$\begin{cases} \text{Minimize} & x + y + z \\ \text{subject to} & x \geq -1, y \geq 2, z \geq 0. \end{cases}$$

In this linear program with three variables x, y, z the optimal solution is $x = -1, y = 2, z = 0$. The optimal value is 1. Note that a solution should contain values for all variables involved. ∎

Example 1.10. *A Nonlinear Problem*

$$\begin{cases} \text{Minimize} & x^2 + y^3 + z^4 \\ \text{subject to} & |x| \geq 1, |y| \leq 3. \end{cases}$$

This is a mathematical program with three variables and two constraints that is not linear because the objective function is not affine and the constraints are not linear. (However, the second constraint can be replaced by two linear constraints, and the feasible region is the disjoint union of two parts; each can be given by three linear constraints.) Nevertheless, using common sense, it is clear that the problem splits into three separate optimization problems with one variable each. So there are exactly two optimal solutions, $x = \pm 1, y = -3, z = 0$ and min $= -26$. ∎

All numbers in linear programming are real numbers. In fact, it is hard to imagine a linear program arising out of business and industrial concerns, with numbers not being actually rational numbers. Why? You might ask yourself if the price of a product could be stated as an irrational number, for example, $\sqrt{2}$. We will see later that to solve a linear programming problem with rational data we do not need irrational numbers. However, this is not the case with nonlinear problems, as you can see when you solve the (nonlinear) equation $x^2 = 2$.

To develop your own appreciation of optimization problems, try to solve the following two problems. Are they linear programs?

Problem 1.11

$$\begin{cases} \text{Maximize} & x \\ \text{subject to} & 2 \leq x \leq 3. \end{cases}$$

Solution. As we noted earlier, the maximal value is 3 (max $= 3$ for short) and it is reached at $x = 3$. ∎

One of the main applications of the first derivative of a function, which you study in calculus, is to find the maxima or minima of a function by looking at the critical points. Yet you can see from Problem 1.11 that first-year calculus is not sufficient to solve linear programs. Suppose you are trying to find the maximum and minimum of the linear form $f(x) = x$ on the interval $2 \leq x \leq 3$ by determining where the first derivative equals zero. You observe that the first derivative, 1, never equals zero. Yet the objective function reaches its maximum, 3, and minimum, 2, on this interval.

Problem 1.12

$$\begin{cases} \text{Maximize} & x + 2y + z \\ \text{subject to} & x + y = 1, \\ & z \geq 0. \end{cases}$$

Solution. The objective function takes arbitrarily large values as y goes to $+\infty$, $x = 1 - y$, $z = 0$. Informally, we can write max $= \infty$. This is an *unbounded* linear program. ■

Problems 1.11 and 1.12 are both linear programs because the objective functions and all the constraints are linear. Note that the optimal (maximal or minimal) value of an optimization problem is unique. (Had we found two different values, one would be better, so the other would not be optimal.) The optimal value always exists if we add the symbols $-\infty$, $+\infty$ to the set of real numbers as possible values for the optimal value. But if it is reached at all, there could be more than one way to reach it. That is, we may have many optimal solutions for the same optimal value.

Now you have encountered the following terms: *linear form, affine function, linear constraint, linear program, objective function, optimal value, optimal solution, feasible solution.* Try to explain the meaning of each of these terms.

Remark. We have already mentioned that linear programming is a part of mathematical programming. In its turn, mathematical programming is a tool in *operations research* (or operational research), which is an application of scientific methods to the management and administration of organized military, governmental, commercial, and industrial processes. Historically, the terms *programming* and *operations* came from planning military operations.

The terms *systems engineering* and *management science* mean almost the same as operations research with less or more stress on the human factor. As a part of operations research, linear programming is concerned not only with solving of linear programs but also with

- acquiring and processing data required to make decisions
- problem formulation and model construction
- testing the models and interpreting solutions
- implementing solutions into decisions
- controlling the decisions
- organizing and interconnecting different aspects of the process

In this book we stress mathematical aspects of linear programming, but we are also concerned with translating word problems into mathematical language, transforming linear programs into differents forms, and making connections with game theory and statistics.

How is linear programming connected with *linear algebra*? The main concern in linear algebra is solving systems of linear equations. We will see in Chapter 5 that solving linear programs is equivalent to finding feasible solutions for systems of linear constraints. Thus, from a mathematical point of view, linear programming is about more general and difficult problems.

Remark. Besides mathematical programming, there are other areas of mathematics and computer science where optimization plays a prominent role. For example, both *control theory* and *calculus of variations* are concerned with optimization problems that cannot be described easily with a finite set of variables. The feasible solutions could be functions satisfying certain conditions. We may ask what is the shortest curve connecting two given points in plane. Or we can ask about the most efficient way to sort data of any size. Sometimes mathematical programming can help to solve those problems.

Historic Remark. The mathematicians mentioned in this book are well known, and their bios can be found in encyclopedias, biographies, history books, and on the Web.

Joseph Fourier, a French mathematician well known also as an Egyptologist and administrator, is famous for his Fourier series, which are very important in mathematical physics and engineering. His work on linear approximation and linear programming is not so well known. A son of a tailor, he had 11 siblings and 3 half-siblings. His mother died when he was nine years old, and his father died the following year. He received military and religious education and was involved in politics. His life was in danger a few times.

Leonid Kantorovich, a Soviet mathematician with very important contributions to economics, was almost unknown in the United States until the simplex method was successfully implemented for computers and widely used. He got his Ph.D. in mathematics at age 18. The author had the pleasure of meeting Kantorovich several times at mathematical talks and at business meetings involving optimization of advanced planning in the former U.S.S.R. One of many things he did in mathematics was introducing the notion of a distance between probability distributions, which was rediscoved later in different forms by other mathematicians, including the author (the Vaserstein distance). This distance is the optimal value for a problem similar to the transportation problem (see Example 2.4 and Chapter 6).

Exercises

1–13. State whether the following are true or false. Explain your reasoning.

1. $1 \le 2$ **2.** $-10 \le -1$

3. $3 \le 3$ **4.** $-5/12 \ge -3/7$

5. $x^2 + |y| \ge 0$ for all numbers x, y

6. $3x \ge x$ for all numbers x

7. $3x^3 \ge 2x^2$ for all numbers x

8. Every linear program should have at least one linear constraint

9. Every linear program has an optimal solution

10. Each variable in a linear program should be nonnegative

11. Any linear program has a unique optimal solution

12. The total number of constraints in a linear program is always larger than the number of variables

13. The constraint $2x + 5 = 6x - 3$ is equivalent to a linear equation for x ■

14–17. Determine whether the following functions of x and y are linear forms.

14. $2x$ **15.** $x + y + 1$

16. $(\sin 1)\, x + e^z\, y$ **17.** $x \sin a + y\, z$ ■

18–23. Is this a linear constraint for x?

18. $x > 2$ **19.** $|x| \le 1$

20. $0 = 1$ **21.** $0 \ge 1$

22. $xy^2 = 3$ **23.** $ax = b$. ■

24–26. Do you agree with the following statements? Why or why not?

24. $|x| \le 1$ is equivalent to a system of two linear constraints

25. $|x| \ge 1$ is equivalent to a system of two linear constraints

26. The equation $(x-1)^2 = 0$ is equivalent to a linear constraint for x ■

27. Solve the equation $ax = b$ for x, where a and b are given numbers.

28–30. Solve the following three linear systems of equations for x and y.

28.

$$\begin{cases} x + 2y = 3 \\ 5x + 9y = 4 \end{cases}$$