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Analysis of Turbulent Flows

Second Revised and Expanded Edition

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by

TUNCER CEBECI
Distinguished Technical Fellow,
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Preface to the Second Edition

The first edition of this book, *Analysis of Turbulent Boundary Layers*, was written in the period between 1970 and early 1974 when the subject of turbulence was in its early stages and that of turbulence modeling in its infancy. The subject had advanced considerably over the years with greater emphasis on the use of numerical methods and an increasing requirement and ability to calculate turbulent two- and three-dimensional flows with and without separation. The tools for experimentation were still the traditional Pitot tube and hot wire-anemometer so that the range of flows that could be examined was limited and computational methods still included integral methods and a small range of procedures based on the numerical solution of boundary layer equations and designed to match the limited range of measured conditions. There have been tremendous advances in experimental techniques with the development of non-intrusive optical methods such as laser-Doppler, phase-Doppler and particle-image velocimetry, all for the measurement of velocity and related quantities and of a wide range of methods for the measurement of scalars. These advances have allowed an equivalent expansion in the range of flows that have been investigated and also in the way in which they could be examined and interpreted. Similarly, the use of numerical methods to solve time-averaged forms of the Navier-Stokes equations, sometimes interactively with the inviscid-flow equations, has expanded, even more so with the rise and sometimes fall of Companies that wished to promote and sell particular computer codes. The result of these developments has been an enormous expansion of the literature and has provided a great deal of information beyond that which was available when the first edition was written. Thus, the topics of the first edition needed to be re-examined in the light of new experiments and calculations, and the ability of calculation methods to predict a wide range of practical flows, including those with separation, to be reassessed.

This second edition, entitled *Analysis of Turbulent Flows*, undertakes the necessary reappraisal, reformulation and expansion, and evaluates calculation methods more extensively but also within the limitations of two-dimensional equations largely because this makes explanations easier and the book of acceptable size. In addition, it is written to meet the needs of graduate students as well as engineers and so includes homework problems that are more sensibly formulated within the constraints of two independent variables. References to more complex flows, and particularly those with separation, are provided and the relative merits of various turbulence models considered. This topic is expanded in a separate book, *Turbulence Models and Their Application*, aimed more at the practicing engineer than at the graduate student and published by Horizons/Springer in 2003. Another book, *An Engineering Approach to the Calculation of Aerodynamic Flows*, also published by Horizons/Springer in 1999, demonstrates the range of applicability of interactive boundary layer theory.

The structure of the first edition is retained but there are changes and expansions in all chapters. Chapters 1 and 4 provide information of turbulence and turbulent flows and have been expanded to consider a selection of the large quantity of new information even though some of it is difficult to use within calculation methods. Chapters 2 and 3, which deal with conservation equations and the reduced forms appropriate to boundary-layer flows respectively are improved but retain their original format. Chapter 4 has been expanded to include the thermal boundary layer and some aspects of compressibility. Chapters 5 and 6 have been re-written to provide comprehensive descriptions of algebraic and transport turbulence models, both in the contexts of boundary-layer and Navier-Stokes equations. Chapter 7 is also new and describes simple methods, including those that solve algebraic and integral equations, for the calculation of momentum and heat-transfer in wall boundary layers with and without pressure gradients and in free shear flows. Chapter 8, also new, describes an efficient and accurate method for the solution of incompressible, two-dimensional boundary layer equations in differential form. This method emphasizes the use of the algebraic eddy-viscosity approach of Chapter 5 and allows appraisal of the method for attached and separated flows. The use of this numerical method with the transport turbulence models of Chapter 6 is considered in Chapter 9 for boundary-layer flows, and the appraisal of the models again includes consideration of attached and separated flows, the latter sometimes within higher-order forms of the Navier-Stokes equations. The last chapter, Chapter 10, describes computer programs that are provided on a CD-ROM and include integral and differential methods for boundary layer flows with and without heat transfer. The differential methods offer algebraic and transport turbulence models.

The content of this new edition should be viewed in the context of new developments such as those associated with large-eddy simulations (LES) and direct numerical solutions (DNS) of the Navier-Stokes equations. LES existed in 1976 as part of the effort to represent meteorological flows and has been rediscovered recently as part of the recognition of the approximate nature of solutions of time-averaged equations as considered here. There is no doubt that LES has a place in the spectrum of methods applied to the prediction of turbulent flows but we should not expect a panacea since it too involves approximations within the numerical method, the filter between time-dependent and time-average solutions and small-scale modeling. DNS approach also has imperfections and mainly associated with the computational expense which implies compromises between accuracy and complexity or, more usually, restriction to simple boundary conditions and low Reynolds numbers. It is likely that practical aerodynamic calculations with and without separation will continue to make use of solutions of the inviscid-flow equations and some reduced forms of the Navier-Stokes equations for many years, and this book is aimed mainly at this approach.

This second edition was written with help from many colleagues. AMO Smith was an enthusiastic catalyst and ideas were discussed with him over the years. Many colleagues and friends from Boeing, the former Douglas Aircraft Company and the McDonnell-Douglas Company have contributed by discussion and advice and include K. C. Chang and J. P. Shao. Similarly, Peter Bradshaw of Stanford University, Herb Keller of Cal Tech and Jim Whitelaw of Imperial College have helped in countless ways. I am also grateful to Arno Schouwenburg of Elsevier Publishing Company.

Indian Wells, November 2003 *Tuncer Cebeci*

Contents

1 Introduction

1.1 Introductory Remarks

Turbulence in viscous flows is described by the Navier–Stokes equations, perfected by Stokes in 1845, and now soluble by Direct Numerical Simulation (DNS). However, computing capacity restricts solutions to simple boundary conditions and moderate Reynolds numbers and calculations for complex geometries are very costly. Thus, there is need for simplified, and therefore approximate, calculations for most engineering problems. It is instructive to go back some eighty years to remarks made by Prandtl [1] who began an important lecture as follows:

> What I am about to say on the phenomena of turbulent flows is still far from conclusive. It concerns, rather, the first steps in a new path which I hope will be followed by many others.
> The researches on the problem of turbulence which have been carried on at Göttingen for about five years have unfortunately left the hope of a thorough understanding of turbulent flow very small. The photographs and kinetographic pictures have shown us only how hopelessly complicated this flow is . . .

Prandtl spoke at a time when numerical calculations made use of primitive devices – slide rules and mechanical desk calculators. We are no longer "hopeless" because DNS provides us with complete details of simple turbulent flows, while experiments have advanced with the help of new techniques including non-obtrusive laser-Doppler and particle-image velocimetry. Also, developments in large-eddy simulation (LES) are also likely to be helpful although this method also involves approximations, both in the filter separating the large (low-wave-number) eddies and the small 'sub-grid-scale' eddies, and in the semi-empirical models for the latter.

Even LES is currently too expensive for routine use in engineering, and a common procedure is to adopt the decomposition first introduced by Reynolds for incompressible flows in which the turbulent motion is assumed to comprise the sum of mean (usually time-averaged) and fluctuating parts, the latter covering the whole range of eddy sizes. When introduced into the Navier–Stokes equations in terms of dependent variables the time-averaged equations provide a basis for assumptions for turbulent diffusion terms and, therefore, for attacking mean-flow problems. The resulting equations and their reduced forms contain additional terms, known as the Reynolds stresses and representing turbulent diffusion, so that there are more unknowns than equations. A similar situation arises in transfer of heat and other scalar quantities. In order to proceed further, additional equations for these unknown quantities, or assumptions about the relationship between the unknown quantities and the mean-flow variables, are required. This is referred to as the "closure" problem of turbulence modeling.

The subject of turbulence modeling has advanced considerably in the last forty years, corresponding roughly to the increasing availability of powerful digital computers. The process started with 'algebraic' formulations (for example, algebraic formulas for eddy viscosity) and progressed towards methods in which partial differential equations for the transport of turbulence quantities (eddy viscosity, or the Reynolds stresses themselves) are solved simultaneously with reduced forms of the Navier–Stokes equations. At the same time numerical methods have been developed to solve forms of the conservation equations which are more general than the two-dimensional boundary layer equations considered at the Stanford Conference of 1968.

The first edition of this book was written in the period from 1968 to 1973 and was confined to algebraic models for two-dimensional boundary layers. Transport models were in their infancy and were discussed without serious application or evaluation. There were no similar books at that time. This situation has changed and there are several books to which the reader can refer. Books on turbulence include those of Tennekes and Lumley [2], Lesieur [3], Durbin and Petterson [5]. Among those on turbulence models the most comprehensive is probably that of Wilcox [6].

The present book has greater emphasis on modern numerical methods for boundary-layer equations than the first edition and considers turbulence models from advanced algebraic to transport equations but with more emphasis on engineering approaches. A second volume entitled *Turbulence Models and Their Application* extends this subject to encompass separated flows within the framework of interactive boundary layer theory.

This chapter provides some of the terminology used in subsequent chapters, provides examples of turbulent flows and their complexity, and introduces some important turbulent-flow characteristics.

1.2 Turbulence – Miscellaneous Remarks

We start this chapter by addressing the question "What is turbulence?" In the 25^{th} Wilbur Wright Memorial Lecture entitled "Turbulence," von Kármán [7] defined turbulence by quoting G. I. Taylor as follows:

> Turbulence is an irregular motion which in general makes its appearance in fluids, gaseous or liquid, when they flow past solid surfaces or even when neighboring streams of the same fluid flow past or over one another.

That definition is acceptable but is not completely satisfactory. Many irregular flows cannot be considered turbulent. To be turbulent, they must have certain stationary statistical properties analogous to those of fluids when considered on the molecular scale. Hinze [8] recognizes the deficiency in von Kármán's definition and proposes the following:

> Turbulent fluid motion is an irregular condition of flow in which the various quantities show a random variation with time and space coordinates, so that statistically distinct average values can be discerned.

In addition turbulence has a wide range of wave lengths. The three statements taken together define the subject adequately.

What were probably the first observations of turbulent flow in a scientific sense were described by Hagen [9]. He was studying flow of water through round tubes and observed two distinct kinds of flow, which are now known as laminar (or Hagen-Poiseuille) and turbulent. If the flow was laminar as it left the tube, it looked clear like glass; if turbulent, it appeared opaque and frosty. The two kinds of flow can be generated readily by many household faucets. Fifteen years later, in 1854, he published a second paper showing that viscosity as well as velocity influenced the boundary between the two flow regimes. In his work he observed the mean* velocity $\bar{u}$ in the tube to be a function of both head and water temperature. (Of course, temperature uniquely determines viscosity.) His results are shown in Fig. 1.1 for several tube diameters. The plot contains implicit variations of $\bar{u}$, r_0, and ν, the velocity, the tube radius, and the kinematic viscosity, respectively. This form of presentation displays no orderliness in the data. About thirty years later, Reynolds [11] introduced the parameter $R_r \equiv \bar{u}r_0/\nu$ an example of what is now known as the Reynolds number (with velocity and length scales depending on the problem). It collapsed Hagen's data into nearly a single curve. The new parameter together with the dimensionless friction factor λ, defined such that the pressure drop $\Delta p = \lambda(\varrho\bar{u}^2/2)(l/r_0)$, transforms the plot of Fig. 1.1 to that of Fig. 1.2. The quantity l is tube length; the other quantities have the usual meaning. Thus was born the parameter, Reynolds number. The

* For now, let "mean" denote an average with respect to time, over a time long compared with the lowest frequencies of the turbulent fluctuations. In Section 2.3 we will give more details of this and other kinds of averaging.