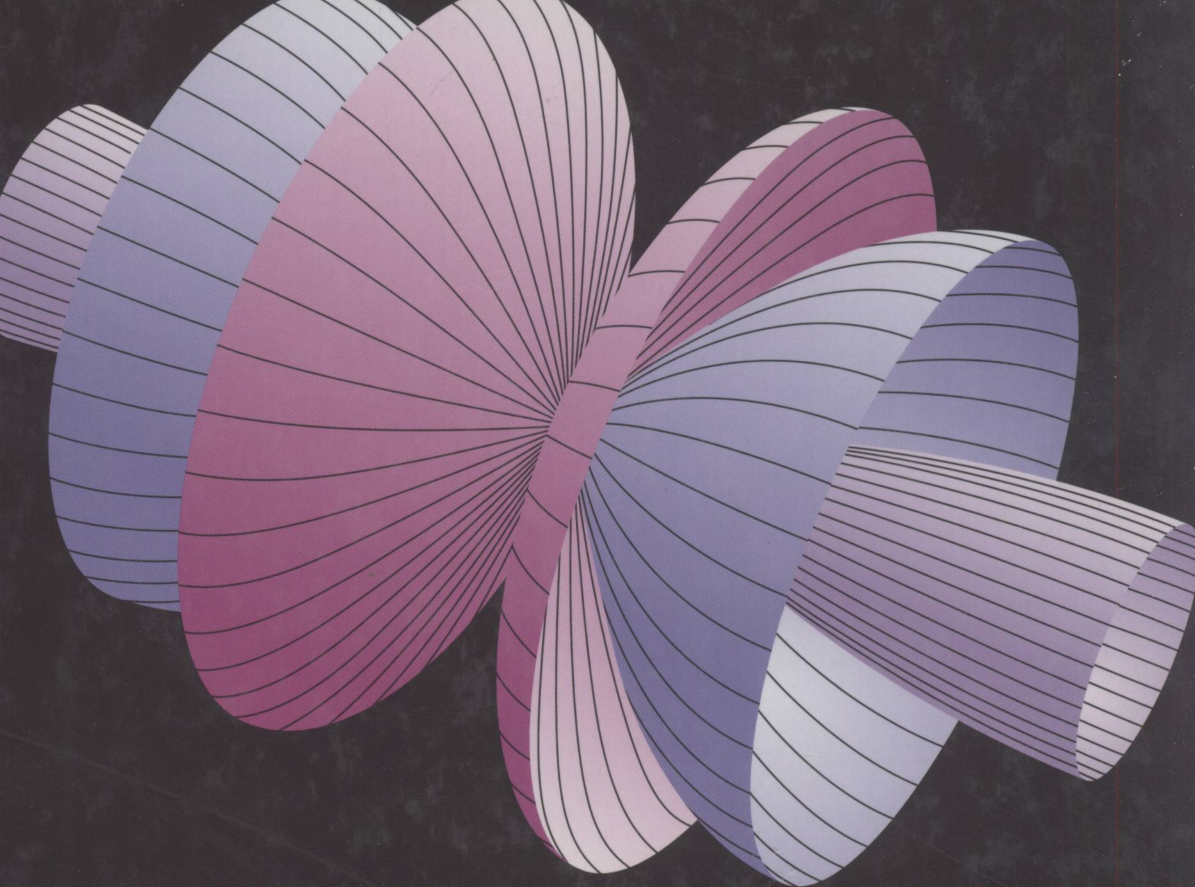

A First Course in String Theory

Barton Zwiebach

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A First Course in String Theory

An accessible introduction to string theory, this book provides a detailed and self-contained demonstration of the main concepts involved. The first part begins with a discussion of special relativity, electromagnetism, and the physics of extra dimensions. D-branes and the classical dynamics of relativistic strings are examined next, followed by the quantization of open and closed bosonic strings in the light-cone gauge, and a brief introduction to superstrings. The second part begins with a detailed study of D-branes followed by string thermodynamics. It discusses possible physical applications: the construction of the Standard Model on D-branes, the calculation of the entropy of black holes, and the string-theory/gauge-theory correspondence. Additionally, it covers T-duality of open and closed strings, electromagnetic fields on D-branes, Born–Infeld electrodynamics, covariant string quantization, and string interactions. Primarily aimed as a textbook for advanced undergraduate and beginning graduate courses, it will also be ideal for a wide range of scientists and mathematicians who are curious about string theory.

Barton Zwiebach is presently Professor of Physics at the Massachusetts Institute of Technology. He was born in Peru and completed his undergraduate studies at the Universidad Nacional de Ingeniería in Lima. His graduate work in physics was done at the California Institute of Technology, obtaining his doctorate in 1983, working under the supervision of Professor Murray Gell-Mann (Nobel Prize Physics, 1969). He held postdoctoral positions at the University of California at Berkeley and at MIT, where he became an Assistant Professor of Physics in 1987, and a permanent member of the faculty in 1994. Professor Zwiebach's speciality is string theory and theoretical particle physics. His central contributions have been in the area of string field theory, where he did the early work on the construction of the field theory of open strings and then developed the field theory of closed strings. He has also made important contributions to the subjects of D-branes with exceptional symmetry and tachyon condensation. This book originated from lectures given by Professor Zwiebach for his new course at MIT, called String Theory for Undergraduates. He has now received two awards for this course: the Everett Moore Baker Award for Excellence in Undergraduate Teaching and the School of Science Award for Undergraduate Education.

To my parents, Oscar and Betty Zwiebach, with gratitude

Foreword

String theory is one of the most exciting fields in theoretical physics. This ambitious and speculative theory offers the potential of unifying gravity and all the other forces of nature and all forms of matter into one unified conceptual structure.

String theory has the unfortunate reputation of being impossibly difficult to understand. To some extent this is because, even to its practitioners, the theory is so new and so ill understood. However, the basic concepts of string theory are quite simple and should be accessible to students of physics with only advanced undergraduate training.

I have often been asked by students and by fellow physicists to recommend an introduction to the basics of string theory. Until now all I could do was point them either to popular science accounts or to advanced textbooks. But now I can recommend to them Barton Zwiebach's excellent book.

Zwiebach is an accomplished string theorist, who has made many important contributions to the theory, especially to the development of string field theory. In this book he presents a remarkably comprehensive description of string theory that starts at the beginning, assumes only minimal knowledge of advanced physics, and proceeds to the current frontiers of physics. Already tested in the form of a very successful undergraduate course at MIT, Zwiebach's exposition proves that string theory can be understood and appreciated by a wide audience.

I strongly recommend this book to anyone who wants to learn the basics of string theory.

David Gross
Director, Kavli Institute For Theoretical Physics
University of California, Santa Barbara

Preface

The idea of having a serious string theory course for undergraduates was first suggested to me by a group of MIT sophomores sometime in May of 2001. I was teaching Statistical Physics, and I had spent an hour-long recitation explaining how a relativistic string at high energies appears to approach a constant temperature (the Hagedorn temperature). I was intrigued by the idea of a basic string theory course, but it was not immediately clear to me that a useful one could be devised at this level.

A few months later, I had a conversation with Marc Kastner, the Physics Department Head. In passing, I told him about the sophomores' request for a string theory course. Kastner's instantaneous and enthusiastic reaction made me consider seriously the idea for the first time. At the end of 2001, a new course was added to the undergraduate physics curriculum at MIT. In the spring term of 2002 I taught *String Theory for Undergraduates* for the first time. This book grew out of the lecture notes for that course.

When we think about teaching string theory at the undergraduate level the main question is, "Can the material really be explained at this level?" After teaching the subject two times, I am convinced that the answer to the question is a definite yes. Although a complete mastery of string theory requires a graduate-level physics education, the basics of string theory can be well understood with the limited tools acquired in the first two or three years of an undergraduate education.

What is the value of learning string theory, for an undergraduate? By exposing the students to cutting-edge ideas, a course in string theory can help nurture the excitement and enthusiasm that led them to choose physics as a major. Moreover, students will find in string theory an opportunity to sharpen and refine their understanding of most of the undergraduate physics curriculum. This is valuable even for students who do not plan to specialize in theoretical physics.

This book was tailored to be understandable to an advanced undergraduate. Therefore, I believe it will be a readable introduction to string theory for any graduate student or, in fact, for any physicist who wants to learn the basics of string theory.

About this book

A First Course in String Theory should be accessible to anyone who has been exposed to special relativity, basic quantum mechanics, electromagnetism, and introductory statistical physics. Some familiarity with Lagrangian mechanics is useful but not indispensable.

Except for the introduction, all chapters contain exercises and problems. The exercises, called *Quick calculations*, are inserted at various points throughout the text. They are control calculations that are expected to be straightforward. Undue difficulty in carrying them out may indicate problems understanding the material. The problems at the end of the chapters are more challenging and sometimes develop new ideas. A problem marked with a dagger† is one whose results are cited later in the text. A mastery of the material requires solving all the exercises and many of the problems. All the problems should be read, at least.

Throughout the book the material is developed in a self-contained way, and very little must be taken on faith. Sections marked with a star* have a somewhat different character. Their aim is to address subjects of current research interest for which a full explanation cannot be provided at the level of this book. The reader will be asked to accept some reasonable facts at face value, but otherwise the material should be *fully* understandable. Starred sections are not addressed to experts, and they typically contain exercises.

This book has two parts. Part I is called "Basics," and Part II is called "Developments." Part I begins with Chapter 1 and concludes with Chapter 13. Part II comprises the rest of the book: it begins with Chapter 14 and it ends with Chapter 23.

Chapter 1 serves as an introduction. Chapter 2 reviews special relativity, but it also introduces concepts that are likely to be new: light-cone coordinates, light-cone energy, compact extra dimensions, and orbifolds. In Chapter 3 we review electrodynamics and its manifestly relativistic formulation. We make some comments on general relativity and study the effect of compact dimensions on the Planck length. We are able at this point to examine the exciting possibility that large extra dimensions may exist. Chapter 4 uses nonrelativistic strings to develop some intuition, to review the Lagrangian formulation of mechanics, and to introduce terminology. Chapter 5 uses the relativistic point particle to prepare the ground for the study of the relativistic string. The power and elegance of the Lagrangian formulation become evident at this point. The first encounter with string theory happens in Chapter 6, which deals with the classical dynamics of the relativistic string. This is a very important chapter, and it must be understood thoroughly. Chapter 7 solidifies the understanding of string dynamics through the detailed study of string motion. Chapters 1 through 7 could comprise a mini-course in string theory.

Chapters 8 through 11 prepare the ground for the quantization of relativistic strings. In Chapter 8, one learns how to calculate conserved quantities, such as the momentum and the angular momentum of free strings. Chapter 9 gives the light-cone gauge solution of the string equations of motion and introduces the terminology that is used in the quantum theory. Chapter 10 explains the basics of quantum fields and particle states, with emphasis on the counting of the parameters that characterize scalar field states, photon states, and graviton states. In Chapter 11 we perform the light-cone gauge quantization of the relativistic particle. It all comes together in Chapter 12, another important chapter that should be understood thoroughly. This chapter presents the light-cone gauge quantization of the open relativistic string. The critical dimension is obtained and photon states are shown to emerge. Chapter 12 contains a starred section on the subject of tachyon condensation. Chapter 13 is the last chapter of Part I. Closed strings are quantized and graviton states emerge. This chapter contains a starred section that gives an introduction to superstrings.

The first part of this book can be characterized as an uphill road that leads to the quantization of the string at the summit. In the second part of this book the climb is over. The pace slows down a little, and the material elaborates upon previously introduced ideas. In Part II one reaps many rewards for the effort exerted in Part I.

The first three chapters of Part II discuss material with physical applications. Chapter 14 deals with the quantization of open strings on various D-brane configurations. The discussion of orientifolds has been relegated to the problems at the end of the chapter. Chapter 15 introduces the concept of string charge and demonstrates that the endpoints of open strings carry Maxwell charge. This chapter also discusses in detail the particle content of the Standard Model and one approach, based on intersecting D6-branes, to the construction of a realistic string model. Chapter 16 begins with string thermodynamics, followed by the subject of black hole entropy. It presents string theory attempts to derive the entropy of Schwarzschild black holes and the successful derivation of the entropy for a supersymmetric black hole. The chapter concludes with an introductory discussion of the AdS/CFT correspondence. Several sections here are starred.

The next four chapters are organized around the fascinating subject of T-duality. Chapters 17 and 18 present the T-duality properties of closed and open strings, respectively. Chapter 19 studies D-branes with electromagnetic fields, using T-duality as the main tool. Chapter 20 introduces the general framework of nonlinear electrodynamics. It demonstrates that electromagnetic fields in string theory are governed by Born–Infeld theory, a nonlinear theory in which the self-energy of point charges is finite.

Chapter 21 gives an introduction to the Lorentz covariant quantization of strings. It also introduces the Polyakov string action. The last two chapters in the book, Chapters 22 and 23, examine string interactions. We learn that the string diagrams which represent the processes of string interactions are Riemann surfaces. These two chapters assume a little familiarity with complex variables and have a mathematical flavor. One important goal here is to provide insight into the absence of ultraviolet divergences in string theory, the fact that made string theory the first candidate for a theory of quantum gravity.

In this book I have tried to emphasize the connections with ideas that students have learned before. The quantization of strings is described as the quantization of an infinite number of oscillators. String charge is visualized as a Maxwell current. The effects of Wilson lines on circles are compared with the Bohm–Aharonov effect. The modulus of an annulus is related to the capacitance of a cylindrical conductor, and so forth and so on. The treatment of topics is generally explicit and detailed, with formalism kept to a minimum.

This book as a textbook

In this book, the choice was made to use light-cone gauge to quantize the strings. This approach to quantization can be understood in full detail by students with some prior exposure to quantum mechanics. The same is *not* true for the Lorentz covariant quantization of strings, where states of negative norms must be dealt with, the Hamiltonian vanishes, and there is no conventional looking Schrödinger equation. The light-cone approach suffices for most physical problems and, in fact, simplifies the treatment of several questions.

Part I of the book is structured tightly. Little can be omitted without hampering the understanding of string quantization. In Part II, the ordering of chapters was guided by one goal: to get to the physical applications as soon as possible. Here variations on the choice of topics are possible and different readers/instructors may take different routes.

My experience suggests that the material can be taught with the following schedules. In a school with an academic year composed of three quarters, Part I can be covered in one quarter, and the whole book can be covered in two quarters. In a school with an academic year composed of two semesters, Part I and three or four chapters from Part II may be covered in one semester. Which chapters to select from Part II is a matter of taste. At MIT we covered Chapters 14–17. Given the relevance of D-branes, Chapter 14 is recommended strongly. Chapters 15 and 16 give an appreciation for current research in string theory, and Chapter 17 deals with T-duality of closed strings. Lecturers who prefer to focus on T-duality and its implications will cover the first three sections of Chapter 15, and then as much as possible from Chapters 17–20. If this book is used to teach exclusively to graduate students, the pace can be quickened and one may be able to cover most of its contents in one semester.

An updated list of corrections can be found at http://xserver.lns.mit.edu/~zwiebach/firstcourse.html. This web page also contains supplementary material that may be of interest to readers and instructors. Solutions to the problems in the book are available to lecturers via solutions@cambridge. org.

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Teaching string theory to a class composed largely of bright undergraduates was both a stimulating and a rewarding experience. I am grateful to the group of students that composed the first class:

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Megha Padi
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Timothy Richards
James Smith
Morgan Sonderegger
David Starr

They were enthusiastic, funny, and lively. My lectures were voice-recorded and three of the students, Gabrielle Magro, Megha Padi, and David Starr, turned the tapes and the blackboard equations into LaTeX files. I am grateful to the three of them for their dedication and for the care they took in creating accurate files. They provided the impetus to start the process of writing a book. I edited the files to produce lecture notes.

Additional files for a set of summer lectures were created by Gabrielle and Megha. In the next six months the lecture notes became the draft for a book. After teaching the course for a second time in the spring term of 2003 and a long summer of edits and revisions, the book was completed in October 2003.

By the time the lecture notes had become a book draft, David Starr offered to read it critically. He basically marked every paragraph, suggesting improvements in the exposition and demonstrating an uncanny ability to spot weak points. His criticism forced me to go through major rewriting. His input was tremendous. Whatever degree of clarity has been achieved, it is in no small measure thanks to his effort.

I am delighted to acknowledge help and advice from my friend and colleague Jeffrey Goldstone. He shared generously his understanding of string theory, and several sections in this book literally grew out of his comments. He helped me teach the course the second time that it was offered. While doing so, he offered perceptive criticism of the whole text. He also helped improve many of the problems, for which he wrote elegant solutions.

The input of my friend and collaborator Ashoke Sen was critical. He believed that string theory could be taught at a basic level and encouraged me to try to do it. I consulted repeatedly with him about the topics to be covered and about the strategies to present them. He kindly read the first full set of lecture notes and gave invaluable advice that helped shape the form of this book.

The help and interest of many people made writing this book a very pleasant task. For detailed comments on all of its content I am indebted to Chien-Hao Liu and to James Stasheff. Alan Dunn and Blake Stacey helped test the problems that could not be assigned in class. Jan Troost was a sounding board and provided advice and criticism. I've relied on the knowledge of my string theory colleagues – Amihay Hanany, Daniel Freedman, and Washington Taylor. I'd like to thank Philip Argyres, Andreas Karch, and Frieder Lenz for testing the lecture notes with their students. Juan Maldacena and Samir Mathur provided helpful input on the subject of string thermodynamics and black holes. Boris Körs, Fernando Quevedo, and Angel Uranga helped and advised on the subject of string phenomenology. Thanks are also due to Tamsin van Essen, editor at Cambridge, for her advice and her careful work during the entire publishing process.

Finally, I would like to thank my wife Gaby and my children Cecile, Evy, Margaret, and Aaron. At every step of the way I was showered with their love and support. Cecile and Evy read parts of the manuscript and advised on language. Questions on string theory from Gaby and Margaret tested my ability to explain. Young Aaron insisted that a ghost sitting on a string would make a perfect cover page, but we settled for strings moving in electric fields.

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