Calculus

Fong Yuen • Wang Yuan

Springer

微積分

杜忠誥書

方源　王元

Fong Yuen
Department of Mathematics
National Cheng Kung University
Tainan
Taiwan 701

Wang Yuan
Institute of Mathematics
Academia Sinica
Beijing 100080
China

Library of Congress Cataloging-in-Publication Data

Fong, Yuen.
Calculus / Yuen Fong, Yuan Wang.
p. cm.
Includes index.
ISBN 9813083018 (softcover) (Singapore)
9813083522 (hardcover)
1. Calculus I. Wang, Yuan, 1930- . II. Title
QA 303.F586 1999
515--dc20 96-29408
CIP

ISBN 981-3083-01-8 (softcover)
ISBN 981-3083-52-2 (hardcover)

Typesetting: Asco Typesetters, Hong Kong
Printed in Singapore

SPIN 10546278 (softcover) 5 4 3 2 1 0
SPIN 10748684 (hardcover) 5 4 3 2 1 0

Springer
Singapore
Berlin
Heidelberg
New York
Barcelona
Hong Kong
London
Milan
Paris
Tokyo

Calculus

Fong Yuen • Wang Yuan

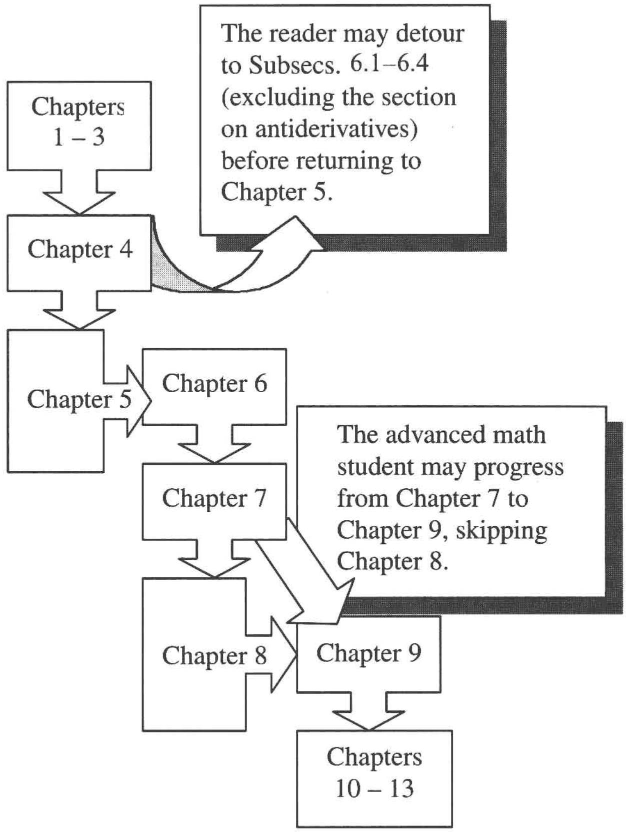
Springer

Preface

The text presented here is designed for a two-semester course on elementary calculus. It has been used in the past 20 years by students in various scientific disciplines throughout Taiwan and mainland China. The main aim of writing this book is to make it accessible to all students—those familiar with the subject or not—but the arguments herein are rigorous. However, attempts have been made to present the main philosophy and thinking behind calculus through various innovative examples and applications so that the abstract notions and concepts of calculus are within reasonable understanding.

Reading and understanding mathematics is an art that is completely different from reading any other literature. Readers need practice reading mathematics. Therefore, a solutions manual containing detailed solutions to all exercises in this book will be provided separately. This hopefully will assist students engaged in self-study or who are not under formal instruction.

Recommended Usage of "Calculus"



Acknowledgements

At this point, the first author would like to credit those having an indelible and positive influence on his mathematical dvelopment and career. Among his best and most influential teachers and friends include Professors I.H. Lin, National Cheng Kung University; J.D.P. Meldrun, University of Edinburgh; A. Oswald, University of Teesside; G. Pilz, Johannes Kepler Universität Linz; the late James R. Clay, University of Arizona; Li Fu-an, Chinese Academy of Sciences; R. Wiegandt, Hungarian Academy of Sciences; K. Kaarli, University of Tartu; K.I. Beidar, Wen-Fong Ke, C.K. Lin, Lam Ngau, and Cheng-I Weng, National Cheng Kung University; A.V. Mikhalev, Moscow State University; Yeong-Nan Yeh, Academia Sinica; and Y.H. Xu, Fudan University. Special mention should go to James R. Clay ("Big Jim"), who the first author the laws of thought and philosophy which are of utmost importance for a learned mathematician. Section 5.6 is dedicated to him for their lifelong academic collaboration.

Further, this book would not come into fruition without the help of many others. Here, acknowledgement should go to those who have encouraged and supported the first author financially and spiritually through the years. These include Fong Ka Wai, Emily Fong Cheng Sau Kwan, Ching-Kwong Chow, Dunnet S.W. Cheung, Lau Fung, William Taui W.Y., Abel Tsui Chun Kong, Ng Kwok Ying, Raleigh Siu Lau, Samuel Lee Wai Hung, Wong Yiu Cheong, Chak Chung Man, and Tina Wong. However, special mention must be made to the first author's brother, Fong Ka wai, who has been supportive of the former's academic career for the past 20 years. Both him and Ching-Kwong Chow have generously purchased and donated copies of this book to university libraries in mainland China. This is in response to the event, the **Hope of Education** for the 21st century, in China.

On the preparation of this book, sincere thanks go to Lee Jen Yea who devoted countless hours typing the manuscript. Thanks are also due to

Professors Long-Shung Shiao, Shian Leou, Y.S.Tsai, H.J. Shyr, J.S. Lin, P.H. Lee, Min-te Chao, and Liao Lung-Chi who have been supportive of this book. Also, the authors are most grateful to Professors Chao and Miao for the use of their figures, especially in the last two chapters.

Finally, gratitude must be given to Maurice Kwong, Managing Director, and Ian Shelley, former Director of Publishing, Springer-Verlag Singapore, for their assistance in the publication of this book.

Fong Yuen	Wang Yuan
Tainan	Beijing

March 1999

Contents

1

Introduction

1.1 What is Calculus?

To today's scientists, calculus is a kind of elementary mathematics (say, algebra, geometry, and trigonometry) enhanced by the **limiting process**, while to people of ancient Rome, calculus was a pebble that was used in gambling and counting. Actually, calculus is the study of the behavior of real functions of real variables by means of a basic notion called **limit**. This very idea of limit immediately leads to the concepts of differentiation and integration, which provide numerous applications in the various disciplines. For example, a physicist may use integral calculus to determine the work done by a variable force, a chemist may employ differential calculus to investigate the results of various chemical reactions, a biologist may use calculus to forecast the outcome of the rate of growth of bacteria in a culture, and finally, an economist may apply it to problems that involve corporate profits and losses.

Calculus obtains its ideas from elementary mathematics and extends them to more general situations. Table 1.1.1 gives a contrast between precalculus (elementary mathematics without calculus) and calculus.

Now we shall give a brief account of the history of calculus. Formally speaking, the actual invention of calculus is due to two eminent mathematicians, Sir Isaac Newton (1642–1727) of England and Gottfried Wilhelm von Leibniz (1646–1716) of Germany. However, one can trace back to the time of ancient Greece where the original notion of calculus was found by Archimedes in the area problem about 2500 years ago. By using the "method of exhaustion", the Greeks were able to determine the area of any polygon by dividing it into triangles as shown in Figure 1.1.1 and adding the areas of these triangles to form the required area A.

It is a more difficult task to determine the area of a curved region. The Greek's method of exhaustion was to inscribe polygons in a curved region and then let the number of sides of polygons increase.

Table 1.1.1

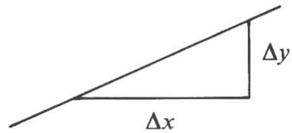
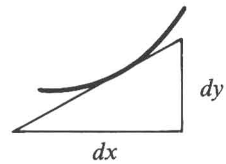
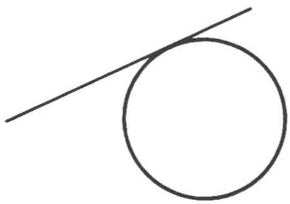
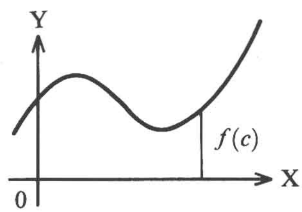
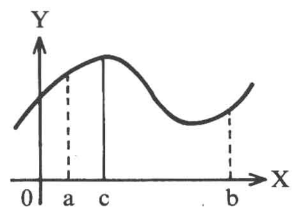
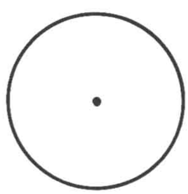
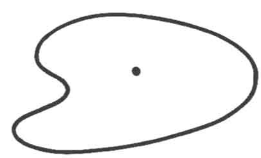
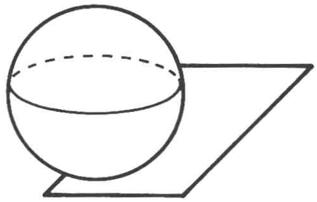
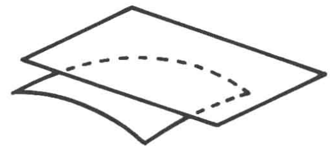
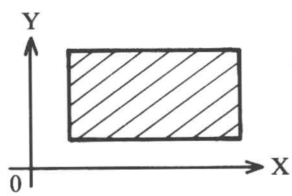
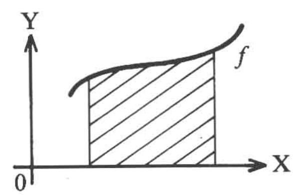
Precalculus	Calculus
 m: slope of a straight line $y = mx + c$ where $m = \Delta y/\Delta x$	 $f'(x)$: slope of a curve $y = f(x)$ where $f'(x) = dy/dx$
 tangent line to a circle	 tangent line to a general curve
 $f(c)$: height of a curve at c	 $f(c)$: maximum height of a curve at $c \in [a, b]$
 center of a circle	 centroid of a region

Table 1.1.1 (cont.)

Precalculus	Calculus
 tangent plane to a sphere	 tangent plane to a general surface
$\sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n$ sum of finite numbers	$\sum_{i=1}^{\infty} a_i = a_1 + a_2 + \cdots + a_n + \cdots$ sum of infinite numbers
 length of a line segment	 length of an arc
 area of a rectangle	 area under a curve

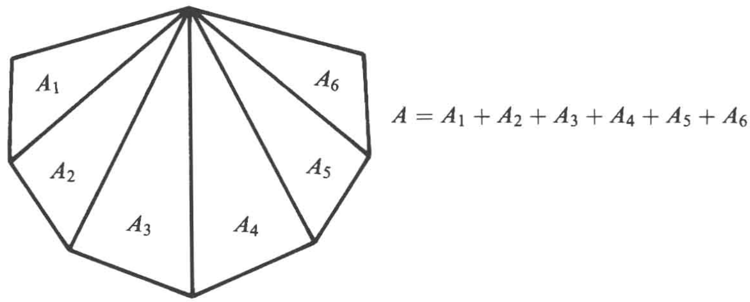


Figure 1.1.1

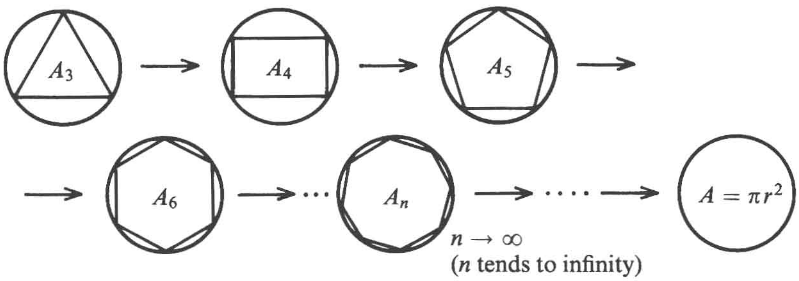


Figure 1.1.2

Figure 1.1.2 shows that the actual area of a circle is in fact derivable by taking a limiting process of inscribed regular polygons, i.e., let A_n be the area of the inscribed regular polygon with n sides, and as n increases, the area of A_n becomes closer to the area of a circle. Here we say that the area of the circle is equal to the **limit** of the areas of the inscribed regular polygons and denote it as

$$A = \lim_{n \to \infty} A_n.$$

One should notice that not only the ancient Greeks used the concept of limit in an ambiguous manner, even Newton and Leibnitz developed their ideas of the derivative and integral with only a vague notion of the limit concept. It was not until more than a century later that Augustin-Louis Cauchy (1789–1857), a famous French methematician, then gave a precise definition for the limit of a function that we all use today. Thus, the results of calculus can be proved in a rigorous manner and have been universally accepted since the 19th century.

1.2 Sets and Functions

Although an excursion into the theory of sets is not necessarily needed at this stage, an intuitive understanding of the basics of set theory will not only be helpful throughout the study of the present text but will also facilitate mathematical communications with your fellow classmates and professors. Here we would like to emphasize that the word *set* is the accepted technical term for such synonyms as *class*, *collection*, *aggregate*, and *assemblage*. In daily life, we use such phrases as a set of pictures, a set of books, a set of programmes, and so on. However, in this book, we are more interested in sets of collections of mathematical objects, such as a *set of real numbers*, a *set of positive integers*, a *set of points in the X-Y plane*, and so on. Thus, all these examples give us an intuitive feeling that a set is a collection of well-defined objects. So, we have the following definition.

Definition 1.2.1 ***Definition of a Set***

A **set** *is a collection of well-defined objects* (*points*, *elements*, or *members*).

We note that the above definition permits us to form the set of all the elements x which satisfy certain constrained conditions or properties, namely $P(x)$, except for the *set of all the sets* x *which satisfy* $P(x)$. This distinction suffices to eliminate the logical paradoxes. Once the semantic paradoxes have been avoided, the set S whose existence is asserted by Definition 1.2.1 will be designated by the symbol

$$S = \{x|P(x)\}.$$

The above expression is read as "S is the set that contains the element x such that x satisfies the property $P(x)$".

We generally designate sets by capital letters X, Y, Z, etc., and elements of a set by small letters x, y, z, etc. If a set S contains the points x, y, z, w, we write

$$S = \{x, y, z, w\}.$$

The notation "$x \in S$" means "x is an element of S" or "x belongs to S"; the notation "$x \notin S$" means "x is not an element of S" or "x does not belong to S".

Example 1.2.2 ***Examples of Sets***

(i) $\mathbb{N} = \{1, 2, 3, \ldots, n, \ldots\}$ is the *set of all natural numbers* (or *positive integers*).