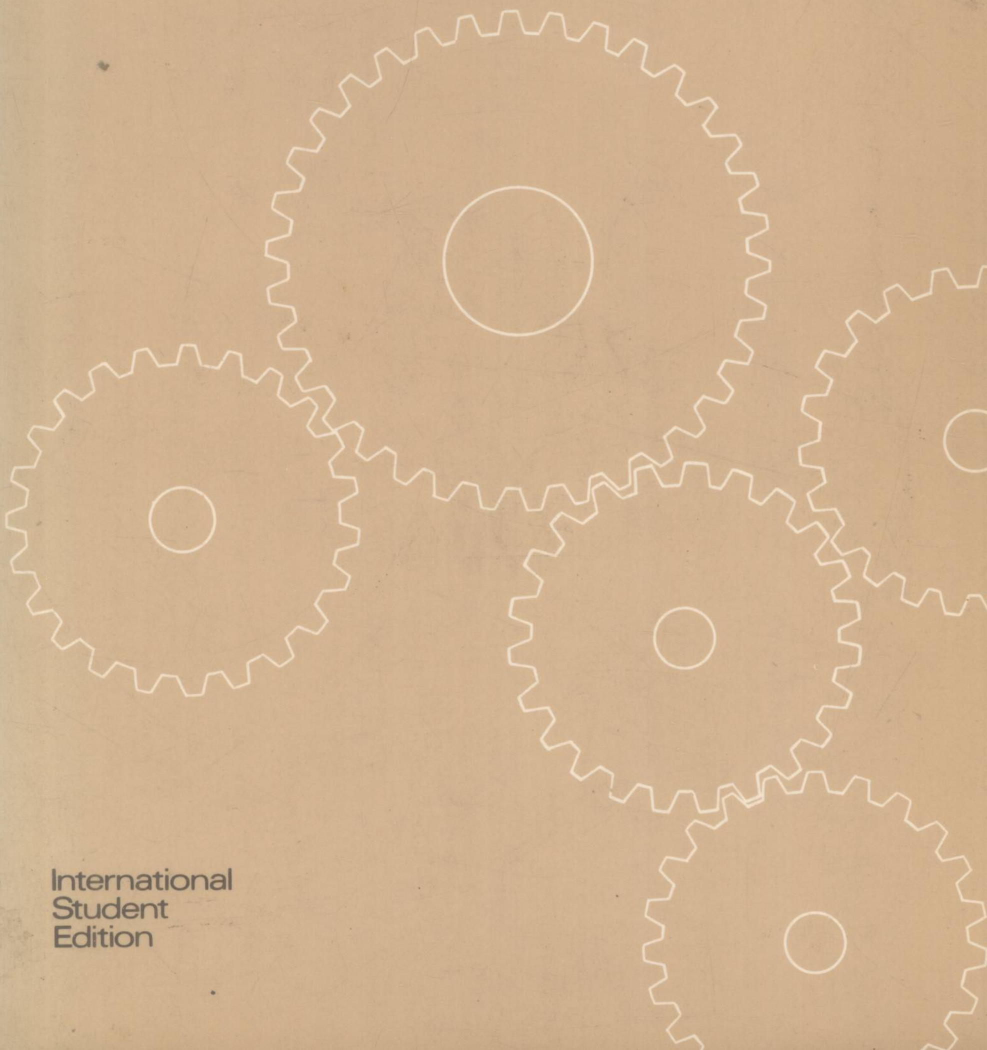
# FLUID MECHANICS

## FRANK M. WHITE



International
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# FLUID MECHANICS

**Frank M. White**
Professor of Mechanical Engineering
University of Rhode Island

INTERNATIONAL STUDENT EDITION

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# FLUID MECHANICS

To Jeanne

# PREFACE

This book is a textbook for a one- or two-term course in undergraduate fluid mechanics. The student should have a knowledge of calculus and particle dynamics. Thermodynamics is very helpful but not strictly necessary except for selected topics. Strength of materials is an excellent introduction to the study of fluid mechanics, but again not strictly necessary.

The coverage is broad and diverse and contains material of interest to mechanical, civil, and aerospace engineers. There are also some topics from the new field of ocean engineering.

The first four chapters cover the basic topics of fluid properties, hydrostatics, control volumes, and differential relations. The remaining seven chapters treat specialized topics: similarity, duct flow, boundary-layer flow, potential flow, compressible flow, open-channel flow, and turbomachinery. Instructors are free to select material appropriate to their students. Whole chapters can be deleted, and each chapter contains advanced material which can be omitted if desired. Civil engineers may omit Chapters 8 and 9 and concentrate on Chapters 10 and 11. Aerospace engineers would be especially interested in Chapters 7 and 9. Mechanical engineers would be strongly oriented toward Chapters 5, 6, and 11. There is plenty of material for a full year. At our university we would cover Chapters 1–6 in the first semester and Chapters 7–11 in the second. The *Instructor's Manual* gives further suggestions on course organization.

The author's strength is in teaching, so every attempt has been to make this book teachable, readable, and interesting—fun, even. There are 140 fully worked examples and 1089 problem exercises. One could repeat the course many times without repeating a problem exercise. Much of the material is traditional, of course, but there are several unorthodox touches, such as the use of three Moody charts in Chapter 6 rather than the usual one. Every attempt has been to use data and theory inventively. There are 218 references for further study, but the book is really self-contained.

The text material uses the SI and British Gravitational unit systems, concurrently, and the examples and problem exercises are equally divided between the two systems. It is a shame that United States schools are moving so slowly toward the SI system, leaving many of us standing in the middle.

This text is broad enough and clear enough that the instructor may depart from the text during the lecture, and thus enrich the student. I have always found it useful to present an alternate approach, if possible, during lectures, so that students may be exposed to the great diversity of engineering techniques which accomplish the same aim of proper design and problem solution.

I am greatly indebted to Professor Ken Schneider of California State Polytechnic University, Pomona, for reading and thoroughly criticizing the entire manuscript. Also, Professors George Brown, Warren Hagist, Dick Lessmann, and Rodger Dowdell of my own university made many helpful suggestions. Any flaws which remain are my own fault, and I would appreciate hearing about them, along with your suggestions for improvement.

Finally, during this three-year writing effort, the support and encouragement of my wife and four daughters have made all the difference.

**Frank M. White**

# CONTENTS



* These sections may be omitted without loss of continuity.

# CHAPTER 1 INTRODUCTION

## 1.1 PRELIMINARY REMARKS

Fluid mechanics is the study of fluids in motion or at rest and the subsequent effects of the fluid on the boundaries, which may be either solid surfaces or other fluids. The essence of the subject of fluid flow is that of a judicious compromise between theory and experiment. Since fluid flow is a branch of mechanics, it satisfies a set of well-documented basic conservation laws and thus a great deal of theoretical treatment is available. The theory is often frustrating, however, because it applies mainly to certain idealized situations which may be invalid in practical problems. The two chief obstacles to a workable theory are geometry and viscosity. The general theory of fluid motion (Chap. 4) is too difficult to enable the user to attack arbitrary geometric configurations, so that most textbooks concentrate on flat plates, circular pipes and other easy geometries. It is possible to apply numerical techniques to arbitrary geometries, and specialized textbooks are now appearing which explain these digital-computer approximations [1].[1] This book will present many theoretical results, keeping their limitations in mind.

The second obstacle to theory is the action of viscosity, which can be neglected only in certain idealized flows (Chap. 8). First, viscosity increases the difficulty of the basic equations, although the boundary-layer approximation found by Ludwig Prandtl in 1904 (Chap. 7) has greatly simplified viscous-flow analyses. Second, viscosity has a destabilizing effect on all fluids, giving rise, at frustratingly small velocities, to a disorderly, random phenomenon called *turbulence*. The theory of turbulent flow is crude and heavily backed up by experiment (Chap. 6), yet it can be quite serviceable as an engineering estimate. Textbooks are beginning to present digital-computer techniques for turbulent-flow analysis [2], but they are based strictly upon empirical assumptions regarding the time mean of the turbulent stress field.

Thus there is theory available for fluid-flow problems, but in all cases it should be backed up by experiment. Often the experimental data provide the main source of information about specific flows, such as the drag and lift of immersed bodies (Chap. 7). Fortunately, fluid mechanics is a highly visual subject, with good in-

[1] Numbered references appear at the end of each chapter.

strumentation [3, 4], and the use of dimensional analysis and modeling concepts (Chap. 5) is widespread. Thus experimentation provides a natural and easy complement to the theory. Appendix C lists a variety of interesting films which have been prepared to visualize fluid-flow phenomena. You should keep in mind that theory and experiment should go hand in hand in all studies of fluid mechanics.

## 1.2 THE CONCEPT OF A FLUID

From the point of view of fluid mechanics, all matter consists of only two states, fluid and solid. The difference between the two is perfectly obvious to the layman, and it is an interesting exercise to ask a lay person to put this difference into words. The technical distinction lies with the reaction of the two to an applied shear or tangential stress. A solid can resist a shear stress by a static deformation; a fluid cannot. Any shear stress applied to a fluid, no matter how small, will result in motion of that fluid. The fluid moves and deforms continuously as long as the shear stress is applied. As a corollary, we can say that a fluid at rest must be in a state of zero shear stress, a state often called the hydrostatic stress condition in structural analysis. In this condition, Mohr's circle for stress reduces to a point, and there is no shear stress on any plane cut through the element under stress.

Given the definition of a fluid above, every layman also knows that there are two classes of fluids, *liquids* and *gases*. Again the distinction is a technical one concerning the effect of cohesive forces. A liquid, being composed of relatively close-packed molecules with strong cohesive forces, tends to retain its volume and will form a free surface in a gravitational field if unconfined from above. Free-surface flows are dominated by gravitational effects and are studied in Chaps. 5 and 10. Since gas molecules are widely spaced with negligible cohesive forces, a gas is free to expand until it encounters confining walls. A gas has no definite volume and left to itself without confinement forms an atmosphere which is essentially hydrostatic. The hydrostatic behavior of liquids and gases is taken up in Chap. 2. Gases cannot form a free surface, and thus gas flows are rarely concerned with gravitational effects other than buoyancy.

Figure 1.1 illustrates a solid block resting on a rigid plane and stressed by its own weight. The solid sags into a static deflection, shown as a highly exaggerated dashed line, resisting shear without flow. A free-body diagram of element $A$ on the side of the block shows that there is shear in the block along a plane cut at an angle $\theta$ through $A$. Since the block sides are unsupported, element $A$ has zero stress on the left and right sides and compression stress $\sigma = -p$ on the top and bottom. Mohr's circle does not reduce to a point, and there is nonzero shear stress in the block.

By contrast, the liquid and gas at rest in Fig. 1.1 require the supporting walls in order to eliminate shear stress. The walls exert a compression stress $-p$ and reduce Mohr's circle to a point with zero shear everywhere, i.e., the hydrostatic condition. The liquid retains its volume and forms a free surface in the container. If the walls are removed, shear develops in the liquid and a big splash results. If the container is tilted, shear again develops, waves form, and the free surface seeks a horizontal configuration, pouring out over the lip if necessary. Meanwhile, the gas

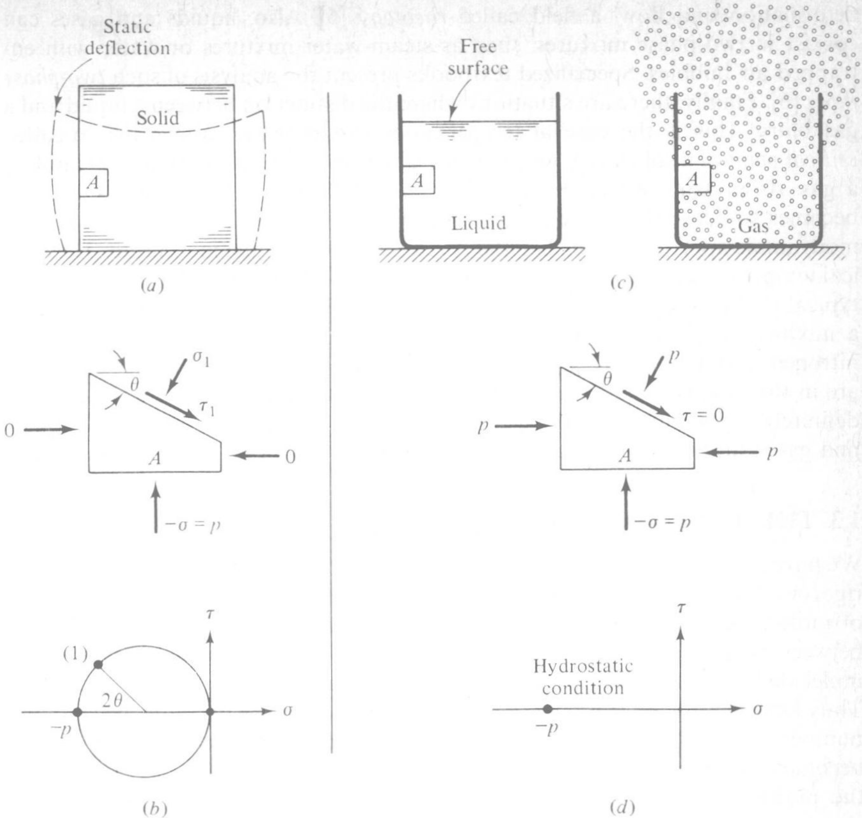


**Fig. 1.1** A solid at rest can resist shear. (*a*) Static deflection of the solid; (*b*) equilibrium and Mohr's circle for solid element *A*. A fluid cannot resist shear. (*c*) Containing walls are needed; (*d*) equilibrium and Mohr's circle for fluid element *A*.

is unrestrained and expands out of the container, filling all available space. Element *A* in the gas is also hydrostatic and exerts a compression stress $-p$ on the walls.

In the above discussion clear decisions could be made about solids, liquids, and gases. Most engineering fluid mechanics problems deal with these clear cases, i.e., the common liquids, such as water, oil, mercury, gasoline, and alcohol, and the common gases, such as air, helium, hydrogen, and steam, in their common temperature and pressure ranges. There are many borderline cases, however, of which you should be aware. Some apparently "solid" substances such as asphalt and lead resist shear stress for short periods but actually deform slowly and exhibit definite fluid behavior over long periods of time. Other substances, notably colloid and slurry mixtures, resist small shear stresses but "yield" at large stress and begin to flow like fluids. Specialized textbooks are devoted to this study of more general deformation and flow, a field called *rheology* [5]. Also, liquids and gases can coexist in two-phase mixtures, such as steam-water mixtures or water with entrapped air bubbles. Specialized textbooks present the analysis of such *two-phase flows* [6]. Finally, there are situations where the distinction between a liquid and a gas blurs. This is the case at temperatures and pressures above the so-called *critical point* of a substance, where only a single phase exists, primarily resembling a gas. As pressure increases far above the critical point, the gaslike substance becomes so dense that there is some resemblance to a liquid and the usual thermodynamic approximations like the perfect-gas law become inaccurate. The critical temperature and pressure of water are $T_c = 647$ K and $p_c = 219$ atm,[1] so that typical problems involving water and steam are below the critical point. Air, being a mixture of gases, has no distinct critical point, but its principal component, nitrogen, has $T_c = 126$ K and $p_c = 34$ atm. Thus typical problems involving air are in the range of high temperature and low pressure where air is distinctly and definitely a gas. This text will be concerned solely with clearly identifiable liquids and gases, and the borderline cases discussed above will be beyond our scope.

## 1.3 THE FLUID AS A CONTINUUM

We have already used technical terms such as fluid pressure and density without a rigorous discussion of their definition. As far as we know, fluids are aggregations of molecules, widely spaced for a gas, closely spaced for a liquid. The distance between molecules is very large compared with the molecular diameter. The molecules are not fixed in a lattice but move about freely relative to each other. Thus fluid density, or mass per unit volume, has no precise meaning, because the number of molecules occupying a given volume continually changes. This effect becomes unimportant if the unit volume is large compared with, say, the cube of the molecular spacing, when the number of molecules within the volume will remain nearly constant in spite of the enormous interchange of particles across the boundaries. If, however, the chosen unit volume is too large, there could be a noticeable variation in the bulk aggregation of the particles. This situation is illustrated in Fig. 1.2, where the "density" as calculated from molecular mass $\delta m$ within a given volume $\delta \mathcal{V}$ is plotted versus the size of the unit volume. There is a limiting volume $\delta \mathcal{V}^*$ below which molecular variations may be important and above which aggregate variations may be important. The density $\rho$ of a fluid is best defined as

$$\rho = \lim_{\delta \mathcal{V} \to \delta \mathcal{V}^*} \frac{\delta m}{\delta \mathcal{V}} \tag{1.1}$$

The limiting volume $\delta \mathcal{V}^*$ is about $10^{-9}$ mm$^3$ for all liquids and for gases at atmospheric pressure. For example, $10^{-9}$ mm$^3$ of air at standard conditions contains approximately $3 \times 10^7$ molecules, which is sufficient to define a nearly constant density according to Eq. (1.1). Most engineering problems are concerned with physical dimensions much larger than this limiting volume, so that density is

[1] One atmosphere equals 2116 lbf/ft$^2$ = 101,300 Pa.

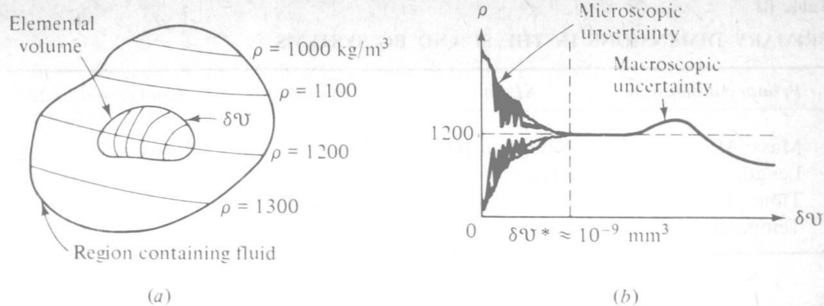


**Fig. 1.2** The limit definition of continuum fluid density. (*a*) An elemental volume in a fluid region of variable continuum density; (*b*) calculated density versus size of the elemental volume.

essentially a point function and fluid properties can be thought of as varying continually in space, as sketched in the left portion of Fig. 1.2. Such a fluid is called a *continuum*, which simply means that its variation in properties is so smooth that the differential calculus can be used to analyze the substance. We shall assume that continuum calculus is valid for all the analyses in this book. Again there are borderline cases for gases at such low pressures that molecular spacing and mean free path[1] are comparable to, or larger than, the physical size of the system. This requires that the continuum approximation be dropped in favor of a molecular theory of rarefied-gas flow [7]. In principle, all fluid-mechanics problems can be attacked from the molecular viewpoint, but no such attempt will be made here. Note that the use of continuum calculus does not preclude the possibility of discontinuous jumps in fluid properties across a free surface or fluid interface or across a shock wave in a compressible fluid (Chap. 9). Our calculus in Chap. 4 must be flexible enough to handle discontinuous boundary conditions.

## 1.4 DIMENSIONS AND UNITS

A *dimension* is the measure by which a physical variable is expressed quantitatively. A *unit* is a particular way of attaching a number to the quantitative dimension. Thus length is a dimension associated with such variables as distance, displacement, width, deflection, and height, while centimeters and inches are both numerical units for expressing length. Dimension is a powerful concept about which a splendid tool called *dimensional analysis* has been developed (Chap. 5), while units are the nitty-gritty, the number which the customer wants as the final answer.

Systems of units have always varied widely from country to country, even after international agreements have been reached. Engineers need numbers and therefore need unit systems, and the numbers must be accurate because the safety of the public is at stake. You cannot design and build a piping system whose diameter is $D$ and whose length is $L$. American engineers have persisted too long in clinging to

[1] The mean distance traveled by molecules between collisions.

**Table 1.1**

**PRIMARY DIMENSIONS IN THE SI AND BG SYSTEMS**

| *Primary dimension* | *SI unit* | *BG unit* | *Conversion factor* |
|---|---|---|---|
| Mass $\{M\}$ | Kilogram (kg) | Slug | 1 slug = 14.5939 kg |
| Length $\{L\}$ | Meter (m) | Foot (ft) | 1 ft = 0.3048 m |
| Time $\{T\}$ | Second (s) | Second (s) | 1 s = 1 s |
| Temperature $\{\Theta\}$ | Kelvin (K) | Rankine (°R) | 1 K = 1.8°R |

British systems of units. There is too much margin for error in most British systems, and many an engineering student has flunked a test because of a missing or improper conversion factor of 12 or 144 or 32.2 or 60 or 1.8. Practicing engineers can make the same errors. The writer is aware from personal experience of a serious preliminary error in the design of an aircraft due to a missing factor of 32.2 to convert pounds of mass into slugs.

In 1872 an international meeting in France proposed a treaty called the Metric Convention, which was signed in 1875 by 17 countries including the United States. It was an improvement over British systems because its use of base 10 is the foundation of our number system, learned from childhood by all. Problems still remained because even the metric countries differed in their use of kiloponds instead of dynes or newtons, kilograms instead of grams, or calories instead of joules. To standardize the metric system, a General Conference of Weights and Measures attended in 1960 by 40 countries proposed the *International System of Units* (SI). We are now undergoing a painful period of transition to the SI system, an adjustment which may take the remainder of this century to complete. The professional societies have led the way. Since July 1, 1974 SI units have been required by all papers published by the American Society of Mechanical Engineers, which prepared a useful booklet explaining the SI system [8]. The present text will use SI units together with British gravitational (BG) units.

In fluid mechanics there are only four *primary dimensions* from which all other dimensions can be derived. They are mass, length, time, and temperature.[1] These dimensions and their units in both systems are given in Table 1.1. Note that the kelvin unit uses no degree symbol. The braces around a symbol like $\{M\}$ mean "the dimension of" mass. All other variables in fluid mechanics can be expressed in terms of $\{M\}$, $\{L\}$, $\{T\}$, and $\{\Theta\}$. For example, acceleration has the dimensions $\{LT^{-2}\}$. The most crucial of these secondary dimensions is that of force, which is directly related to mass, length, and time by Newton's second law

$$\mathbf{F} = m\mathbf{a} \tag{1.2}$$

From this we see that, dimensionally, $\{F\} = \{MLT^{-2}\}$. A constant of proportionality is avoided by defining the force unit exactly in terms of the primary units. Thus

[1] If electromagnetic effects are important, a fifth primary dimension must be included, electric current $\{I\}$, the SI unit of which is the ampere (A).