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Gravitation: Foundations and Frontiers

引力

——基础与前沿

(影印版)

〔印度〕帕德马纳班

(T. Padmanabhan) 著



北京大学出版社
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举报电话:010-62752024 电子信箱:fd@pup.pku.edu.cn

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序 言

物理学是研究物质、能量以及它们之间相互作用的科学。她不仅是化学、生命、材料、信息、能源和环境等相关学科的基础,同时还是许多新兴学科和交叉学科的前沿。在科技发展日新月异和国际竞争日趋激烈的今天,物理学不仅囿于基础科学和技术应用研究的范畴,而且在社会发展与人类进步的历史进程中发挥着越来越关键的作用。

我们欣喜地看到,改革开放三十多年来,随着中国政治、经济、教育、文化等领域各项事业的持续稳定发展,我国物理学取得了跨越式的进步,做出了很多为世界瞩目的研究成果。今日的中国物理正在经历一个历史上少有的黄金时代。

在我国物理学科快速发展的背景下,近年来物理学相关书籍也呈现百花齐放的良好态势,在知识传承、学术交流、人才培养等方面发挥着无可替代的作用。从另一方面看,尽管国内各出版社相继推出了一些质量很高的物理教材和图书,但系统总结物理学各门类知识和发展,深入浅出地介绍其与现代科学技术之间的渊源,并针对不同层次的读者提供有价值的教材和研究参考,仍是我国科学传播与出版界面临的一个极富挑战性的课题。

为有力推动我国物理学研究、加快相关学科的建设与发展,特别是展现近年来中国物理学者的研究水平和成果,北京大学出版社在国家出版基金的支持下推出了“中外物理学精品书系”,试图对以上难题进行大胆的尝试和探索。该书系编委会集结了数十位来自内地和香港顶尖高校及科研院所的知名专家学者。他们都是目前该领域十分活跃的专家,确保了整套丛书的权威性和前瞻性。

这套书系内容丰富,涵盖面广,可读性强,其中既有对我国传统物理学发展的梳理和总结,也有对正在蓬勃发展的物理学前沿的全面展示;既引进和介绍了世界物理学研究的发展动态,也面向国际主流领域传播中国物理的优秀专著。可以说,“中外物理学精品书系”力图完整呈现近现代世界和中国物理

科学发展的全貌,是一部目前国内为数不多的兼具学术价值和阅读乐趣的经典物理丛书。

“中外物理学精品书系”另一个突出特点是,在把西方物理的精华要义“请进来”的同时,也将我国近现代物理的优秀成果“送出去”。物理学科在世界范围内的重要性不言而喻,引进和翻译世界物理的经典著作和前沿动态,可以满足当前国内物理教学和科研工作的迫切需求。另一方面,改革开放几十年来,我国的物理学研究取得了长足发展,一大批具有较高学术价值的著作相继问世。这套丛书首次将一些中国物理学者的优秀论著以英文版的形式直接推向国际相关研究的主流领域,使世界对中国物理学的过去和现状有更多的深入了解,不仅充分展示出中国物理学研究和积累的“硬实力”,也向世界主动传播我国科技文化领域不断创新的“软实力”,对全面提升中国科学、教育和文化领域的国际形象起到重要的促进作用。

值得一提的是,“中外物理学精品书系”还对中国近现代物理学科的经典著作进行了全面收录。20世纪以来,中国物理界诞生了很多经典作品,但当时大都分散出版,如今很多代表性的作品已经淹没在浩瀚的图书海洋中,读者们对这些论著也都是“只闻其声,未见其真”。该书系的编者们在这方面下了很大工夫,对中国物理学科不同时期、不同分支的经典著作进行了系统的整理和收录。这项工作具有非常重要的学术意义和社会价值,不仅可以很好地保护和传承我国物理学的经典文献,充分发挥其应有的传世育人的作用,更能使广大物理学人和青年学子切身体会我国物理学研究的发展脉络和优良传统,真正领悟到老一辈科学家严谨求实、追求卓越、博大精深的治学之美。

温家宝总理在2006年中国科学技术大会上指出,“加强基础研究是提升国家创新能力、积累智力资本的重要途径,是我国跻身世界科技强国的必要条件”。中国的发展在于创新,而基础研究正是一切创新的根本和源泉。我相信,这套“中外物理学精品书系”的出版,不仅可以使所有热爱和研究物理学的人们从中获取思维的启迪、智力的挑战和阅读的乐趣,也将进一步推动其他相关基础科学更好更快地发展,为我国今后的科技创新和社会进步做出应有的贡献。

“中外物理学精品书系”编委会 主任

中国科学院院士,北京大学教授

王恩哥

2010年5月于燕园

GRAVITATION

Foundations and Frontiers

T. PADMANABHAN

*IUCAA, Pune,
India*



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Dedicated to the fellow citizens of India

Preface

There is a need for a *comprehensive, advanced level*, textbook dealing with all aspects of gravity, written *for the physicist* in a *contemporary* style. The italicized adjectives in the above sentence are the key: most of the existing books on the market are either outdated in emphasis, too mathematical for a physicist, not comprehensive or written at an elementary level. (For example, the two unique books – L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, and C. W. Misner, K. S. Thorne and J. A. Wheeler (MTW), *Gravitation* – which I consider to be masterpieces in this subject are more than three decades old and are out of date in their emphasis.) The current book is expected to fill this niche and I hope it becomes a standard reference in this field. Some of the features of this book, including the summary of chapters, are given below.

As the title implies, this book covers both Foundations (Chapters 1–10) and Frontiers (Chapters 11–16) of general relativity so as to cater for the needs of different segments of readership. The Foundations acquaint the readers with the basics of general relativity while the topics in Frontiers allow one to ‘mix-and-match’, depending on interest and inclination. This modular structure of the book will allow it to be adapted for different types of course work.

For a specialist researcher or a student of gravity, this book provides a comprehensive coverage of all the contemporary topics, some of which are discussed in a textbook for the first time, as far as I know. The cognoscenti will find that there is a fair amount of originality in the discussion (and in the Exercises) of even the conventional topics.

While the book is quite comprehensive, it also has a structure which will make it accessible to a wide target audience. Researchers and teachers interested in theoretical physics, general relativity, relativistic astrophysics and cosmology will find it useful for their research and adaptable for their course requirements. (The section *How to use this book*, just after this Preface, gives more details of this aspect.) The discussion is presented in a style suitable for physicists, ensuring that it caters

for the current interest in gravity among physicists working in related areas. The large number (more than 225) of reasonably nontrivial Exercises makes it ideal for self-study.

Another unique feature of this book is a set of Projects at the end of selected chapters. The Projects are advanced level exercises presented with helpful hints to show the reader a direction of attack. Several of them are based on research literature dealing with key open issues in different areas. These will act as a bridge for students to cross over from textbook material to real life research. Graduate students and grad school teachers will find the Exercises and Projects extremely useful. Advanced undergraduate students with a flair for theoretical physics will also be able to use parts of this book, especially in combination with more elementary books.

Here is a brief description of the chapters of the book and their inter-relationship.

Chapters 1 and 2 of this book are somewhat unique and serve an important purpose, which I would like to explain. A student learning general relativity often finds that she simultaneously has to cope with (i) conceptual and mathematical issues which arise from the spacetime being curved and (ii) technical issues and concepts which are essentially special relativistic but were never emphasized adequately in a special relativity course! For example, manipulation of four-dimensional integrals or the concept and properties of the energy-momentum tensor have nothing to do with general relativity *a priori* – but are usually not covered in depth in conventional special relativity courses. The first two chapters give the student a rigorous training in four-dimensional techniques in flat spacetime so that she can concentrate on issues which are genuinely general relativistic later on. These chapters can also usefully serve as modular course material for a short course on advanced special relativity or classical field theory.

Chapter 1 introduces special relativity using four-vectors and the action principle right from the outset. Chapter 2 introduces the electromagnetic field through the four-vector formalism. I expect the student to have done a standard course in classical mechanics and electromagnetic theory but I do *not* assume familiarity with the relativistic (four-vector) notation. Several topics that are needed later in general relativity are introduced in these two chapters in order to familiarize the reader early on. Examples include the use of the relativistic Hamilton–Jacobi equation, precession of Coulomb orbits, dynamics of the electromagnetic field obtained from an action principle, derivation of the field of an arbitrarily moving charged particle, radiation reaction, etc. Chapter 2 also serves as a launch pad for discussing spin-0 and spin-2 interactions, using electromagnetism as a familiar example.

Chapter 3 attempts to put together special relativity and gravity and explains, in clear and precise terms, why it does not lead to a consistent picture. Most textbooks I know (except MTW) do not explain the issues involved clearly and with adequate

detail. For example, this chapter contains a detailed discussion of the spin-2 tensor field which is not available in textbooks. It is important for a student to realize that the description of gravity in terms of curvature of spacetime is inevitable and natural. This chapter will also lay the foundation for the description of the spin-2 tensor field h_{ab} , which will play an important role in the study of gravitational waves and cosmological perturbation theory later on.

Having convinced the reader that gravity is related to spacetime geometry, Chapter 4 begins with the description of general relativity by introducing the metric tensor and extending the ideas of four-vectors, tensors, etc., to a nontrivial background. There are two points that I would like to highlight about this chapter. First, I have introduced every concept with a physical principle rather than in the abstract language of differential geometry. For example, direct variation of the line interval leads to the geodesic equation *through* which one can motivate the notion of Christoffel symbols, covariant derivative, etc., in a simple manner. During the courses I have taught over years, students have found this approach attractive and simpler to grasp. Second, I find that students sometimes confuse issues which arise when curvilinear coordinates are used in flat spacetime with those related to genuine curvature. This chapter clarifies such issues.

Chapter 5 introduces the concept of the curvature tensor from three different perspectives and describes its properties. It then moves on to provide a complete description of electrodynamics, statistical mechanics, thermodynamics and wave propagation in curved spacetime, including the Raychaudhuri equation and the focusing theorem.

Chapter 6 starts with a clear and coherent derivation of Einstein's field equations from an action principle. I have provided a careful discussion of the surface term in the Einstein–Hilbert action (again not usually found in textbooks) in a manner which is quite general and turns out to be useful in the discussion of Lanczos–Lovelock models in Chapter 15. I then proceed to discuss the general structure of the field equations, the energy-momentum pseudo-tensor for gravity and the weak field limit of gravity.

After developing the formalism in the first six chapters, I apply it to discuss four key applications of general relativity – spherically symmetric spacetimes, black hole physics, gravitational waves and cosmology – in the next four chapters. (The only other key topic I have omitted, due to lack of space, is the physics of compact stellar remnants.)

Chapter 7 deals with the simplest class of exact solutions to Einstein's equations, which are those with spherical symmetry. The chapter also discusses the orbits of particles and photons in these spacetimes and the tests of general relativity. These are used in Chapter 8, which covers several aspects of black hole physics, concentrating mostly on the Schwarzschild and Kerr black holes. It also introduces

important concepts like the maximal extension of a manifold, Penrose–Carter diagrams and the geometrical description of horizons as null surfaces. A derivation of the zeroth law of black hole mechanics and illustrations of the first and second laws are also provided. The material developed here forms the backdrop for the discussions in Chapters 13, 15 and 16.

Chapter 9 takes up one of the key new phenomena that arise in general relativity, viz. the existence of solutions to Einstein's equations which represent disturbances in the spacetime that propagate at the speed of light. A careful discussion of gauge invariance and coordinate conditions in the description of gravitational waves is provided. I also make explicit contact with similar phenomena in the case of electromagnetic radiation in order to help the reader to understand the concepts better. A detailed discussion of the binary pulsar is included and a Project at the end of the chapter explores the nuances of the post-Newtonian approximation.

Chapter 10 applies general relativity to study cosmology and the evolution of the universe. Given the prominence cosmology enjoys in current research and the fact that this interest will persist in future, it is important that all general relativists are acquainted with cosmology at the same level of detail as, for example, with the Schwarzschild metric. This is the motivation for Chapter 10 as well as Chapter 13 (which deals with general relativistic perturbation theory). The emphasis here will be mostly on the geometrical aspects of the universe rather than on physical cosmology, for which several other excellent textbooks (e.g. mine!) exist. However, in order to provide a complete picture and to appreciate the interplay between theory and observation, it is necessary to discuss certain aspects of the evolutionary history of the universe – which is done to the extent needed.

The second part of the book (Frontiers, Chapters 11–16) discusses six separate topics which are reasonably independent of each other (though not completely). While a student or researcher specializing in gravitation should study all of them, others could choose the topics based on their interest after covering the first part of the book.

Chapter 11 introduces the language of differential forms and exterior calculus and translates many of the results of the previous chapters into the language of forms. It also describes briefly the structure of gauge theories to illustrate the generality of the formalism. The emphasis is in developing the key concepts rapidly and connecting them up with the more familiar language used in the earlier chapters, rather than in maintaining mathematical rigour.

Chapter 12 describes the $(1 + 3)$ -decomposition of general relativity and its Hamiltonian structure. I provide a derivation of Gauss–Codazzi equations and Einstein's equations in the $(1 + 3)$ -form. The connection between the surface term in the Einstein–Hilbert action and the extrinsic curvature of the boundary is also

spelt out in detail. Other topics include the derivation of junction conditions which are used later in Chapter 15 while discussing the brane world cosmologies.

Chapter 13 describes general relativistic linear perturbation theory in the context of cosmology. This subject has acquired major significance, thanks to the observational connection it makes with cosmic microwave background radiation. In view of this, I have also included a brief discussion of the application of perturbation theory in deriving the temperature anisotropies of the background radiation.

Chapter 14 describes some interesting results which arise when one studies standard quantum field theory in a background spacetime with a nontrivial metric. Though the discussion is reasonably self-contained, some familiarity with simple ideas of quantum theory of free fields will be helpful. The key result which I focus on is the intriguing connection between thermodynamics and horizons. This connection can be viewed from very different perspectives not all of which can rigorously be proved to be equivalent to one another. In view of the importance of this result, most of this chapter concentrates on obtaining this result using different techniques and interpreting it physically. In the latter part of the chapter, I have added a discussion of quantum field theory in the Friedmann universe and the generation of perturbations during the inflationary phase of the universe.

Chapter 15 discusses a few *selected* topics in the study of gravity in dimensions other than $D = 4$. I have kept the discussion of models in $D < 4$ quite brief and have spent more time on the $D > 4$ case. In this context – after providing a brief, but adequate, discussion of brane world models which are enjoying some popularity currently – I describe the structure of Lanczos–Lovelock models in detail. These models share several intriguing features with Einstein’s theory and constitute a natural generalization of Einstein’s theory to higher dimensions. I hope this chapter will fill the need, often felt by students working in this area, for a textbook discussion of Lanczos–Lovelock models.

The final chapter provides a perspective on gravity as an emergent phenomenon. (Obviously, this chapter shows my personal bias but I am sure that is acceptable in the *last* chapter!) I have tried to put together several peculiar features in the standard description of gravity and emphasize certain ideas which the reader might find fascinating and intriguing.

Because of the highly pedagogical nature of the material covered in this textbook, I have not given detailed references to original literature except on rare occasions when a particular derivation is not available in the standard textbooks. The annotated list of Notes given at the end of the book cites several other text books which I found useful. Some of these books contain extensive bibliographies and references to original literature. The selection of books and references cited here clearly reflects the bias of the author and I apologize to anyone who feels their work or contribution has been overlooked.

Discussions with several people, far too numerous to name individually, have helped me in writing this book. Here I shall confine myself to those who provided detailed comments on earlier drafts of the manuscript. Donald Lynden-Bell and Aseem Paranjape provided extensive and very detailed comments on most of the chapters and I am very thankful to them. I also thank A. Das, S. Dhurandar, P. P. Divakaran, J. Ehlers, G. F. R. Ellis, Rajesh Gopakumar, N. Kumar, N. Mukunda, J. V. Narlikar, Maulik Parikh, T. R. Seshadri and L. Sriramkumar for detailed comments on selected chapters.

Vince Higgs (CUP) took up my proposal to write this book with enthusiasm. The processing of this volume was handled by Laura Clark (CUP) and I thank her for the effort she has put in.

This project would not have been possible without the dedicated support from Vasanthi Padmanabhan, who not only did the entire TEXing and formatting but also produced most of the figures. I thank her for her help. It is a pleasure to acknowledge the library and other research facilities available at IUCAA, which were useful in this task.

How to use this book

This book can be adapted by readers with varying backgrounds and requirements as well as by teachers handling different courses. The material is presented in a fairly modular fashion and I describe below different sub-units that can be combined for possible courses or for self-study.

1 Advanced special relativity

Chapter 1 along with parts of Chapter 2 (especially Sections 2.2, 2.5, 2.6, 2.10) can form a course in advanced special relativity. No previous familiarity with four-vector notation (in the description of relativistic mechanics or electrodynamics) is required.

2 Classical field theory

Parts of Chapter 1 along with Chapter 2 and Sections 3.2, 3.3 will give a comprehensive exposure to classical field theory. This will require familiarity with special relativity using four-vector notation which can be acquired from specific sections of Chapter 1.

3 Introductory general relativity

Assuming familiarity with special relativity, a basic course in general relativity (GR) can be structured using the following material: Sections 3.5, Chapter 4 (except Sections 4.8, 4.9), Chapter 5 (except Sections 5.2.3, 5.3.3, 5.4.4, 5.5, 5.6), Sections 6.2.5, 6.4.1, 7.2.1, 7.4.1, 7.4.2, 7.5. This can be supplemented with selected topics in Chapters 8 and 9.

4 Relativistic cosmology

Chapter 10 (except Sections 10.6, 10.7) along with Chapter 13 and parts of Sections 14.7 and 14.8 will constitute a course in relativistic cosmology and perturbation theory from a contemporary point of view.

5 Quantum field theory in curved spacetime

Parts of Chapter 8 (especially Sections 8.2, 8.3, 8.7) and Chapter 14 will constitute a first course in this subject. It will assume familiarity with GR but not with detailed properties of black holes or quantum field theory. Parts of Chapter 2 can supplement this course.

6 Applied general relativity

For students who have already done a first course in GR, Chapters 6, 8, 9 and 12 (with parts of Chapter 7 not covered in the first course) will provide a description of *advanced* topics in GR.

Exercises and Projects

None of the Exercises in this book is trivial or of simple ‘plug-in’ type. Some of them involve extending the concepts developed in the text or understanding them from a different perspective; others require detailed application of the material introduced in the chapter. There are more than 225 exercises and it is strongly recommended that the reader attempts as many as possible. Some of the nontrivial exercises contain hints and short answers.

The Projects are more advanced exercises linking to original literature. It will often be necessary to study additional references in order to comprehensively grasp or answer the questions raised in the projects. Many of them are open-ended (and could even lead to publishable results) but all of them are presented in a graded manner so that a serious student will be able to complete most parts of any project. They are included so as to provide a bridge for students to cross over from the textbook material to original research and should be approached in this light.

Notation and conventions

Throughout the book, the Latin indices a, b, \dots, i, j, \dots , etc., run over 0, 1, 2, 3 with the 0-index denoting the time dimension and (1, 2, 3) denoting the standard space dimensions. The Greek indices α, β, \dots , etc., will run over 1, 2, 3. Except when indicated otherwise, the units are chosen with $c = 1$.

We will use the vector notation for both three-vectors and four-vectors by using different fonts. The four-momentum, for example, will be denoted by \mathbf{p} while the three-momentum will be denoted by \mathbf{p} .

The signature is $(-, +, +, +)$ and curvature tensor is defined by the convention $R^a_{bcd} \equiv \partial_c \Gamma^a_{bd} - \dots$ with $R_{bd} = R^a_{bad}$.

The symbol \equiv is used to indicate that the equation defines a new variable or notation.

Contents

<i>List of exercises</i>	<i>page</i> xiii
<i>List of projects</i>	xix
<i>Preface</i>	xxi
<i>How to use this book</i>	xxvii
1 Special relativity	1
1.1 Introduction	1
1.2 The principles of special relativity	1
1.3 Transformation of coordinates and velocities	6
1.3.1 Lorentz transformation	8
1.3.2 Transformation of velocities	10
1.3.3 Lorentz boost in an arbitrary direction	11
1.4 Four-vectors	13
1.4.1 Four-velocity and acceleration	17
1.5 Tensors	19
1.6 Tensors as geometrical objects	23
1.7 Volume and surface integrals in four dimensions	26
1.8 Particle dynamics	29
1.9 The distribution function and its moments	35
1.10 The Lorentz group and Pauli matrices	45
2 Scalar and electromagnetic fields in special relativity	54
2.1 Introduction	54
2.2 External fields of force	54
2.3 Classical scalar field	55
2.3.1 Dynamics of a particle interacting with a scalar field	55
2.3.2 Action and dynamics of the scalar field	57
2.3.3 Energy-momentum tensor for the scalar field	60
2.3.4 Free field and the wave solutions	62