NUCLEAR SIZES AND STRUCTURE

BY

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PREFACE

When we first discussed the need for a new book on nuclear sizes in 1970, nine years after the publication of Elton's *Nuclear sizes*, the topic had become established as a major area of physics. There had been a number of conferences devoted entirely to such topics as 'Electromagnetic sizes of nuclei' and 'Intermediate energy physics', and a continuing series with the title 'High energy physics and nuclear structure'. A number of review articles and books had appeared on subjects such as electron scattering, optical isotope shifts, muonic atoms, nucleon-nucleus scattering, pionic atoms, and theories of the nuclear ground state.

It appeared that the main task would be to draw together established methods and to show the relation between results obtained from the various electromagnetic and strong interaction processes. By the time we had actually started work it was evident that the subject was going through a phase of major development and renewal. New experimental techniques and completely new accelerators have produced fresh data with vastly increased accuracy. This has completely changed our expectation of what can be deduced about nuclear sizes and shapes. At the same time new methods of analysis have led to a much better awareness of what properties are actually being studied in a particular process and what uncertainties are associated with the parameters deduced from the analyses. In response to these developments we have endeavoured to cover the study of nuclear sizes and shapes by means of a very wide range of electromagnetic, strong, and weak interaction processes, and have placed particular emphasis on the progress in experimental measurement and in theoretical techniques used to predict nuclear behaviour or to extract detailed information from the measurements.

A principal aim of the book is to explain precisely which properties of the nucleus can be obtained from the various experiments and to indicate the extent to which previously published parameters are really determined by the measurements. As far as possible we have tried to include sufficient formalism and background information to serve as an introduction to each topic for postgraduate students. We have taken critical discussion of results and comparison of methods to be of greater importance than the tabulation of parameters, although many results of analyses are presented in tables and figures.

The major part of this book was completed early in 1975 although a very few topics have been extended to include work reported in 1976. This means that the treatment of certain fast-developing topics will inevitably appear inadequate by the time the book is in the hands of the reader. Almost every week a new preprint arrives which could lead to an additional comment or revision. We hope that the shortcomings of the book in this respect will be taken as an indication of the vitality of the subject and act as a stimulus to further interest and work in the various fields.

Guildford, Surrey R.C.B.
September 1976 D.F.J.

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CONTENTS

1
INTRODUCTION AND DEFINITIONS

It will be seen that this theory makes the radius of the uranium nucleus very small, about 7×10^{-13} cm,.... It sounds incredible but may not be impossible.

Rutherford 1929

1.1 INTRODUCTION TO THE STUDY OF NUCLEAR SIZES

The determination of nuclear shapes and sizes is one of the traditional problems of nuclear physics. The extent to which we are able to make precise and meaningful statements about the nuclear matter distribution and the nuclear charge distribution and the variation in both quantities from one nucleus to another reveals quite clearly the state of our understanding of much more fundamental issues, such as the nature of the interactions between various types of particles and the role of these interactions in scattering phenomena, the subtle balance between various features of the nucleon-nucleon interactions in bound states, and the difference between the average properties of nuclei described by macroscopic models and the specific nuclear structure properties described by microscopic models.

The study of nuclear sizes involves both the study of the nuclear charge distribution by means of processes dominated by electromagnetic interactions and the study of the nuclear matter distribution by means of strong-interaction processes. By combining the information so obtained, comparison of the proton and neutron distributions can be made. Most of the discussion will be devoted to the determination of the radial shape of the distributions in spherical nuclei, but the angular dependence of the shape of nuclei which are not spherical will also be considered. One of our principal aims will be to try to determine and explain precisely which properties of the relevant distributions can be obtained from the various experiments and to indicate the extent to which previously published parameters are really determined by the measurements as opposed to being merely consistent with them.

The interaction between charged leptons (i.e. electrons, positrons, and muons) and nucleons consists of an electromagnetic and a weak term, but the latter has a completely negligible effect in the processes considered here. The first direct determination of nuclear charge radii came from the measurements on elastic electron scattering by Lyman, Hansen, and Scott (1951). This was many years after the suggestion (Guth 1934) that, for fast electrons, the finite size of the nuclear charge distribution would produce large deviations from the differential cross-section (Mott 1929) for elastic scattering from a point charge. Experimental techniques subsequently improved very rapidly, and electron scattering has been extensively used in the study of nuclear charge distributions and other properties of nuclei. Two years after the experiments of Lyman *et al.*, Fitch and Rainwater (1953) measured the energies of X-rays from muonic atoms, which provided another means of studying nuclear charge radii. The idea of stopping negative muons in matter to form muonic atoms and using the X-ray energies to obtain nuclear size information was originally due to Wheeler (1947). The accuracy of experimental measurements on muonic atoms improved slowly at first and then very rapidly after about 1960. A much older method exists for the determination of differences between nuclear charge radii of different isotopes of an element. This involves measurement of the isotope shift of spectral lines in electronic (i.e. ordinary) atoms. A similar shift, called the isomer shift, gives the change in the charge radius when a long-lived nuclear state is excited. The nuclear size contribution to the shift was first pointed out by Rosenthal and Breit (1932) long after the first measurement of an isotope shift in which finite nuclear size effects made a substantial contribution had been done by Merton (1919). The electromagnetic interaction with the nuclear magnetic moment is observable in elastic electron scattering at 180°, in inelastic electron scattering, and in hyperfine splitting of certain atomic levels, and it is therefore possible to determine the magnetic moment distribution of nuclei.

It is now known that the proton and neutron each have an intrinsic electromagnetic structure and that processes such as high-energy electron-nucleon scattering cannot be described adequately in terms of a point charge and a point magnetic moment. The theory of the electromagnetic structure of nucleons will not be discussed here, and the reader is referred to reviews such as that by Drell and Zachariasen (1961). It will, however, be necessary to use the information so far obtained for nucleons in order to make a connection between the predictions of theories for nuclear distributions due to point nucleons and the observed nuclear charge and magnetic moment distributions due to nucleons with finite electromagnetic size.

The effect of a strong-interaction radius for nuclei was seen in early experiments on α-particle scattering. The scattering data, together with the results from α-decay of heavy nuclei, were interpreted in terms of an attractive nuclear potential plus a repulsive Coulomb barrier (Rutherford 1929) and an estimate of the nuclear potential radius was obtained. Following the development of the cyclotron, the energy dependence of α-particle scattering from heavy nuclei was studied up to 40 MeV (Farwell and Wegner 1954) and the abrupt departure from pure Coulomb scattering beyond a critical energy was interpreted in terms of a radius parameter (Blair 1954). This radius parameter cannot be directly interpreted in terms of a nuclear matter radius, since the range of the potential must be connected with the finite range of nuclear forces and the size of the projectile.

The description of nucleon scattering from nuclei in terms of a complex one-body potential was placed on a sound theoretical foundation by Feshbach, Porter, and Weisskopf (1954), who showed that such a potential can be related to the averaged or gross-structure properties of the compound nuclear system. They succeeded in reproducing neutron scattering data up to a few MeV with a complex square-well potential, but fits to angular distributions for 20 MeV protons required a potential with a diffuse surface resembling the surface of the nuclear charge distribution (Woods and Saxon 1954). For scattering at ~100 MeV, Serber (1947) had suggested that the collision of an incident nucleon with the nucleus could be interpreted in terms of collisions with individual nucleons, and Fernbach, Serber, and Taylor (1949) analysed total neutron cross-sections effectively with a square-well potential whose imaginary (absorptive) part was related to the total cross-sections for nucleon-nucleon scattering. The long mean free paths derived in such work were explained by Weisskopf (1951) in terms of the exclusion principle which limits nucleon-nucleon collisions in the nucleus to those in which bound nucleons are raised above the Fermi surface. The scattering of nucleons and pions from nuclei at energies ~1 GeV were interpreted in terms of a potential which was related in a fairly intuitive way to the nuclear matter distribution (Coor *et al.* 1955, Williams 1955, Abashian, Cool, and Cronin 1956).

In recent years the data obtained in many experiments on scattering of medium-energy nucleons have been analysed in terms of a complex potential and potential parameters have been determined, while analyses of the scattering of strongly absorbed projectiles, such as α-particles, in terms of diffraction models have yielded values of diffraction radii. In addition there have been many developments in the theory of potential scattering for nuclei and in our understanding of the connection between the potential, the nucleon-nucleon force, and the nuclear ground state (Feshbach 1958, 1962, Goldberger and Watson 1964). Some progress has been made in understanding the relation between diffraction radii and potential radii (Blair 1966).

As in the case of the study of electromagnetic interactions, our intention in the study of nuclear scattering is to establish what size parameters can be determined from the detailed fits to data, and to examine the relation between these parameters and the nuclear matter distribution.

In addition to elastic scattering, many other processes yield information on nuclear sizes, usually by somewhat indirect means. These processes include inelastic scattering, certain direct nuclear reactions, and high-energy processes such as pion production. The measurement of Coulomb energy differences between appropriate nuclear states and of X-ray energies in π-mesonic and K-mesonic atoms also yields valuable information. All these topics have been studied with increasing accuracy in recent years.

In the earliest years of the work mentioned above it was sufficient to regard the nucleus as an object with a sharp radius R and uniform density as shown in Fig. 1.1(a). By 1954

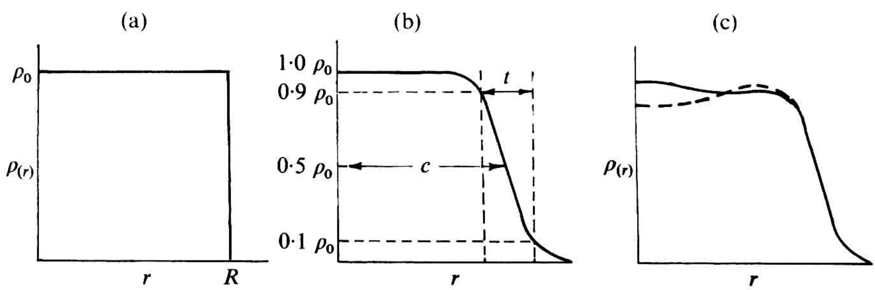

FIG. 1.1. Examples of nuclear distribution functions. (a) Billiard-ball nucleus with a well-defined radius R. (b) The Fermi distribution. (c) Shell model distributions. (From Jackson 1974.)

it was known that the nucleus had a diffuse surface which was quite well represented by the distribution shown in Fig. 1.1(b), and more recent work has suggested distributions of the form shown in Fig. 1.1(c). Thus it is no longer possible to define a single 'nuclear radius', and we are now concerned with the set of size parameters or spatial moments of certain distribution functions for nuclei.

1.2 DEFINITIONS

1.2.1 *Distribution functions and form factors*

In nuclear theory, we require density distributions or one-particle density functions for the protons, neutrons, and nucleons. There are distributions of *point* particles in the nucleus. If the ground state of a nucleus with Z protons and $N = A - Z$ neutrons is denoted by $|0\rangle$, then the proton distribution is given by

$$Z\ \rho_p(\underset{\sim}{r}) = \langle 0|\ \sum_{i=1}^{Z} \delta(\underset{\sim}{r} - \underset{\sim}{r}_i)\ |0\rangle \tag{1.1}$$

and the neutron distribution is given by

$$N\ \rho_n(\underset{\sim}{r}) = \langle 0|\ \sum_{i=1}^{N} \delta(\underset{\sim}{r} - \underset{\sim}{r}_i)\ |0\rangle \tag{1.2}$$

where the sums run only over protons or neutrons, respectively, and the normalization chosen here is

$$\int \rho_p(\underset{\sim}{r})\ d^3r = \int \rho_n(\underset{\sim}{r})\ d^3r = 1. \tag{1.3}$$

The matter or nucleon distribution is then given by

$$A\ \rho_m(\underset{\sim}{r}) = Z\ \rho_p(\underset{\sim}{r}) + N\ \rho_n(\underset{\sim}{r}). \tag{1.4}$$

In many situations involving electromagnetic interactions we actually require the nuclear charge distribution $\rho_{ch}(\underset{\sim}{r})$ of the nucleus instead of the distribution of point protons. Since the proton is not a point charge but has a finite size in the electromagnetic sense, ρ_{ch} can be obtained from ρ_p by folding in the charge distribution of the proton, i.e.

$$\rho_{ch}(\underset{\sim}{r}') = \int \rho_p(\underset{\sim}{r})\ \rho_d(|\underset{\sim}{r} - \underset{\sim}{r}'|)\ d^3r \tag{1.5}$$

where ρ_d represents the charge distribution of the proton also normalized to unity. It follows from the folding integral (1.5) that the r.m.s. radius of ρ_{ch} is increased relative to that of ρ_p, according to the formula

$$\langle r^2\rangle_{ch} = \langle r^2\rangle_p + \langle r^2\rangle_d, \tag{1.6}$$

the diffuseness is increased, and any irregularities in the shape of ρ_p are smoothed by the folding procedure.

It is also possible to define two-particle and higher density functions. These give the probability of finding a point nucleon at $\underset{\sim}{r}$ if there is another nucleon at $\underset{\sim}{r}'$, and so on. For example, the two-nucleon density function is given by

$$A(A-1)\,\rho_{\mathrm{m}}(\underset{\sim}{r},\underset{\sim}{r}') = \langle 0|\sum_{i\neq j}^{A} \delta(\underset{\sim}{r}-\underset{\sim}{r}_i)\,\delta(\underset{\sim}{r}'-\underset{\sim}{r}_j)\,|0\rangle. \tag{1.7}$$

If $\rho_{\mathrm{m}}(\underset{\sim}{r},\underset{\sim}{r}') = \rho_{\mathrm{m}}(\underset{\sim}{r})\,\rho_{\mathrm{m}}(\underset{\sim}{r}')$, the system is said to be uncorrelated, but in general $\rho_{\mathrm{m}}(\underset{\sim}{r},\underset{\sim}{r}')$ is written as

$$\rho_{\mathrm{m}}(\underset{\sim}{r},\underset{\sim}{r}') = C(\underset{\sim}{r},\underset{\sim}{r}') + \rho_{\mathrm{m}}(\underset{\sim}{r})\,\rho_{\mathrm{m}}(\underset{\sim}{r}') \tag{1.8}$$

where $C(\underset{\sim}{r},\underset{\sim}{r}')$ is the pair correlation function. In an extreme single-particle model only Pauli correlations arising from the exclusion principle are present, but in a more realistic model, e.g. including configuration mixing or clustering in the ground state, additional correlations of medium range arise. Short-range correlations due to the short-range behaviour of the nuclear force are also present.

The Fourier transforms of the various distributions are known as form factors. For example, the nuclear form factor is

$$F_{\mathrm{m}}(q) = \int \exp(\mathrm{i}\underset{\sim}{q}\cdot\underset{\sim}{r})\,\rho_{\mathrm{m}}(\underset{\sim}{r})\,\mathrm{d}^3 r \tag{1.9}$$

and the charge form factor is

$$F_{\mathrm{ch}}(q) = \int \exp(\mathrm{i}\underset{\sim}{q}\cdot\underset{\sim}{r})\,\rho_{\mathrm{ch}}(\underset{\sim}{r})\,\mathrm{d}^3 r. \tag{1.10}$$

Using eqn (1.5) for ρ_{ch} it follows that

$$F_{\mathrm{ch}}(q) = f_{\mathrm{d}}(q)\,F_{\mathrm{p}}(q) \tag{1.11}$$

where f_{d}, F_{p} are the form factors of a single proton and of the point protons in the nucleus, respectively.

1.2.2 *Some functional forms*

The distribution functions can be parametrized directly or indirectly. The direct parametrization involves the choice of a suitable functional form with parameters which may be varied to fit the experimental data. The functional form most widely used is the Fermi distribution