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Introduction

With this volume, we rename this continuing series *Symposia on Theoretical Physics and Mathematics* and include contributions in pure mathematics presented at the scientific meetings of the Institute. This volume, comprising the lectures of the Fourth Anniversary Symposium, presents a rich and varied fare ranging from experimental high-energy physics to mathematical analysis.

The symposium was inaugurated by Sir C.P. Ramaswami Aiyer, whose references in critical detail to Indian contributions to mathematics were characteristic of the versatility of this statesman, scholar, and educator. The scientific session of the symposium commenced with the introductory lecture of Professor Gunnar Källén of Sweden, who gave a masterly survey of the development of modern physics from Schrödinger to Gell-Mann.

The contributions in elementary particle physics of Sudarshan, Ruegg, and Gruber echoed the triumphant march of unitary symmetry during the years 1961–1964. The participation of the experimental physicists Gerson Goldhaber and B.J. Moyer from California was a natural sequel to the introduction of "the heady atmosphere of Berkeley into the placid environs of my family home in Madras" a few years ago when the group of theoretical physicists forming the nucleus of the present Institute first gathered for discussions on high-energy physics. Theoretical papers closely related to experimental research were presented at the symposium by Källén, Ananthanarayanan, and Nair.

Correspondingly, there were contributions relating to mathematical aspects of physics by Takahashi of Dublin, Rzewuski of Poland, and Kotani of Japan. The papers of Narlikar, Daniel, Narasimhan, Pičman, and Mathews contributed to the variety of subjects discussed in the volume.

With the participation of Professor Hayman in the symposium a significant beginning was made in the faculty of pure mathematics which started functioning under Professor Unni, a member of our permanent staff. Professor Hayman dealt with Nevanlinna theory, and Unni with functions of exponential type.

The range of subjects demonstrates the most striking feature of fundamental research today—the vanishing of frontiers that hitherto separated various domains of knowledge.

Alladi Ramakrishnan

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On Locally Isomorphic Groups and Cartan–Stiefel Diagrams

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1. INTRODUCTION

The groups considered in this review article are the semisimple compact and connected Lie groups. Given such a group, all the (semisimple compact connected Lie) groups which are locally isomorphic to it are determined. The relations which hold among the members of such a set of groups—called a family—are given, i.e., the relations between a group, its covering groups, and its universal covering group are established.[1,2] For the semisimple compact connected Lie groups, diagrams can be derived—the so called Cartan–Stiefel diagrams. The connections between these diagrams and the results stated for the families of locally isomorphic groups will be established in the second part of this article.[3,4]

It is the intention of the author to keep this article understandable to the physicist not very familiar with group theory. For this reason all proofs are omitted and a more "intuitive" argumentation is used. For someone wanting more details or rigorous proofs, references are given to supplement the article.

It is well known that a Lie algebra determines the corresponding Lie group only in the neighborhood of the identity.[5] Therefore,

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if two Lie groups are locally isomorphic,[6,7] i.e., if they are identical in a conveniently chosen neighborhood of the identity, then they have the same Lie algebra. Thus, to one and the same Lie algebra, correspond, in general, several Lie groups, namely, all the Lie groups which are locally isomorphic. So, for instance, $SU(2)$, $SO(3)$, and $O(3)$ are locally isomorphic and therefore have the same Lie algebra.[8] However, whereas $SU(2)$ and $SO(3)$ are connected Lie groups, $O(3)$ is not; $O(3)$ consits of two "pieces" which are disconnected from each other.[9] One piece corresponds to all elements of $O(3)$ having determinant $+1$, the other to all elements having determinant -1. It will become clear later why these two pieces are disconnected. However, as already stated, there will be considered, only locally isomorphic Lie groups which are (linear)[10] connected. Thus, if we define a family of groups to consist of the set of all locally isomorphic connected Lie groups, then only $SO(3)$ and $SU(2)$ will belong to a family. It will turn out that this family does not contain more members.

2. TOPOLOGICAL GROUPS, TOPOLOGICAL SPACES, AND PATHS

Out of the properties of the Lie groups we shall need the property that the Lie groups are topological groups and, in particular, that they are topological spaces. In this section we shall be concerned with these aspects of the Lie groups.

A topological group is defined[11] to be a set G of elements such that G is (a) an abstract group, (b) a topological space, and (c) the group operation is continuous in the topological space.

Condition (c) requires, for instance, that if the group product $g_3 = g_1 \cdot g_2$ of two elements g_1 and g_2 is formed and if g is an element in the neighborhood of g_1 (or g_2), then $g \cdot g_2$ (or $g_1 \cdot g$) is in the neighborhood of g_3. Thus it is condition (c) which interconnects the group and space aspects of a topological group. It is clear that in a given abstract group one cannot arbitrarily introduce a topology; it has to be done in such a manner that condition (c) is satisfied in that topology.

For the moment we shall forget that G is a group and consider only the space G. As an example, we consider again $SO(3)$ and $SU(2)$. The topological space which underlies $SO(3)$ is a unit sphere in a three-dimensional Euclidean space, R^3, with its center in the origin. Thereby, diametral boundary points have to be identified[12,13] (they correspond to the same rotation). Each point p of this sphere corresponds to a group element g of $SO(3)$. The identification of point p and group element g is given as follows: The point p represents that group element g of $SO(3)$ for which the straight line through the origin and p is the rotation axis and the distance between p and the origin is the magnitude of the rotation around that axis.

The topological space underlying the group $SU(2)$ however is a unit sphere in R^3 which consists of two sheets (two sheets in the sense of Riemannian sheets). The first sphere (sheet) is the topological space of $SO(3)$ apart from the fact that now a boundary point of this sphere has to be identified with the diametral point on the second sphere; i.e., at the boundary one enters from one sphere into the other. Therefore, as long as the two spaces are considered near the origin (the identity of the group), they are homeomorphic,[14] i.e., identical. Only if one goes away far enough from the origin the two spaces became different; namely, on the boundary either the second sphere is entered [$SU(2)$] or the original sphere is entered at the diametral boundary point [$SO(3)$]. Another way of looking at the situation for $SU(2)$ is the following. The second sphere is "taken out" while the first sphere remains fixed with its center at the origin. Whenever a point on the boundary of the first sphere is considered, the point corresponding to it on the second sphere is brought into coincidence with it (see Fig. 1).

Another example is the one-dimensional toroid T^1 and the additive group of real numbers, the real line R^1. The topological

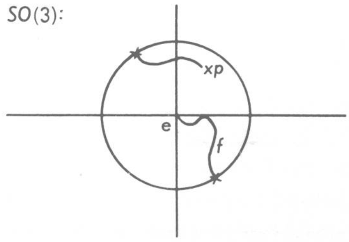

Fig. 1a. The topological space of $SO(3)$ is a unit sphere in the Euclidean space R^3 with its center at the origin. A path f in the space of $SO(3)$ is indicated. The path begins at the origin (the identity of the group) and ends at the point p, i.e., $f(0) = e$, $f(1) = p$. At the boundary of the sphere, the path jumps to the boundary point lying diametral.

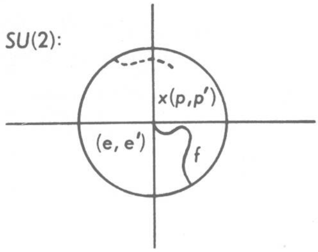

Fig. 1b. The topological space of $SU(2)$. Each point of the unit sphere is double valued. The sphere has two sheets; the points of the two sheets are denoted by unprimed and primed symbols, respectively. The two sheets are connected on the boundary of the sphere. Again, a path f in the space of $SU(2)$ has been drawn, beginning at the origin e (the identity of the group). However, at the boundary of the sphere one enters now the *second* sheet (at the diametral point). Thus, the end point of the path f is the point p'.

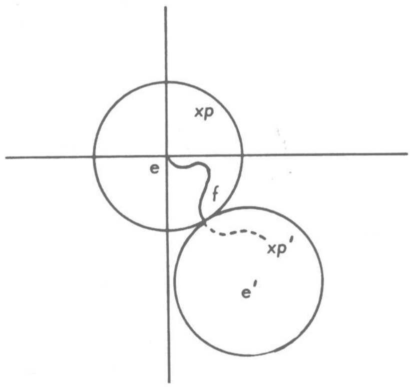

Fig. 1c. Same as Fig. 1b; however, the second sheet (sphere) has now been "taken out."

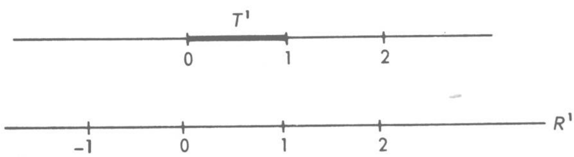

Fig. 2. The topological spaces underlying the toroid T^1 $(0 \leq x < 1)$ and its universal covering group R^1, the real line. The set of all integers N is an invariant subgroup of R^1 and $R^1/N \sim T^1$.