

# Theoretical Foundations of Computer Graphics and CAD

Directed by J. E. Bresenham, R. A. Earnshaw, A. R. Forrest,  
R. J. Lansdown, M. L. V. Pitteway

Edited by R. A. Earnshaw

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Edited by

**R.A. Earnshaw**

University of Leeds, UK

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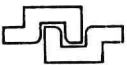
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# About the Directors

## *Dr Jack E. Bresenham*

Dr Bresenham is a Professor of Computer Science on the faculty of Winthrop College in Rock Hill, South Carolina, USA. He earned his BSEE in 1959 from the University of New Mexico and MS and PhD degrees in Industrial Engineering in 1960 and 1964 from Stanford University. He joined IBM in 1960 at San Jose, California and spent the last three years as a Senior Technical Staff member in CPD-HQ System Design Support at IBM's Development Laboratory in Research Triangle Park, North Carolina. His work has been recognised within IBM by receipt of a 1967 SDD Outstanding Contribution Award for management of three RPG compilers for System/360; a 1983 First Invention Plateau Award; a 1984 CPD Outstanding Contribution Award for "Bresenham's Algorithms"; and a 1985 Hursley Special Contribution Award for his algorithmic and microcode contribution to the IBM PC/GX Graphics Display. Pixel-level algorithms for use in raster graphics displays and programming support has been his technical speciality. He has worked as an engineer, manager, planner, programmer, and administrator at IBM Laboratories in Hursley, Research Triangle Park, Milan, Mohansic and San Jose, and at Headquarter Operations in SCD-Harrison, and World-Trade, White Plains.

## *Dr Rae A. Earnshaw*

Dr Earnshaw holds the BSc and PhD degrees from the University of Leeds, the latter in computer graphics. He has been a faculty staff member since 1968 and is now Head of Computer Graphics. He is a Fellow of the British Computer Society and has been Chairman of the Computer Graphics and Displays Group since 1981. He has been a Visiting Professor at Illinois Institute of Technology, Chicago, USA, Northwestern Polytechnical University, Xian, China, and George Washington University, Washington DC, USA. Dr Earnshaw has acted as a consultant to US companies and the College CAD/ CAM Consortium and given seminars at a variety of UK and US institutions and research laboratories. He was Director of 1985 NATO ASI on "Fundamental Algorithms for Computer Graphics" and Co-Chair of the International Summer Institute on "State of the Art in Computer Graphics" held in Scotland in 1986. He is also Co-Chair of the International Conference on "Parallel Processing for Computer Vision and Display" at the University of Leeds in 1988. His current interests are graphics algorithms, integrated graphics and text, display technology, CAD/CAM, and human-computer interface issues.

## *Professor A. Robin Forrest*

Professor Forrest earned his BSc from the University of Edinburgh in 1965 and PhD from the University of Cambridge in 1968 for work on curves and surfaces for computer-aided design. He has been an Assistant Director of Research in the Computer Laboratory, University of Cambridge, Visiting Professor at the Universities of Syracuse and Utah, and Visiting Scientist at Beijing Institute of Aeronautics and Astronautics, China, and Xerox PARC Image Science Laboratory, USA. He has been Professor of Computing Studies at the University of East Anglia, UK, since 1980. He has acted as a consultant to numerous companies and institutions in the USA and the UK, and given seminars

and lectures at many universities around the world. He is a Fellow of the British Computer Society, and a Fellow of the Institute of Mathematics and its Applications. Current research interests include computational geometry – theoretical and applied, geometric modelling for computer-aided design and manufacture, geometric languages, geometric machines, colour raster graphics, and illustration systems.

*Mr R. John Lansdown*

John Lansdown is Chairman of System Simulation Ltd, UK, a UNIX software and systems house specialising in work for designers and other decision makers. Since 1960 he has been using computers in art and design – initially in his work as an architect – but, more recently, in the fields of choreography, film-making, graphic and product design. He has been the Secretary of the Computer Arts Society since its inception in 1968 and has organised a number of exhibitions and conferences dealing with the impact of computing on the arts. Until June 1986 he was a Senior Research Fellow and Tutor at the Royal College of Art, London, and is currently a Senior Research Fellow at City University, London. He is also a Senior Visiting Fellow at the Department of Architectural Science, University of Sydney, and the Department of Media and Communication Studies, Dorset Institute of Higher Education. With Gillian Crampton-Smith he teaches a post-graduate course in Computer Graphics for Graphics Designers at St Martin's School of Art, London. His publications include works on computer-aided design, architecture, art, graphics, knowledge-based systems, and choreography, whilst his regular column in the Computer Bulletin, "Not Only Computing – Also Art" has been running since 1974.

*Professor Mike L.V. Pitteway*

Professor Pitteway was educated at Felsted School and Queens' College, Cambridge. With the assistance of the EDSAC computers he completed his PhD in 1956 for work in radio physics and then spent two years in the USA as a Harkness Fellow at MIT, Boulder Research Laboratories, Cornell and Stanford Universities. After three years at the Radio Research Station in Slough, UK, he became Director of the Cripps Computing Centre at the University of Nottingham, and was then appointed Professor and Head of the Computer Science Department at Brunel University. At the time of the 1985 Ilkley NATO Institute he was allowed to relinquish his administrative responsibilities and is now a full-time Research Professor of Computer Science. He is a Fellow of the British Computer Society, the Institute of Mathematics and its Applications, the Institute of Physics, and Associate Editor of Computer Graphics, Vision and Image Processing. He has published many papers, and current research interests include pixel-level algorithms, anti-aliasing, analysis of algorithms and computer modelling.

# Acknowledgements

The Directors and Authors gratefully acknowledge the encouragement and assistance provided by the NATO Scientific Affairs Division, Brussels, Belgium, and also the following co-sponsors who supported the venture: British Computer Society (BCS) Technical Committee on Computers in Graphics, Design and Manufacture, BCS Computer Graphics and Displays Group, and the Computer Graphics Society. In addition, Digital Equipment Corporation provided industrial sponsorship in conjunction with their Paris Research Laboratory. Without the generous moral, professional, and financial support provided by these organisations, this Advanced Study Institute on "Theoretical Foundations of Computer Graphics and CAD" would not have been possible. We are very grateful to Dr Craig Sinclair, Director of the NATO ASI Programme, for his advice and guidance during the preparations for the Advanced Study Institute.

Thanks and appreciation are also due to the University of Leeds, the University of East Anglia, Brunel University, System Simulation Ltd and IBM for their encouragement and moral support of the Directors. A special word of thanks is due to Mrs Frances Johnson and Mr Brian Booker in the Commercial Office of the University of Leeds for handling the administrative arrangements during the period leading up to the Institute, and to Mrs Johnson for all her help and assistance during the two weeks of the Conference. Many kinds of problems were dealt with patiently and with a sense of perspective and good humour. The exceptionally good spirit at Il Ciocco was due in large measure to Mrs Johnson's assistance, hospitality and personality. We also thank the staff of Il Ciocco for their support and cooperation in providing the venue and infrastructure for the Conference.

The Invited and Contributing Lecturers spent many hours preparing their lectures and also their written papers included in this volume, and in discussion at the Institute – we are very grateful to them for their contributions and support, without which there would have been no Institute and no book. A number of papers have been revised and extended in the light of discussions in Il Ciocco and we thank the authors for spending even more time to ensure a quality contribution to this volume. We also thank all the delegates who attended and contributed their papers and ideas to the programme; a number of delegate papers are included in this book.

Leeds, England  
1 September 1987

Jack E. Bresenham  
Rae A. Earnshaw  
A. Robin Forrest  
R. John Lansdown  
Mike L.V. Pitteway



# Preface

An Advanced Study Institute on the theme "Theoretical Foundations of Computer Graphics and CAD" was held in Il Ciocco, Italy, 4–17 July, 1987 under the auspices of the Scientific Affairs Division of NATO. The Institute was organised by a Scientific Committee consisting of Dr J.E. Bresenham, Winthrop College, USA, Dr R.A. Earnshaw, University of Leeds, UK, Professor A.R. Forrest, University of East Anglia, UK, and Professor M.L.V. Pitteway, Brunel University, UK. This book contains the formal presentations given at the Institute.

Some 100 participants attended the Institute, representing over 20 countries. These included Australia, Austria, Belgium, Brazil, Canada, China, France, Germany, Greece, India, Israel, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, United Kingdom, USA. Academia, industry, government, and research laboratories were all represented. This contributed greatly to the success of the Institute since it promoted effective interchange of information from one constituency to another, and encouraged the generation of new ideas and perspectives.

The primary objectives of the Institute were to provide an analysis and exposition of the theoretical foundations and bases of computer graphics and computer-aided design in order to give our understanding and exploitation of them a more rigorous and comprehensive basis. This bridging of the gap between theory and practice in a systematic and detailed way is of great interest and significance at the present time. Computer graphics and CAD are rapidly moving from disciplines based on pragmatics to those based on formal methods and the application of greater rigour in association with certification and validation criteria. Software systems and display hardware are now so sophisticated that the deficiencies in the underlying models of the displayed images can be more clearly seen. Consistency, accuracy, robustness, and reliability are some key issues upon which attention is now focussing. Vendors and practitioners may feel satisfied if the results are correct most of the time, but theoreticians and mathematicians are concerned at the lack of rigour and formalism in some of the ad hoc approaches adopted in current work.

As parallel processing methods and techniques bring the disciplines of computer graphics and image processing closer together, each can learn lessons from the other. In addition, it is becoming clear that interface modelling, expert systems techniques, software engineering, and VLSI design, are all areas where a more formal and rigorous approach can produce significant benefits. All these impinge upon the traditional areas of computer graphics and CAD to an increasing extent.

The principal areas of study in this Institute were data structures, geometric algorithms, drawing algorithms, theory and formal methods, geometry and robotics, hardware architectures, curves and surfaces, modelling and CAD/CAM, image generation and reconstruction, graphics systems, human-computer interface, design, and image processing. All this material was covered principally by the Invited and Contributing Lecturers – each being a recognised authority in their particular area. In addition, delegate papers were invited on subjects related to the themes of the Institute. Some 26 papers were accepted after review, and they also appear in this volume. The ordering of the material is divided into twelve main sections, beginning with data structures and then moving on through algorithms, theory and foundations, hardware, modelling; and then on to the higher level areas of image generation, graphics systems, HCI, design, and image processing. It is envisaged that the tutorial and review nature of many of the Invited Lecturers' papers will be very useful in bringing

together a comprehensive summary of current work under one cover. The subject matter of this book is therefore suitable for a wide range of interests, ranging from the advanced student through to researchers and implementors concerned with current issues in computer graphics and CAD. Portions of the book may be used as a standard text since the sections are fairly self-contained and self-explanatory.

The following Invited Lecturers contributed to the programme:

Professor Umberto Cugini, Polytechnic of Milan, Italy  
 Professor David P. Dobkin, Princeton University, USA  
 Professor James D. Foley, George Washington University and CGC, USA  
 Professor Henry Fuchs, University of North Carolina at Chapel Hill, USA  
 Professor Leonidas J. Guibas, Stanford University and DEC SRC, USA  
 Dr Paul J.W. ten Hagen, CWI, Amsterdam, The Netherlands  
 Dr Mark H. Overmars, University of Utrecht, The Netherlands  
 Professor Theo Pavlidis, State University of New York, USA  
 Professor Hanan Samet, University of Maryland, USA  
 Professor Dr Wolfgang Strasser, University of Tubingen, FRG  
 Dr John V. Tucker, University of Leeds, UK

The following Contributing Lecturers each gave at least one presentation:

Dr Varol Akman, CWI, Amsterdam, The Netherlands  
 Professor David Avis, McGill University, Canada  
 Dr David A. Duce, Rutherford Appleton Laboratory, UK  
 Dr Alan E. Middleditch, Polytechnic of Central London, UK  
 Dr Lyle H. Ramshaw, DEC Systems Research Center, USA  
 Professor Dr Alfred Schmitt, Universitat Karlsruhe, FRG  
 Professor Micha Sharir, New York University, USA  
 Professor Godfried T. Toussaint, McGill University, Canada  
 Professor Yehoshua Y. Zeevi, Technion, Israel

The Institute thus had a full programme of lectures and presentations. The Invited Lecturers' papers included in this volume contain the substance of these lectures. In some cases the original paper has been revised in the light of discussions at the Institute. Only the revised and updated papers are included in this volume.

One of the objectives behind the Contributing Lectures was to explore fairly well-defined topic areas in the light of the more broadly based material of the Invited Lecturers. These areas could then be seen as particular instances of the methodology and paradigms that were being developed. Such topic areas included robotics, collision avoidance, stabbing problems, convex polygons, multi-affine maps, ray tracing, Davenport-Schinzel sequences, and vision aspects.

Much of interest was discussed and presented at the Institute and this volume is commended to the reader as a synthesis of current work in the field and a synoptic view of a number of key areas of interest. However, much remains to be done. The Scientific Organising Committee is conscious that their initial goal of providing a comprehensive theoretical framework to under-pin the whole area of computer graphics and CAD was a very ambitious one, and they have only been partly successful. This is in part due to the limitations of time and other resources, but perhaps more importantly, to the fact that the formulation of theoretical bases in computer graphics and CAD has still a long way to go. The application of formal methods and proofs to anything other than the simplest of algorithms or systems is hard and difficult.

The Directors wish to thank the NATO Scientific Affairs Division for their financial support of the original proposal, and also Dr Sinclair for his advice and guidance during the period leading up to the

Advanced Study Institute. We thank the academic co-sponsors: British Computer Society (BCS) Technical Committee on Computers in Graphics, Design and Manufacture, BCS Computer Graphics and Displays Group, and the Computer Graphics Society. We also thank Digital Equipment who provided generous industrial sponsorship in conjunction with their Paris Research Laboratory. Thanks and appreciation are also due to the University of Leeds, the University of East Anglia, Brunel University, System Simulation Ltd and IBM for their encouragement and moral support of the Directors; to Mrs Frances Johnson and Mr Brian Booker in the Commercial Office of the University of Leeds for handling all the administrative arrangements in connection with the Institute; to the staff at Il Ciocco International Centre, Italy, for their support and cooperation; to the Invited and Contributing Lecturers for giving of their time so freely (and also their employers for allowing them to come); and to all the delegates who attended and contributed their ideas to the Institute.

R.A. Earnshaw  
1 September 1987

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 Mr R. John Lansdown, System Simulation Ltd, UK  
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