McGRAW-HILL ELECTRICAL AND ELECTRONIC ENGINEERING SERIES

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Electrical Engineering Science

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PREFACE

This book provides an introduction to electrical engineering as an applied science. The treatment presumes a familiarity with the concepts of differential and integral calculus and a moderate competence in differentiating and integrating simple functions. An elementary college-level physics course that includes electricity and magnetism is desirable as a background.

The aims of this volume are to establish the scientific foundation for electrical engineering via the concepts of electricity and magnetism, to apply these concepts in developing the fundamentals of energy conversion and circuit theory, and to carry forward, in a continuous and integrated way, a modern treatment of network analysis. It is intended to serve as a foundation for subsequent courses such as electronic circuits, energy conversion, advanced network analysis, and network synthesis, and has been written with the purpose of providing the student with a unifying point of view for these varied topics. Although this book starts with electricity and magnetism, a course based upon it will not serve as a substitute for a regular course in electromagnetic theory. In fact, such a course can profitably be taken by the student either concurrently or later in his program. In the treatment of electromagnetic theory given in this text, the physical ideas are emphasized and, by the use of elementary calculus, are formulated in the simplest manner consistent with correctness. Such a treatment, which emphasizes accurate understanding of the physical ideas, should help the student when he later studies the subject at a level where the physical ideas are manipulated more adroitly with the aid of more powerful mathematical techniques.

The book starts with a concise but careful treatment of electromagnetic theory, presented with due regard for the limited mathematical background of the students, and leading to Maxwell's equations in the integral form. The ideas of electromagnetic theory are then applied to two areas: to the conversion of energy, with emphasis on electromechanical transducers, and to the theory of lumped circuits, with particular attention to the relation between the physical concepts and their mathematical formulation. Thus, the treatment proceeds from field ideas to circuits and physical apparatus, and to their mathematical models. With the circuit relations formulated, attention turns to the analysis of networks, *starting with network topology and extending through pole-zero ideas.* The treatment stops just short of the Laplace transform.

There is an attempt to keep the mathematical treatment in proper relationship with the physical ideas. Qualitative analyses are used side by side with quantitative ones. Simple nonlinear circuits are treated wherever possible so as to keep linear circuit theory in its proper perspective. The dynamics of electromechanical systems are treated briefly, but perhaps enough so that the student is at least aware of the subject.

The material has been used in class by the authors in substantially its present form for several years. A major acknowledgment of thanks must go to the students who provided the feedback that guided the final writing of the manuscript.

Certain sections, not essential to the understanding of subsequent portions of the text, may be omitted at the discretion of the instructor. These are:

Sections 2.3 through 2.5, which treat semiconductors, the *p-n* junction, and the semiconductor rectifier.

Section 3.16, which analyzes the propagation of a simple electromagnetic wave.

Sections 4.15 and 4.16, which discuss in detail the limitations of small-circuit theory as applied to a simple inductor and a simple capacitor.

In Chapter 5, the analysis of d-c machines (Secs. 5.15 through 5.19); or the entire treatment of rotating machines (Secs. 5.11 through 5.19); or Chapter 5 in its entirety.

Section 8.11, which treats the output of a linear system as the superposition of responses to step inputs.

Section 13.6, which expresses the real and reactive powers at the terminals of a network in terms of the real and reactive powers of the elements.

Section 15.2, which deals with the construction of the frequency response characteristics for circuits with multiple zeros and poles on the negative real axis.

Chapter 17, if the instructor wishes to postpone the treatment of the transformer to another course.

Chapter 18, if polyphase circuits are to be treated elsewhere.

The authors acknowledge the aid given to them by their colleagues, particularly I. B. Pyne, W. H. Surber, Jr., G. Warfield, and P. J. Warter of the department of electrical engineering, and C. O. Alley, Jr., of the department of physics. The authors also thank Miss Florence Armstrong, who not only typed the manuscript but also aided in its preparation in numerous other ways.

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CONTENTS

CHAPTER 1

ELECTRICAL FORCES AND FIELDS

1.1. Forces and Charges. One of the basic laws of mechanics concerns the gravitational attraction between material bodies. Experiment shows that this force can be expressed in the form $F = Gm_1m_2/d^2$, where m_1 and m_2 are the masses of the respective bodies, d is the distance between their centers of mass, and G is the so-called gravitational constant. It was Newton who first established the fact that the gravitational force varies inversely as the square of the distance, and it was Cavendish along with others who measured the value of the constant G. It has been found, however, that under certain conditions material bodies exert forces on each other which are many times greater than the forces that can be attributed to gravitation. Furthermore, forces of repulsion as well as attraction are observed. When such forces occur, the agents responsible for the forces are called *electric charges.*

Electric charges are known only by the forces that they exert on each other. The forces are found to depend on the velocities of the charges as well as on their relative positions. By means of these forces, one set of charges can do work on another set, and the resulting transfer of energy can be controlled so as to perform useful functions.

All of the phenomena called electrical or magnetic, and all of the applications based on these phenomena, can be attributed directly to the forces exerted between charges.

The spinning electrons in the atoms of a bar magnet exert forces on the spinning electrons in a piece of iron, with the result that the two bodies are drawn toward each other.

In a direct-current (d-c) generator, moving electrons in the field winding exert forces on the free electrons within the moving armature winding and thus cause a flow of current in the armature circuit. In a motor, the moving electrons in the field and armature circuits react on each other and cause a torque which turns the armature.

The electrons flowing in a resistive conductor exert forces on the charges of the atoms in the solid and transfer some of their energy to the atomic lattice, where it takes the form of vibrations which we sense as heat.

In a vacuum tube, electrons are liberated from the cathode by heat and are attracted by charges on the anode. Their flight to the anode is further controlled by forces that are exerted by charges on the grid. The electrons flowing through the tube exert forces on the free electrons in the circuits connected to the vacuum tube, and the resulting motion of charge can be controlled so as to provide amplification, oscillation, and so on. An oscillatory motion can be imparted to the free electrons in an antenna, which is a structure designed to permit the charges to exert appreciable forces on other charges a considerable distance away. After an appropriate time has gone by (for the forces are felt only after a time lag corresponding to the speed of light in free space), the electrons in various receiving antennas are set into oscillation, and they in turn impress forces on free electrons in the connecting wires, and so on.

In any utilization of electricity, whether in power, communication, control, or other fields, the problems are basically the production and control of the forces and energy exchange between charges. Through these forces, electrical energy is transmitted and controlled and is transformed into other forms, such as, for example, mechanical energy.

Electromagnetic theory is concerned with the formulation of the laws of these forces as deduced from experiment and then with the consequences of these laws as applied to various phenomena. It is concerned with physically demonstrable facts and with useful methods of visualizing and handling these facts.

1.2. Forces and Fields. The force exerted between charges depends on whether or not the charges are in motion. In determining these forces, it is convenient to introduce a concept which forms a halfway stopping point in the thinking (and computing) process. This concept is that of *fields*. The word *field* implies an effect which is distributed throughout a region of space, as distinguished from an effect which is concentrated at a point.

A stationary charge is visualized as the source of a field of influence, known as the *electric field*, which extends through the region surrounding the charge. The forces exerted on other charges are then visualized as being caused by this field. In computing the force between two charges, one computes the electric field caused by one charge and then finds the force that this field exerts on a second charge.

If the charges are set into motion, the forces are changed. To describe this change, we introduce a second kind of field, known as the *magnetic field*, to supplement the action of the electric field. The magnetic field is then visualized as exerting an *additional* force on any charge that is in motion. Instead of computing the forces between moving charges directly, one first computes the electric and magnetic fields caused by one moving charge and then finds the force which each type of field exerts on the second moving charge.

When viewed in this way, the fields seem to have no reality and appear as only convenient fictions. Whether we consider the fields to have reality or not, the advantages of the concept are most apparent when we consider the forces exerted by an accelerating charge on other charges which perhaps are far away. Out of the field concept comes quite naturally the idea of the propagation of energy in an electromagnetic wave. The wave consists of electric and magnetic fields traveling together at a finite velocity and is detectable by the forces that it exerts on electric charges. Here one might think of "retarded action at a distance" without the interposition of the traveling wave, but the concepts of waves and fields are most convenient. Hereafter we shall not worry about the reality of the fields but shall accept them as conveniences and even talk about them as realities.

1.3. The MKS System of Units. In this book we shall use the meter-kilogram-second (mks) system of units. In the mks system the unit of length is the *meter*, the unit of mass is the *kilogram*, and the unit of time is the *second*. The unit of force is that which will give a mass of one kilogram an acceleration of one meter per second per second, and this is named the *newton*. The unit of energy is obtained by multiplying the unit of force times the unit of length, giving the *newton-meter*, which turns out to be exactly the same as the *watt-second*, or *joule*, of electrical energy. This is convenient when one deals, as we must, with both electrical and mechanical energy and is one of the reasons for our choice of the system. Also, the sizes of the units in the mks system are particularly convenient for many purposes.

We shall use the mks system in all equations. Sometimes, in specifying data, particularly in problems, it will be more suitable to use other units, but we shall always convert them to the mks system before substituting in equations. Because the mks system does not satisfy all needs perfectly and is not in universal use, the student should gain some familiarity with other systems. A comparison of a few units is given below. A table of conversion factors will be found in Appendix A.

Length:	1 meter = 10^2 centimeters
Mass:	1 kilogram = 10^3 grams
Force:	1 newton = 10^5 dynes = 0.2248 pound

1.4. Forces between Stationary Charges in a Vacuum. Coulomb's Law. Imagine two small bodies that are charged electrically and are isolated in a region where there are no other electrical charges, either free or bound in atoms. This region, therefore, exclusive of the small charged bodies, must be a vacuum. If the charged bodies are held stationary, one finds that they exert equal and opposite electric forces on each other and that these forces act on a line joining the two bodies, as shown in Fig. 1.1. Here we have represented each force by a vector whose length represents the magnitude of the force and whose direction shows the direction in which the force acts. The forces will repel the bodies if the charges are of like sign, which is the case shown in the figure. If the charges are of opposite signs, the forces are in such a direction as to attract the bodies toward one another.

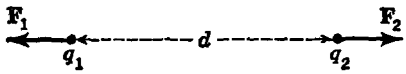

FIG. 1.1. Forces between two isolated stationary charged bodies. The charges q_1 and q_2 are of the same sign.

Experiment shows that, if the size of the charged bodies is small compared with the distance between them, the force is proportional to the product of the two charges and varies inversely with the square of the distance. The geometry of the situation is frequently idealized by calling the charged bodies "point charges." Measurement of the constant of proportionality shows that, to six significant figures, it is equal to

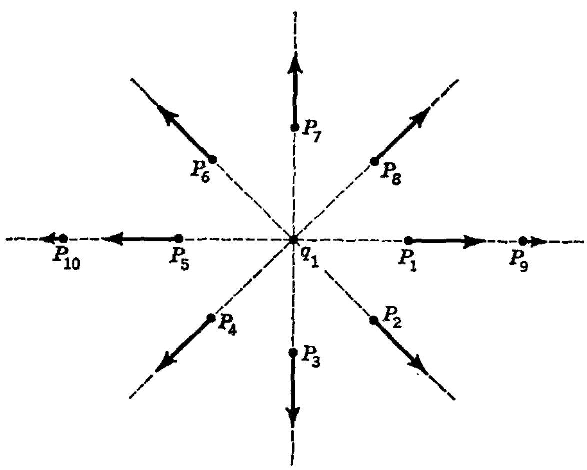

FIG. 1.2. The electric forces exerted at several positions on a second charge q_2 by a stationary charge q_1.

8.98740×10^9 mks units. Within 2 parts in 1000 we can write this as 9×10^9 mks units and express the force between "point" charges as

$$F = 9 \times 10^9 \frac{q_1 q_2}{d^2} \qquad \text{newtons} \tag{1.1}$$

where q_1 and q_2 are the charges in coulombs and d is the distance between them in meters. Equation 1.1, together with the remarks concerning the direction of the force, are known as *Coulomb's law*. It should be noted that each electron has a negative charge of 1.602×10^{-19} coulomb. Therefore, 1 coulomb of charge corresponds to the charge on an enormously large number of electrons, namely, 6.24×10^{18} electrons.

Imagine that one of the charges, q_1, is held stationary and that the second charge is moved to various positions. The force exerted on the second charge at each of several points marked P_1, P_2, etc., is shown schematically in Fig. 1.2. In each case the magnitude of the force is given by Eq. 1.1, and its direction is indicated by the direction of the vector.

1.5. The Electric Field. We now use the concept of the *electric field.* As was mentioned in Sec. 1.2, the charge q_1 is visualized as the source of a field of influence which extends through the region surrounding the charge. The force exerted on q_2 is regarded as being caused by this field.

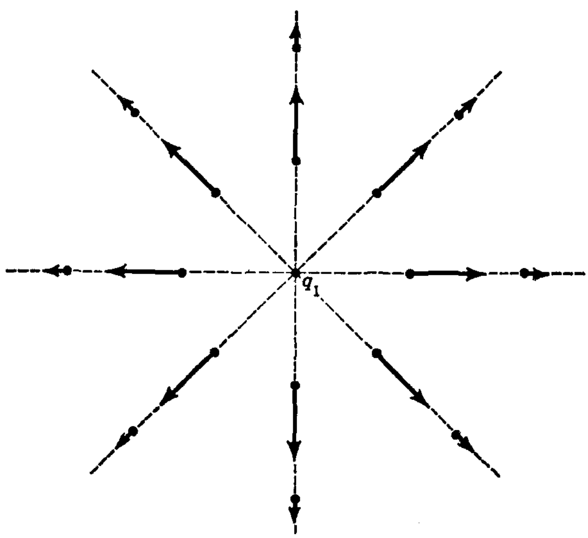

FIG. 1.3. The electric field at a number of points due to one point charge q_1.

The strength of the electric field of q_1 will be defined as the force which q_1 exerts on a unit amount (1 coulomb) of q_2; that is, it is the force per unit q_2. The force was given by Eq. 1.1. Dividing this by q_2, we obtain the electric field at a distance d, caused by the isolated point charge q_1:

$$\mathcal{E}_1 = \frac{F_2}{q_2} = 9 \times 10^9 \frac{q_1}{d^2} \qquad \text{newtons/coulomb} \tag{1.2}$$

The name given to $\mathcal{E}$ is *electric field intensity.*

The force exerted on q_2 by the field of q_1 is now given by

$$F_2 = q_2\mathcal{E}_1 \qquad \text{newtons} \tag{1.3}$$

We shall show later that the unit of electric field intensity, newtons/coulomb, is identical with volts/meter. Like the force itself, the electric field intensity at any point is a vector and will, in general, vary in magnitude and direction from point to point. It is called a *field* because it extends through a region of space. Figure 1.3 shows vectors representing the electric field of the single charged particle q_1 as measured by the force on a test charge at a number of points (compare Fig. 1.2).

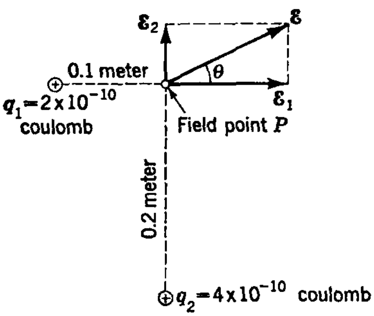

FIG. 1.4. The resultant field of two point charges.

If there are a number of charges, the resultant electric field can be found by adding together vectorially the fields for each of the separate charges. Several examples of this are given below.

Example 1. *Point Charges.* Figure 1.4 shows two point charges q_1 and q_2. We wish to find the resultant field at the *field point P*, as caused by the two charges. The field of q_1 at the field point is

$$\mathcal{E}_1 = 9 \times 10^9 \frac{2 \times 10^{-10}}{(0.1)^2} = 180 \text{ newtons/coulomb}$$

The field of q_2 at the field point is

$$\mathcal{E}_2 = 9 \times 10^9 \frac{4 \times 10^{-10}}{(0.2)^2} = 90 \text{ newtons/coulomb}$$

The magnitude of the total electric field at the field point is

$$\mathcal{E} = \sqrt{(180)^2 + (90)^2} = 200.3 \text{ newtons/coulomb, or volts/meter}$$

The inclination of the resultant field from the horizontal is found from

$$\tan\theta = \tfrac{90}{180} = 0.500$$

from which $\theta = 26.57°$. If a third point charge, say $q_3 = 3 \times 10^{-10}$ coulomb, is placed at the point P, and if the presence of this charge does not change the positions of q_1 and q_2, the force on the third charge caused by the first two will be

$$F_3 = q_3\mathcal{E} = 3 \times 10^{-10} \times 200.3 = 6.01 \times 10^{-8} \text{ newton}$$

This force will act at angle $\theta = 26.57°$ from the horizontal.

Example 2. *A Line Charge.* In Fig. 1.5, a set of charges are arranged uniformly in a straight line. The charge per meter length of line will be designated by ρ_L coulombs/meter. We are interested in finding the electric field at a field point located at a distance r from the line. We shall presume that the individual charges in the line are very close together compared with the distance r in which we are interested; then, without great error, we can analyze a *model* in which the charge is distributed along the line in a uniform, continuous way. Next, we shall assume that the line of charge is very long compared with the distance r and that the field point is well away from the ends; then, we can get a result that is nearly correct by letting the model be of infinite length. Because we have assumed a continuous distribution of charge, the techniques of the calculus are appropriate to the analysis.

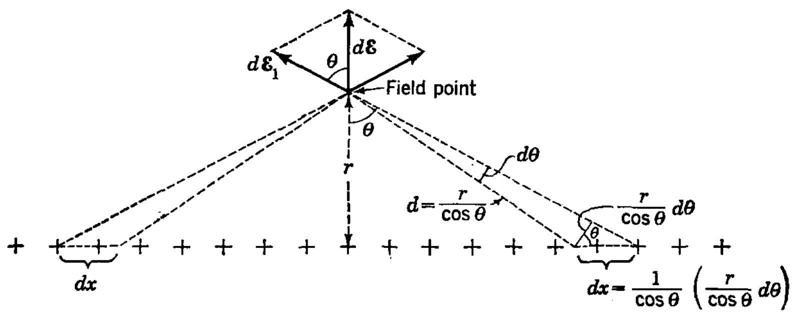

FIG. 1.5. The electric field of a uniform line charge.

Now, we consider two symmetrically disposed elements of length dx, as shown in Fig. 1.5. The charge included in each is $\rho_L\,dx$. When we add their fields vectorially at the field point, the horizontal components cancel out and the radial components add. The electric field caused by one element dx is, by applying Eq. 1.2,

$$\begin{aligned} d\mathcal{E}_1 &= 9 \times 10^9 \frac{\rho_L(r/\cos^2\theta)\,d\theta}{(r/\cos\theta)^2} \\ &= 9 \times 10^9 \frac{\rho_L\,d\theta}{r} \end{aligned}$$

The field caused by both charged elements is then

$$\begin{aligned} d\mathcal{E} &= 2\,d\mathcal{E}_1 \cos\theta \\ &= 18 \times 10^9 \frac{\rho_L}{r} \cos\theta\,d\theta \end{aligned}$$

Now we sum up the contributions of all elements by integrating this function of θ from 0 to $\pi/2$ (we take account of both halves of the line in this way because we have included the effects of both elements dx simultaneously):

$$\begin{aligned} \mathcal{E} &= 18 \times 10^9 \frac{\rho_L}{r} \int_0^{\pi/2} \cos\theta\,d\theta \\ &= 18 \times 10^9 \frac{\rho_L}{r} \sin\theta \Big]_0^{\pi/2} \\ &= 18 \times 10^9 \frac{\rho_L}{r} \qquad \text{newtons/coulomb, or volts/meter} \end{aligned}$$