

MATHEMATICAL WRITINGS OF L. C. HSU

Vol. II Combinatorics & Computing

徐利治数学作品集
第II卷 组合分析与计算



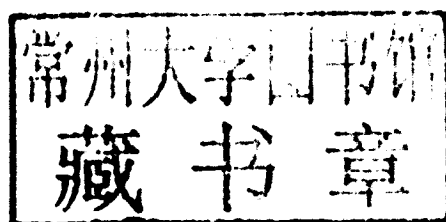
大连理工大学出版社
DALIAN UNIVERSITY OF TECHNOLOGY PRESS

MATHEMATICAL WRITINGS OF L. C. HSU

Vol. II Combinatorics & Computing

徐利治数学作品集

第II卷 组合分析与计算



大连理工大学出版社

2010年7月

图书在版编目(CIP)数据

徐利治数学作品集. 第Ⅱ卷: 组合分析与计算 =
Mathematical Writings of L. C. Hsu——Vol. II :
Combinatorics & Computing; 英文 / 徐利治著. — 大
连: 大连理工大学出版社, 2010. 7
ISBN 978-7-5611-5631-5

I. ①徐… II. ①徐… III. ①数学—文集—英文
IV. ①O1-53

中国版本图书馆 CIP 数据核字(2010)第 131431 号

大连理工大学出版社出版

地址: 大连市软件园路 80 号 邮政编码: 116023

电话: 0411-84708842 邮购: 0411-84703636 传真: 0411-84701466

E-mail: dutp@dutp.cn URL: <http://www.dutp.cn>

大连金华光彩色印刷有限公司印刷

大连理工大学出版社发行

幅面尺寸: 185mm×260mm 插页: 1 印张: 26.75 字数: 612 千字
2010 年 7 月第 1 版 2010 年 7 月第 1 次印刷

责任编辑: 刘新彦 王 伟

责任校对: 婕 琳

封面设计: 李昕阳

ISBN 978-7-5611-5631-5

定 价: 268.00 元(共两卷)



徐利治近照

Editorial Committee

Chairman Wang R. H.

Vice-Chairman Lu Y. F. Jin Y. W.

Committee Members

Li Z. K.	Shi G. Y.	Wang J.
Wang Y.	Wu W.	Yu B.
Zhang H. Q.	Zhang L. W.	

Editors Liu X. Y. Yu H. D.

Editor's Preface

We have the great pleasure to edit Hsu's mathematical writings which had been published within the years 1944—2009. Professor L. C. Hsu was born in the September of 1920, and has been doing and teaching mathematics for more than 60 years. A proper compilation of Hsu's papers into 2 volumes is now first published just before the birth-date of Hsu's 90 years old, so Hsu gladly asks us to convey his sincere thanks to the university authority of Dalian University of Technology for the effective support of publication of his mathematics works. As editors we also sincerely wish the interested readers would possibly get some imaginations as well as inspirations from some selective reading of Hsu's writings.

Liu Xin-yan, Yu Hua-dong
July, 2010

Name List of some Co-authors and Joint Authors Appearing in Vol. I

Chou Y. -S, Chui C. K. , Guo S. -S. , He T. -X. , Hsu L. -P. ,
Lin L. -W. , Shiue P. J. -S. , Sun G. -R. , Tomkins R. J. , Torney D. C. ,
Wang C. -L. , Wang X. -H. , Wang Y. -J. , Yang J. -X. , Yin D. -S.

Name List of some Co-authors and Joint Authors Appearing in Vol. II

Brown T. C. , Bundschuh P. , Chou W. -S. , Chu W. -C. , Chung K. L. ,
Corcino C. B. , Corcino R. B. , Gould H. W. , He T. -X. , Hsu F. -J. ,
Jiang M. -S. , Luo X. -N. , Mullen G. L. , Shiue P. J. -S. , Tan E. L. ,
Wang J. , Wang Y.

Main Topics Concerned in Vol. I

- (1) Asymptotics
- (2) Approximation
- (3) Numerical Integration
- (4) Integral Representation
- (5) Series Summations
- (6) Interpolation
- (7) Others

Main Topics Concerned in Vol. II

- (1) Inversion Formulas
- (2) Möbius Inversions
- (3) Stirling Numbers, etc
- (4) Series Summation Methods
- (5) Others

Contents

Vol. I Analysis & Computing

L. C. Hsu. A survey of some recent developments of approximation theory in China. Approximation Theory IV (Proceeding of Texas International Conference), 1983; 123-151	3
L. C. Hsu. A theorem on the asymptotic behavior of a multiple integral. Duke Mathematical Journal, 1948, 15(3); 623-632	32
L. C. Hsu. Approximations to a class of double integrals of functions of large numbers. American Journal of Mathematics, 1948, 70(4); 698-708	42
L. C. Hsu. On the asymptotic evaluation of a class of multiple integrals involving a parameter. American Journal of Mathematics, 1951, 73; 625-634	53
L. C. Hsu. An asymptotic expression for an integral involving a parameter. Science Record, 1949, 2(4); 339-345	63
L. C. Hsu. Concerning a kind of integrals of complex-valued functions of large numbers. J. Math. Res. & Exposition, 1995, 15(2); 159-166	70
L. C. Hsu. A general approximation method of evaluating multiple integrals. Tôhoku Mathematical Journal, 1957, 9; 45-55	78
L. C. Hsu. A refinement of the line integral approximation method and its application. Science Record, 1958, 2(6); 193-196	89
L. C. Hsu. A theorem concerning an asymptotic integral. Bulletin Calcutta Math. Society, 1951, 43; 109-112	93
L. C. Hsu, L. W. Lin. Two new methods for the approximate calculation of multiple integrals. Acta Mathematica Academiae Scientiarum Hungaricae, 1958, 9; 279-290	97
L. C. Hsu. Concerning the numerical integration of periodic function of several variables. Acta Scientiarum Mathematicarum, 1959, 20(4); 230-233	109
L. C. Hsu. Some approximation formulas for the integration of violently oscillating	

functions and of periodic functions. Science Record, 1959, 3(11); 544-549	113
L. C. Hsu. Note on the numerical integration of periodic functions and of partially periodic functions. Numerische Mathematik, 1961(3); 169-173	119
L. C. Hsu. On the numerical integration of non-periodic functions of several variables. Numerische Mathematik, 1962(4); 329-335	124
L. C. Hsu. Concerning an expansion formula for a type of integrals. Annales Polonici Mathematici, 1961, 11; 7-12	131
L. C. Hsu. A few expansion formulas for the approximate integration over plane regions. Acta Mathematica Academiae Scientiarum Hungaricae, 1962, 13; 387-392 ...	137
L. C. Hsu. A reduction formula for the numerical integration of periodic functions of several variables. Acta Mathematica Academiae Scientiarum Hungaricae, 1962, 13; 383-386	143
L. C. Hsu. On a method for expanding multiple integrals in terms of integrals in lower dimensions. Acta Mathematica Academiae Scientiarum Hungaricae, 1963, 14; 359-367 ...	147
L. C. Hsu, Y. S. Chou. Two classes of boundary type cubature formulas with algebraic precision. Calcolo (Italy), 1986, 23(3); 227-248	156
L. C. Hsu. Approximate integration of rapidly oscillating functions and of periodic functions. Proc. Camb. Phil. Soc., 1963, 59; 81-88	178
L. C. Hsu. Asymptotic expansions of multiple integrals of rapidly oscillating functions. Approximation, Optimization and Computing: Theory and Applications, IMACS, A. G. Law and C. L. Wang (eds), Elsevier Science Publishers B. B. (North-Holland), 1990; 9-11	186
L. C. Hsu, R. J. Tomkins, C. -L. Wang. A quadrature method for a class of strongly oscillatory infinite integrals. BIT (Sweden), 1990, 30; 114-125	189
L. C. Hsu, J. X. Yang. A class of multivariate rational interpolation formulas. Journal of Computational Mathematics, 1984, 2(2); 164-169	201
L. C. Hsu, T. -X. He. On a kind of multivariate rational interpolation. Numerical Mathematics; A Journal of Chinese Universities, 1986, 2; 144-152	207
L. C. Hsu, J. X. Yang. On a method of constructing interpolation formulas via inverse series relations. J. Math. Res. & Exposition, 1982, 2(2); 113-126	216
T. -X. He, L. C. Hsu, P. J. -S. Shiue. On an extension of Abel-Gontscharoff's expansion formula. Analysis in Theory and Applications, 2005, 21(4); 359-369	230
T. -X. He, L. C. Hsu, P. J. -S. Shiue. Multivariate expansions associated with Sheffer-type polynomials and operators. Bulletin of the Institute of Mathematics (Academia Sinica), 2006, 1(4); 451-473	241
T. -X. He, L. C. Hsu, P. J. -S. Shiue. Symbolization of generating functions; an application of the Mullin-Rota theory of binomial enumeration. Computers & Mathematics	

with Applications, 2007, 54:664-678	264
L. C. Hsu. Power-type generating functions. Colloquia Mathematica Societatis János Bolyai, 1990, 58:405-412	279
L. C. Hsu. Certain asymptotic expansions for Laguerre polynomials and Charlier polynomials. Approximation Theory and its Applications, 1995, 11(1):94-104	287
T.-X. He, L. C. Hsu, P. J.-S. Shiue. On Abel-Gontscharoff-Gould's polynomials. Analysis in Theory and Applications, 2003, 19(2):166-184	298
X. H. Wang, L. C. Hsu. A summation formula for power series using Eulerian fractions. The Fibonacci Quarterly, 2003, 41(1):23-30	317
T.-X. He, L. C. Hsu, P. J.-S. Shiue, D. C. Torney. A symbolic operator approach to several summation formulas for power series. Journal of Computational and Applied Mathematics, 2005, 177:17-33	325
T.-X. He, L. C. Hsu, P. J.-S. Shiue. A symbolic operator approach to several summation formulas for power series II. Discrete Mathematics, 2008, 308:3427-3440	342
T.-X. He, L. C. Hsu, P. J.-S. Shiue. Convergence of the summation formulas constructed by using a symbolic operator approach. Computers & Mathematics with Applications, 2006, 51:441-450	356
T.-X. He, L. C. Hsu, D. S. Yin. A pair of operator summation formulas and their applications. Computers & Mathematics with Applications, 2009, 58:1340-1348	366
L. C. Hsu. The representation of functions of bounded variation by singular integrals. Duke Mathematical Journal, 1951, 18(4):837-844	375
L. C. Hsu. A generalization of the Riemann-Lebesgue theorem. Bulletin of the Calcutta Mathematical Society, 1956, 48(4):191-195	383
L. C. Hsu. Concerning the condition of uniform boundedness for a type of scalar-to-vector transformations. Journal of the Indian Mathematical Society, 1957, 21(3&4):115-126	388
L. C. Hsu. An estimation for the first exponential formula in the theory of semi-groups of linear operations. Czechoslovak Mathematical Journal, 1960, 10(85):323-328 ...	400
L. C. Hsu. Approximation of non-bounded continuous functions by certain sequences of linear positive operators or polynomials. Studia Mathematica, 1961, 21:37-43	406
L. C. Hsu. On a kind of extended Fejér-Hermite interpolation polynomials. Acta Mathematica Academiae Scientiarum Hungaricae, 1964, 15:325-328	413
L. C. Hsu. The polynomial approximation of continuous functions defined on $(-\infty, +\infty)$. Czechoslovak Mathematical Journal, 1959, 9(84):574-578	417
L. C. Hsu. Note on an asymptotic expansion of the n th difference of zero. Annals of Mathematical Statistics, 1948, 19:273-277	422
L. C. Hsu, L. P. Hsu. On a new class of approximating polynomials for real functions. Science Record, 1959, 3(2):65-70	427

C. K. Chui, T. -X. He, L. C. Hsu. On a general class of multivariate linear smoothing operators. <i>Journal of Approximation Theory</i> , 1988, 55(1):35-48	433
C. K. Chui, T. -X. He, L. C. Hsu. Asymptotic properties of positive summation-integral operators. <i>Journal of Approximation Theory</i> , 1988, 55(1):49-60	447
S. -S. Guo, L. C. Hsu. Inverse theorems for a certain class of operators. <i>Demonstratio Mathematica</i> , 1988, 21(3):745-760	459
L. C. Hsu, G. -R. Sun. The quasi-Duhamel principle and its applications. <i>J. Math. Res. & Exposition</i> , 1990, 10(4):495-499	475
L. C. Hsu, Y. J. Wang. A refinement of Hilbert's double series theorem. <i>J. Math. Res. & Exposition</i> , 1991, 11(1):143-144	480
L. C. Hsu, G. -R. Sun. On Poincaré's remark and a kind of nonstandard measure defined on $^*(R^n)$. <i>J. Math. Res. & Exposition</i> , 2001, 21(2):159-164	482
L. C. Hsu. On Poincaré-type continuum and certain of its basic properties. <i>J. Math. Res. & Exposition</i> , 2008, 28(1):11-21	488

Vol. II Combinatorics & Computing

L. C. Hsu. Abstract theory of inversion of iterated summations. <i>Duke Mathematical Journal</i> , 1947, 14(2):465-473	501
H. W. Gould, L. C. Hsu. Some new inverse series relations. <i>Duke Mathematical Journal</i> , 1973, 40(4):885-891	510
L. C. Hsu. Self-reciprocal functions and self-reciprocal transforms. <i>J. Math. Res. & Exposition</i> , 1981(2):119-138	517
F. J. Hsu, L. C. Hsu. A unified treatment of a class of combinatorial sums. <i>Discrete Mathematics</i> , 1991(90):191-197	537
W. C. Chu, L. C. Hsu. On some classes of inverse series relations and their applications. <i>Discrete Mathematics</i> , 1993, 123:3-15	544
W. -S. Chou, L. C. Hsu, P. J. -S. Shiue. Application of Faà di Bruno's formula in characterization of inverse relations. <i>Journal of Computational and Applied Mathematics</i> , 2006, 190:151-169	557
L. C. Hsu. Note on an abstract inversion principle. <i>Proc. Edinburgh Math. Soc.</i> , 1954, 9(2):71-73	576
L. C. Hsu. Generalized Möbius-Rota inversion theory associated with non-standard analysis. <i>Science Exploration</i> , 1983, 3(1):1-7	579
P. Bundschuh, L. C. Hsu, P. J. -S. Shiue. Generalized Möbius inversion-theoretical and computational aspects. <i>The Fibonacci Quarterly</i> , 2006, 44(2):109-116	586

L. C. Hsu. Generalized Möbius inversion with applications to integral equations and interpolation process. <i>J. Math. Res. & Exposition</i> , 1987(2); 335-350	594
T. -X. He, L. C. Hsu, P. J. -S. Shiue. On generalized Möbius inversion formulas. <i>Bull. Austral. Math. Soc.</i> , 2006, 73; 79-88	610
K. L. Chung, L. C. Hsu. A combinatorial formula and its application to the theory of probability of arbitrary events. <i>The Annals of Mathematical Statistics</i> , 1945, 16; 91-95	620
L. C. Hsu, J. Wang. Some Möbius-type functions and inversions constructed via difference operators. <i>Tamkang Journal of Mathematics</i> , 1998, 29(2); 89-99	625
L. C. Hsu. Generalized Stirling number pairs associated with inverse relations. <i>The Fibonacci Quarterly</i> , 1987, 25(4); 346-351	636
L. C. Hsu. On Stirling-type pairs and extended Gegenbauer-Humbert-Fibonacci polynomials. <i>Applications of Fibonacci Numbers</i> , 1993, 5; 367-377	642
L. C. Hsu. Theory and application of generalized Stirling number pairs. <i>J. Math. Res. & Exposition</i> , 1989, 9(2); 211-220	653
L. C. Hsu. Some theorems on Stirling-type pairs. <i>Proceedings of the Edinburgh Mathematical Society</i> , 1993, 36; 525-535	663
L. C. Hsu, G. L. Mullen, P. J. -S. Shiue. Dickson-Stirling numbers. <i>Proceedings of the Edinburgh Mathematical Society</i> , 1997, 40; 409-423	674
L. C. Hsu, P. J. -S. Shiue. A unified approach to generalized Stirling numbers. <i>Advances in Applied Mathematics</i> , 1998, 20; 366-384	689
C. B. Corcino, L. C. Hsu, E. L. Tan. Asymptotic approximations of r -Stirling numbers. <i>Approx. Theory & its Appl.</i> , 1999, 15(3); 13-25	708
T. C. Brown, L. C. Hsu, J. Wang, P. J. -S. Shiue. On a certain kind of generalized number-theoretical Möbius function. <i>Math. Scientist</i> , 2000(25); 1-6	721
R. B. Corcino, L. C. Hsu, E. L. Tan. Combinatorial and statistical applications of generalized Stirling numbers. <i>J. Math. Res. & Exposition</i> , 2001, 21(3); 337-343	727
L. C. Hsu. A summation rule using Stirling numbers of the second kind. <i>The Fibonacci Quarterly</i> , 1933, 31(3); 256-262	734
T. -X. He, L. C. Hsu, P. J. -S. Shiue. The Sheffer group and the Riordan group. <i>Discrete Applied Mathematics</i> , 2007, 155; 1895-1909	741
L. C. Hsu. Note on a combinatorial algebraic identity and its application. <i>The Fibonacci Quarterly</i> , 1973, 11(5); 480-484	756
L. C. Hsu, M. S. Jiang. Comments on Egorychev's book "integral representation and the computation of combinatorial sums". <i>Applied Mathematics</i> , 1988(1-2); 163-165 ...	761
L. C. Hsu. Finding some strange identities via Faa di Bruno's formula. <i>J. Math. Res. & Exposition</i> , 1993, 13(2); 159-165	764
L. C. Hsu, P. J. -S. Shiue. On a combinatorial expression concerning Fermat's last theo-	

Mathematical Writings of L. C. Hsu

rem. <i>Advances in Applied Mathematics</i> , 1997, 18; 216-219	771
L. C. Hsu, P. J. -S. Shiue. On certain summation problems and generalizations of Eulerian polynomials and numbers. <i>Discrete Mathematics</i> , 1999, 204; 237-247	775
L. C. Hsu, E. L. Tan. A refinement of de Bruyn's formulas for $\sum a^k k^p$. <i>The Fibonacci Quarterly</i> , 2000, 38(1); 56-60	786
L. C. Hsu, P. J. -S. Shiue. Cycle indicators and special functions. <i>Annals of Combinatorics</i> , 2001, 5; 179-196	791
L. C. Hsu, P. J. -S. Shiue, Y. Wang. Notes on a conjecture of singmaster. <i>The Fibonacci Quarterly</i> , 1995, 33(5); 392-397	809
L. C. Hsu. Some remarks on a generalized Newton interpolation formula. <i>The Mathematics Student</i> , 1951, 19(1&2); 25-29	815
L. C. Hsu. A combinatorial proof of an inequality due to Hua. <i>The Mathematics Student</i> , 1955, 23(3); 97-100	820
L. C. Hsu, X. -N Luo. On a two-sided inequality involving Stirling's formula. <i>J. Math. Res. & Exposition</i> , 1999, 19(3); 491-494	824
L. C. Hsu. Some combinatorial formulas with applications to probable values of a polynomial-product and to differences of zero. <i>The Annals of Mathematical Statistics</i> , 1944, 15; 399-413	828
Appendix:	
Abstracts from <i>Mathematical Reviews</i>	843

Vol. II

Combinatorics & Computing

ABSTRACT THEORY OF INVERSION OF ITERATED SUMMATIONS

BY L. C. Hsu

1. **Introduction.** It is known that the Möbius or Dedekind inversion formula has been extended very abstractly by L. Weisner [5] and P. Hall [3]. These writers have not only given directions on how to find the appropriate Möbius μ -function for a general system, but also found some applications to the theory of groups by the formulas they derived.

The main purpose of this paper is to investigate inversion formulas for iterated summations over sets of a general system. Duality and multiplicative property of the m -th Möbius function are discussed.

Weisner defined as "hierarchy" the general system which satisfies six axioms. It was first noticed by Hall that the hierarchy axioms 4 and 5 may be obviated by his enumeration principle [3]. We may therefore define the general system by only four axioms.

Let $\mathfrak{S}$ be any finite system of sub-sets of a given set G which is not empty (G itself is a member of $\mathfrak{S}$). Let the set-theoretic relation $\subseteq$ be defined in $\mathfrak{S}$ and satisfy the following axioms (A, B, X , etc., denote the members of $\mathfrak{S}$).

- (1) The relation $\subseteq$ is reflexive: $A \subseteq A$.
- (2) The relation $\subseteq$ is asymmetric: $A \subseteq B$ and $B \subseteq A$ imply $A = B$.
- (3) The relation $\subseteq$ is transitive: $A \subseteq B$ and $B \subseteq C$ imply $A \subseteq C$.
- (4) For every pair A, B of $\mathfrak{S}$, only a finite number of members X of $\mathfrak{S}$ exist such that $A \subseteq X \subseteq B$.

The system with the relation so defined may be called simply an $\mathfrak{S}$ -system. Clearly, there is a large number of important cases satisfying these four axioms, e.g. all the sub-groups of a finite group G form an $\mathfrak{S}$ -system with respect to the sub-group relation. We shall establish theorems with respect to the general system $\mathfrak{S}$.

2. **Number of chains.** This section is a preparation for the next. We shall now consider the number of chains. If $A \subseteq B$ and $A \neq B$, then we may write $A \subset B$ and say that A is properly contained in B . A system of members $A_0, A_1, \dots, A_s$ of $\mathfrak{S}$ is called a chain of length s , if $A_0 \subseteq A_1 \subseteq \dots \subseteq A_s$. Comparatively, $A_0 \subset A_1 \subset \dots \subset A_s$ is called a proper chain of length t .

Let the number of distinct chains $A \subseteq X_1 \subseteq \dots \subseteq X_{m-1} \subseteq B$ of length m be denoted by $\tau^{(m)}(A | B)$; and the number of proper chains $A \subset X_1 \subset \dots \subset X_{t-1} \subset B$ of length t by $\lambda^{(t)}(A | B)$, where A and B are fixed members of $\mathfrak{S}$. Then (notice that each proper chain of length t ($\leq m$) can produce $\binom{m}{t}$ chains of length m) we have

Received March 15, 1945, and in revised form January 16, 1947.