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Observers for Linear Systems

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OBSERVERS FOR LINEAR SYSTEMS

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Preface

In 1963, David G. Luenberger initiated the theory of observers for the state reconstruction of linear dynamical systems. Since then, owing to its utility and its intimate connection with fundamental system concepts, observer theory continues to be a fruitful area of research and has been substantially developed in many different directions. In view of this, the observer has come to take its pride of place in linear multivariable control alongside the optimal linear regulator and the Kalman filter. Notwithstanding the importance of the observer and its attendant vast literature, there exists, at the time of writing, no single text dedicated to the subject.

My aim, in writing this monograph, has been to remedy this omission by presenting a comprehensive and unified theory of observers for continuous-time and discrete-time linear systems. The book is intended for post-graduate students and researchers specializing in control systems, now a core subject in a number of disciplines. Forming, as it does, a self-contained volume it should also be of service to control engineers primarily interested in applications, and to mathematicians with some exposure to control problems.

The major thrust in the development of observers for multivariable linear causal systems came from the introduction of state–space methods in the time-domain by Kalman in 1960. In the state–space approach, the dynamic behaviour of a system at any given instant is completely described in a finite-dimensional setting by the system state vector. The immediate impact of state–space methods was the strikingly direct resolution of many long-standing problems of control in a new multivariable system context; for example, pole-shifting compensation, deadbeat control, optimal linear regulator design and non-interacting control. These controllers are normally of the linear state feedback type and, if they are to be implemented, call for the complete availability of the state vector of the system. It is frequently the case, however, that even in low-order systems it is either impossible or inappropriate, from practical considerations, to measure all the elements of the system state vector. If one is to retain the many useful properties of linear state feedback control, it is necessary to overcome this problem of incomplete state information. The observer provides an elegant and practical solution to this problem. Now, an observer is an auxiliary dynamic system that reconstructs the state vector of the original system on the basis of the inputs and outputs of the original system. The reconstructed state vector is then substituted for the inaccessible system state in the usual linear state feedback control law.

In keeping with the title "observers for linear systems", the framework is a finite-dimensional linear system one. Although the theory is mainly described in a linear state–space setting, frequent opportunity is taken to develop multivariable transfer-function methods and interpretations; so important if the designer is to fully exploit the structural properties of observers in a unified manner. Bearing in mind that an observer is itself a dynamic system and that it invariably constitutes the dynamic part of an otherwise static feedback control scheme, there is a marked interplay between observers, linear system theory and dynamic feedback compensation. This interaction is exploited in order to take full advantage of the latest and most significant advances in these subject areas. In particular, much use is made of a recurrent duality between state feedback control and state observation, and the fact that, for the most part, continuous-time and discrete-time problems are algebraically equivalent.

The text is organized as follows. Chapter 1 reviews the fundamental structural properties, namely observability and state reconstructability, that a system must possess for a corresponding state observer to exist. The basic theory of full-order observers, minimal-order observers and a special type of controller known as a dual-observer is introduced. In Chapter 2, the redundancy inherent in the structure of the minimal-order state observer is reduced by exhibiting the original system in various appropriate state–spaces. Chapter 3 examines the reconstruction of a linear function of the system state vector, typically a linear feedback control law, by an observer of further reduced dimension. In common with other chapters, the problem has two main aspects: the determination of the minimal order of the observer and stabilization of the observer. Chapter 4 explores further the possibilities of linear feedback control for systems with inaccessible state. Of particular interest is the construction of a dynamical controller based on the minimal-order state observer. The problem of observer design in order to reconstruct either the state vector or a linear state function of a discrete-time linear system in a minimum number of time steps is the subject of Chapter 5. Chapter 6 considers the problem of estimating the state of continuous-time and discrete-time linear stochastic systems in a least-square error sense, particularly where some but not all of the system measurements are noise-free. An important special case is when all the measurements contain additive white noise, in which case the optimal estimator is identical to the Kalman filter. In Chapter 7, adaptive observers and adaptive observer-based controllers are developed for continuous-time linear systems where *a priori* knowledge of the system parameters is lacking. The basic idea is that the observer estimates the unknown system parameters as well as the state variables of the system.

Chapter 8 undertakes a thorough examination of the complementary role multivariable frequency-response methods and state–space techniques have to play in observer-based system compensation. Using a complex-variable approach, some of the difficulties that may arise in the exclusive pursuit of time-domain methods of design, from the point of view of system robustness and controller instabilities, are highlighted. In Chapter 9, a polynomial-matrix approach is adopted for the synthesis of an observer-based compensator that further serves as a unifying link between transfer-function methods and state–space techniques. Chapter 10 establishes synthesis properties of state observers and linear function observers in terms of a few basic system concepts exhibited in a geometric state–space setting. The book closes in Chapter 11 with a brief discussion of extensions and applications.

Pains have been taken to make the text accessible to both engineers and mathematicians. Some acquaintance with linear algebra, the rudiments of linear dynamic systems and elementary probability theory is assumed. For ease of reference, however, a brief review of the more relevant background material is presented in two appendixes. Theorems, Propositions, etc. have been used to convey major results and summaries in a concise and self-contained fashion. The guiding idea is not rigour *per se*, but rather clarity of exposition. Proofs are usually given unless precluded by excessive length or complexity, in which case the appropriate reference is cited. It is intended that the notes and references which form an integral part of the text, should place the reader in a favourable position to explore the journal literature.

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Acknowledgements

My debt to other investigators is obvious. Where possible I have endeavoured to discharge this debt by specific reference to other works, original papers, etc. This account of observers for linear systems is, in the last analysis, a personal one and will alas, inevitably overlook some contributions. To those who, unknown to me, have helped to shape the present perspective, I express my sincere gratitude if not by name.

Of those who have been most closely associated with my investigations, I feel especially grateful to my former supervisor, Dr M. M. Newmann, who introduced me to the fascinating study of observers. I also thank Professor A. P. Roberts, Dr G. W. Irwin and Dr J. W. Lynn for their encouragement and interest. Part of the manuscript was written during a short stay at the Coordinated Science Laboratory of the University of Illinois, for which I acknowledge the hospitality of Professor J. B. Cruz, Jr, Professor W. R. Perkins and Professor P. V. Kokotovic.

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Finally, I gratefully acknowledge the support of my family and friends in the face of the stresses radiating from me as the centre of this extra activity.

To my mother, my sister Ursula
and the memory of my father

Contents

Chapter 1

Elementary System and Observer Theory

1.1 INTRODUCTION

Since the re-emergence of state–space methods to form a direct multivariable approach to linear control system synthesis and design, a host of controllers now exist to meet various qualitative and quantitative criteria including system stability and optimality. A common feature of these control schemes is the assumption that the system state vector is available for feedback control purposes. The fact that complex multivariable systems rarely satisfy this assumption necessitates either a radical revision of the state–space method, at the loss of its most favourable properties, or the reconstruction of the missing state variables.

Adopting the latter approach, the state observation problem centres on the construction of an auxiliary dynamic system, known as a state reconstructor or observer, driven by the available system inputs and outputs. A block diagram of the open-loop system state reconstruction prcess is presented in Fig. 1.1. If, as is usually the case, the control strategy is of the linear state feedback type $u(t) = Fx(t)$, the observer can be regarded as forming part of a linear feedback compensation scheme used to generate the desired control approximation $F\hat{x}(t)$. This closed-loop observer-system configuration is depicted in Fig. 1.2.

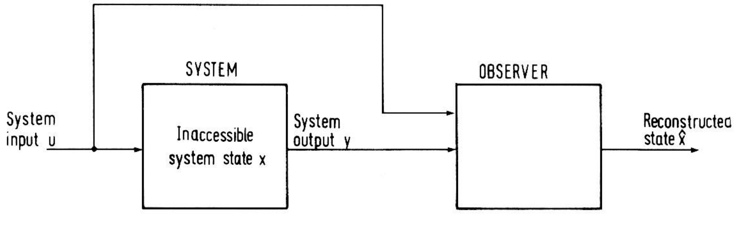

Fig. 1.1 Open-loop system state reconstruction.

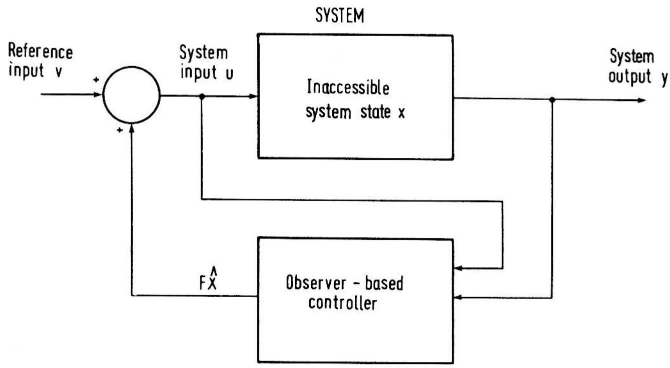

Fig. 1.2 Closed-loop observer-based control system.

The present chapter begins with an introduction to the state–space description of linear dynamical systems. Section 1.3 reviews the fundamental structural properties, namely controllability, reachability, observability and state reconstructability, that a linear system must possess for state feedback control and asymptotic state reconstruction by an observer. In Section 1.5 and Secton 1.6, the repective problems of linear state feedback control with accessible state vector and with inaccessible state vector are discussed. The resolution of the latter problem involves the asymptotic reconstruction of the inaccessible state variables by an observer of dynamic order equal to that of the original system. Section 1.7 sees a major simplification in the reduction of observer order by the number of available measurements of the system state variables to yield a state observer of minimal order. Fortunately, especially from implementation considerations, the parameters of most systems can reasonably be assumed to be constant. In this case, the appropriate minimal-order or full-order observer is time-invariant. Minimal-order observers for discrete-time linear systems are treated in Section 1.8. Finally, in Section 1.9 we reverse the fundamental process of one system observing another system to obtain a special type of controller known as a dual-observer.

1.2 LINEAR STATE–SPACE SYSTEMS

The dynamic behaviour of many systems at any time can be described by the continuous-time finite-dimensional linear system model

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \quad x(t_0) = x_0 \tag{1.1}$$

$$y(t) = C(t)x(t)^* \tag{1.2}$$

* The more general output description $y = Cx + Hu$ is readily accommodated by redefining Equation (1.2) as $\bar{y} \triangleq y - Hu = Cx$.

where $x(t) \in R^n$ is the system state, $x(t_0) \in R^n$ is the state at the initial time t_0, $u(t) \in R^r$ is the control input, and the output $y(t) \in R^m$ represents those linear combinations of the state $x(t)$ available for measurement. The matrices $A(t)$, $B(t)$ and $C(t)$ are assumed to have compatible dimensions and to be continuous and bounded. Throughout the text, the term "linear system" is taken to mean a finite-dimensional linear dynamical system, it being understood that such a linear system is in fact an idealized (mathematical) model of an actual physical system. A solution of the vector differential equation (1.1) is given by the well-known variation of constants formula

$$x(t) = \Phi(t, t_0)x(t_0) + \int_{t_0}^{t} \Phi(t, \lambda)B(\lambda)u(\lambda)\, d\lambda \tag{1.3}$$

where the *transition matrix* $\Phi(t, t_0)$ is the solution of the matrix differential equation

$$\dot{\Phi}(t, t_0) = A(t)\Phi(t, t_0), \quad \Phi(t_0, t_0) = I_n. \tag{1.4}$$

It is remarked that (1.3) holds for *all* t and t_0, and not merely for $t \geqslant t_0$. For the most part we shall deal with linear constant systems, otherwise known as linear *time-invariant* systems in which the defining matrices A, B and C are independent of time t.

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1.5}$$

$$y(t) = Cx(t). \tag{1.6}$$

The transition matrix of (1.5) is given by

$$\Phi(t, t_0) = \exp A(t - t_0) \tag{1.7}$$

where the exponential function $\exp A(t - t_0)$ is defined by the absolutely convergent power series

$$\exp A(t - t_0) = \sum_{i=0}^{\infty} \frac{A^i(t - t_0)}{i!}. \tag{1.8}$$

1.2.1 Linearization

It is invariably the case that the true dynamical system, for which (1.1) and (1.2) represents a linear model, is in fact non-linear and is typically of the form

$$\dot{x}(t) = f[x(t), u(t), t] \tag{1.9}$$

$$y(t) = g[x(t), t]. \tag{1.10}$$

Nonetheless, a linear model of the form of (1.1) and (1.2) can be made to serve as an extremely useful approximation to the non-linear system (1.9) and (1.10) by linearizing (1.9) and (1.10) about a nominal state trajectory $x_0(t)$ and a nominal input $u_0(t)$ where

$$\dot{x}_0(t) = f[x_0(t), u_0(t), t] \tag{1.11}$$

$$y_0(t) = y[x_0(t), t]. \tag{1.12}$$

That is, if one considers *small* perturbations $\delta x(t) \triangleq x(t) - x_0(t)$ and $\delta u \triangleq u(t) - u_0(t)$, one has from the Taylor's series expansions of (1.9) and (1.10) about $[x_0(t), u_0(t), t]$ that

$$\begin{aligned} f[x(t), u(t), t] &= f[x_0(t), u_0(t), t] + A_0(t)\,\delta x(t) \\ &\quad + B_0(t)\,\delta u(t) + \alpha_0[x(t), u(t), t] \end{aligned} \tag{1.13}$$

$$g[x(t), t] = g[x_0(t), t] + C_0(t)\,\delta x(t) + \beta_0[\delta x(t), t] \tag{1.14}$$

where $\alpha_0[\delta x(t), \delta u(t), t]$ and $\beta_0[\delta x(t), t]$ denote second and higher-order terms in the Taylor series expansions. The matrices

$$A_0(t) \triangleq \left.\frac{\partial f}{\partial x}\right|_{[x_0, u_0, t]}, \; B_0(t) \triangleq \left.\frac{\partial f}{\partial u}\right|_{[x_0, u_0, t]} \tag{1.15}$$

$$C_0(t) \triangleq \left.\frac{\partial g}{\partial x}\right|_{[x_0, u_0, t]} \tag{1.16}$$

are Jacobian matrices of appropriate dimensions, evaluated at the known system nominal values $[x_0(t), u_0(t), t]$. From (1.9) to (1.14), neglecting second and higher-order expansion terms, it is readily deduced that

$$\delta\dot{x}(t) = A_0(t)\,\delta x(t) + B_0\,\delta u(t) \tag{1.17}$$

$$\delta y(t) = C_0(t)\,\delta x(t). \tag{1.18}$$

The linearized perturbation model (1.17) and (1.18) is of the same linear form as (1.1) and (1.2), and yields a close approximation to the true non-linear system provided the higher-order expansion terms $\alpha_0[\delta x(t), \delta u(t), t]$ and $\beta_0[x(t), t]$ are *small* for all time t. We shall see presently how the validity of the linearized perturbation model, as characterized by the "smallness" of α_0 and β_0 is reinforced by the (linear) control objective of choosing $\delta u(t)$ so as to regulate $\delta x(t)$ to zero. Henceforth, our attention is focussed on the linear system model (1.1) and (1.2), bearing in mind that it may have arisen in the first place from the linearization of a non-linear process model just described.

1.2.2 Stability

A crucial question in systems theory is whether solutions of the non-linear system of differential equations (1.9) or the linear system of differential