C. W. Gardiner

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In Memory of my Mother

Nellie Muriel Olive Gardiner

1908–1989

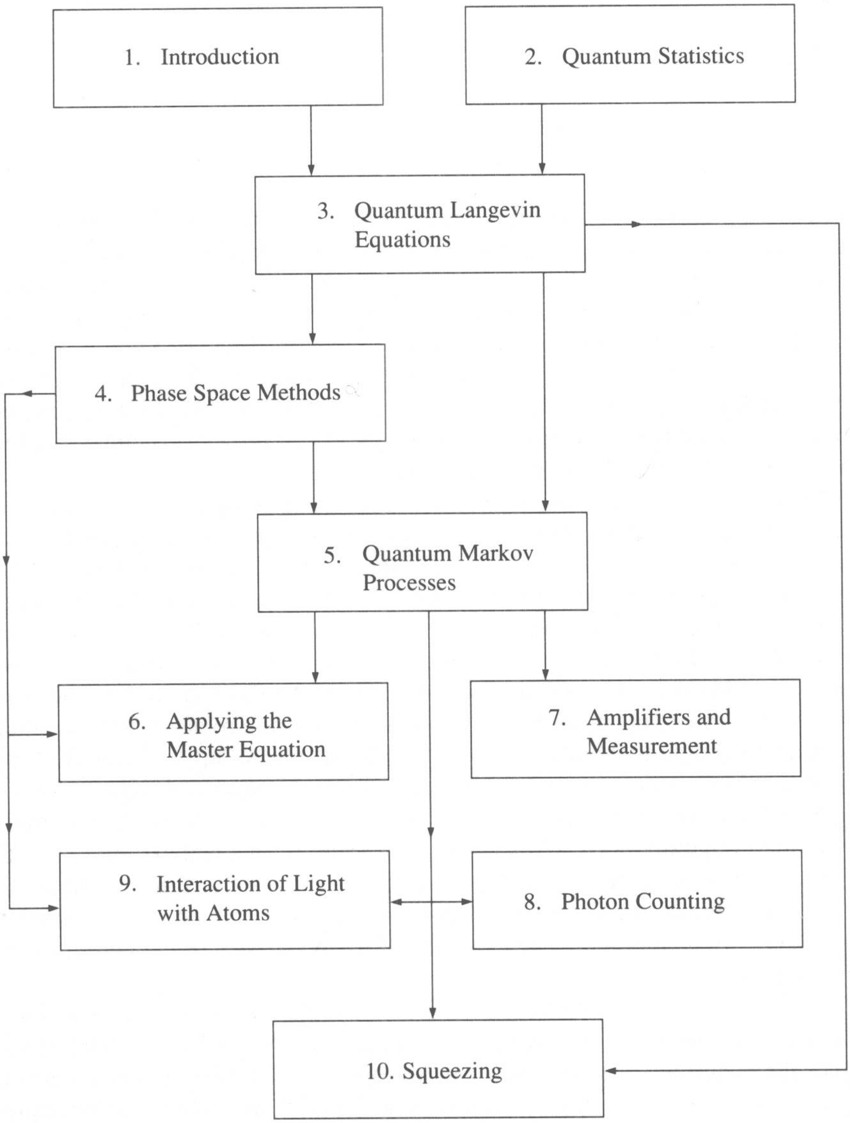
Preface

The term "Quantum Noise" covers a number of separate concepts, and has a variety of manifestations in different realms of physics. The principal experimental areas now covered by the term arise in quantum optics and in the study of Josephson junctions. Although there has been some interaction between these two fields, it is essentially true to say that the two fields have developed independently, and have their own body of techniques for theoretical investigation. This book is largely about the quantum optical field. By this I mean that the techniques presented here are based on the methods used in quantum optics—I certainly do not mean that these methods are applicable only to quantum optics.

My aim in this book is to give a systematic and consistent exposition of those quantum stochastic methods which have developed over the last thirty years in quantum optics. In my previous book, "A Handbook of Stochastic Methods", I concluded with a chapter on quantum mechanical Markov processes. This was only a brief outline of those methods which harmonised most naturally with the classical theory of Markov processes. Since that time the realisation has grown that the study of *squeezing* and other more exotic properties of light fields required a much more careful description of quantum noise, and in particular the relation between inputs and outputs to quantum optical systems. After Matthew Collett and myself worked out such a formalism for describing the production of squeezed light beams, we realised that the same basic approach had within it the foundation for a complete description of quantum noise. This book is the result of this realisation.

This book is not merely a collection of methods drawn from the literature—there is a significant amount of new work, and the interconnections between the various descriptions are new. The structure of the book is as illustrated in the following diagram.

The introduction and Chap. 2 on quantum statistics set the background for the real development of the theory, which takes place mainly in Chap. 3. This chapter develops the quantum Langevin equation of a system in interaction with a heat bath of harmonic oscillators in a form which is rather general, and which can be applied. The *adjoint equation*, which is analogous to a Schrödinger picture version of the quantum Langevin equation is also introduced, and it is shown how all the standard quantum optical techniques can be derived as limits of the adjoint equation. The great advantage of the adjoint equation comes from the remarkable fact that the quantum noise which arises can be exactly represented by a c-number stochastic process. To emphasise this point, a final section in Chap. 3 shows how to apply it to the problem of macroscopic quantum coherence in a low temperature two level system, which can perhaps be realised with a SQUID—that is, it is an application of a quantum optical method to a problem in superconductivity. This section, written


1. Introduction
2. Quantum Statistics
3. Quantum Langevin Equations
4. Phase Space Methods
5. Quantum Markov Processes
6. Applying the Master Equation
7. Amplifiers and Measurement
9. Interaction of Light with Atoms
8. Photon Counting
10. Squeezing

in collaboration with Scott Parkins, shows how quantum noise can be numerically simulated to give slightly more general results than are achievable from the path integral methods usually employed.

The fourth chapter is essentially a compendium of phase space methods, by which c-number representations of quantum processes can be developed. It contains a systematic explanation of coherent states, and the three principal phase space methods, the P-, Q- and Wigner representations.

Chapters 5, 6, and 7 form a group. Chapter 5 contains the theory of the Markovian limit of systems driven by quantum noise, and also shows that this limit occurs in a wider range of systems than those described in Chap. 3. It contains, for the first time in any book, a dual description similar to that which occurs in classical stochastic processes. Thus we have the master equation, which corresponds classically to the Fokker-Planck equation, and *quantum* stochastic differential equations, which are in exact correspondence to their classical counterparts. Basically, these are all results of a quantum white noise limit, though in the case of linear systems, non-white noise can be handled. Thus Chap. 5 contains all that is necessary for a theory of quantum Markov processes.

It is perhaps appropriate to add a word of caution here. The white noise limit is very simple and elegant and has led to some very rigorous mathematical theory. There have been claims that quantum mechanics as usually presented (i.e., using only unitary time evolution) is less general than the theory of quantum Markov processes, and even that because of this conventional quantum mechanics is incomplete. However there is no physical evidence that nature makes use of this more general theory—rather, it appears that the only way in which quantum Markov processes turn up in practice is as a *limiting* description of the motion of a subsystem of a larger system, from which all knowledge of a large number of degrees of freedom is eliminated. This is certainly the point of view I take throughout the book, though I acknowledge that in the field of quantum measurement theory there is a widespread opinion that the theory is not complete. To emphasise—*everything in this book can be understood as a reformulation or a limit of conventional quantum mechanics*.

Chapter 6 is very largely concerned with joining together the results of the previous two chapters, and in developing the ideas of generalised P-representations, introduced by Peter Drummond and myself ten years ago. These methods have gradually become standard techniques, even though there are still some fundamental existence questions to be answered.

On the other hand Chap. 7 depends largely on quantum stochastic differential equations, and presents a way of formulating amplifier theory with inputs and outputs, which is the logical continuation of the 1982 formulation of Carl Caves. I feel that this point of view can be developed much more, and could lead to a "quantum engineering" formulation of amplifiers and related quantum systems. The section on measurement is included here because it is another application of the quantum stochastic differential equation approach, and because, in the model described, it is shown that an understanding of the von Neumann collapse of the wavefunction can be understood as the result of quantum noise and amplifier gain.

My chapter on photon counting is an almost entirely new formulation, in which the techniques of quantum Markov processes are utilised to develop a fully spatially dependent description of the photodetection process. Of course no new physical results arise from this, but I believe the derivation of the conditions under which the quantum Mandel formula is valid, and the relation to the theory of continuous measurements is very illuminating. The latter part of the chapter puts together a way of looking at photodetectors from the point of view of inputs and outputs—that is, we look at input photons and output electrons. Almost incidentally, a Fermionic form of quantum white noise is introduced to describe electron fields.

Chapter 9 is a rather brief summary of the interaction of light and the two level atom, including gas laser theory and optical bistability. The presentation is again unconventional, since it is an application of the quantum Markov process description developed in the earlier parts of the book.

The final chapter is a brief summary of some aspects of squeezed light. This field is still developing, and I did not feel a more intensive discussion would be appropriate.

In order to understand this book, it is necessary to have some knowledge of field quantisation, and a thorough knowledge of non-relativistic quantum mechanics. I have chosen not to put in a description of classical stochastic processes, since this is well covered in my previous book, "A Handbook of Stochastic Methods", to specific sections of which I shall frequently refer by using the abbreviation S.M. In that sense, this book does demand a lot of preparation, but I think that is unavoidable. To understand the full range of physical noise phenomena requires a thorough understanding of both the classical and the quantum fields, and I hope this book and my previous one will provide that.

I have not designed this book primarily as a textbook for a course, though I have included some exercises, which are not necessarily always easy. However, I have given a short course of eight two-hour lectures at the University of Linz, in which I covered the material in Chaps. 1,3 (two lectures), Chap. 4, Chap. 5 and the first three sections of Chap. 9. A very suitable course of about 24 one hour lectures could be made from selections from "Handbook of Stochastic Methods", say Chaps. 1–7, and Chap. 9, and from "Quantum Noise", Chaps. 3–7, and Chap. 9. Other material would be optional. For a balanced view, the student should also attend a course on quantum optics with a more applied point of view, including information on experiments.

Hamilton, New Zealand *Crispin Gardiner*
May 1991

Acknowledgements

The material in this book, as well as the realisation that such a book was possible, arose because of the work done by Matthew Collett in his M.Sc. thesis in 1983, which resulted in the first description of the input-output formalism, and the first correct description of travelling wave squeezed light. This was a period of remarkable productivity, and I wish to acknowledge here the central rôle of Matthew's thinking in the form that this book now takes.

The parts of the book dealing with simulations of the adjoint equation were largely carried out by Scott Parkins, who has also assisted me immensely by reading and checking the proofs. Andrew Smith developed the parts of Chap. 9 involving unconventional phase space methods, and Moira Steyn-Ross carried out some of the work on quantum Brownian motion in Chap. 3.

My most heartfelt thanks go to Heidi Eschmann, who gladly took up the challenge of producing a manuscript in TeX, and, as the reader can see, has managed some typesetting of very great complexity with great success.

The book was written over a period of five years, during which time I have travelled extensively, and benefited greatly from the points of view of many colleagues, including Nico van Kampen, Carl Caves, Jeff Kimble, Howard Carmichael, Gerard Milburn, Peter Drummond, Urbaan Titulaer, Peter Zoller, Marc Levenson, Bob Shelby, Dick Slusher, Bernard Yurke, Fritz Haake, Robert Graham, and my quantum optics colleagues here in New Zealand, Margaret Reid, Matthew Collett, and Dan Walls, who first introduced me to the field.

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