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Stochastic Programming

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Edited by

M. A. H. DEMPSTER

Balliol College, University of Oxford



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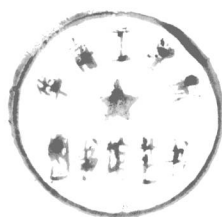
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Preface

The idea of holding the conference upon whose proceedings this book is based arose at the Eighth International Symposium on Mathematical Programming, held in August 1973 at Stanford University. In the autumn of 1973 the Institute of Mathematics and Its Applications kindly agreed to sponsor the first International Conference on Stochastic Programming and a Programme Committee consisting of Oliver Jacobs, Morton Lane, Steven Littlechild, Steven Vajda and myself was formed to organize the programme and invite the speakers. To the other members of the Committee is owed the credit for the large number of presented papers concerned with applications and computational techniques. The Institute organized a most successful residential conference at Balliol College, Oxford with programme sessions in the Mathematical Institute, Oxford, 15–17 July, 1974. At the Conference it was suggested that the editor attempt to write a comprehensive introduction tying the contributions together. It was also the general feeling that several researchers, unable for various reasons to attend the Conference, should be solicited for articles to be included in the Proceedings. The great majority of the contributors also generously volunteered to act, at the editor's discretion, as referees. In the end, updated versions of all but four of the papers presented at the Conference are represented in this volume, together with three of those solicited. All of the papers appearing here in their final form have been extensively refereed and edited.

It is axiomatic that the more interdisciplinary a subject in the mathematical sciences, the more difficult is the provision of a clear, consistent and unambiguous notation for it. I have attempted to overcome the basic notational difficulties and ambiguities in stochastic system theory by enforcing on the authors represented here the use of boldface type to distinguish random variables, vectors and matrices from their realizations. A common symbol ■ is also used to denote the end of proofs. An original plan to collect together at the end of the book and classify all references was abandoned upon the appearance of the excellent bibliography of Stancu-Minasian and Wets. However, an index was judged to be appropriate to a book meant to serve as a supplementary reader to recent texts.

Like these texts, the research reported here presupposes the mastery of beginning graduate level courses in optimization and probability. Although some knowledge of the subject matter is desirable, the introduction has been written with a view to sketching its outlines to a reader with this level of mathematical sophistication.

Much of the editorial work was done in 1974–75 while I was a Fellow at the Center for Advanced Study in the Behavioral Sciences, Stanford, California. It has been completed at Balliol College in the University of Oxford, England over a rather longer period of time than had originally been hoped due to unforeseen circumstances, both personal and general. I should like to express my gratitude to both the above institutions for their generous support.

Thanks are due on behalf of all the participants in the Conference, to Barbara Mayne of the Institute of Mathematics and Its Applications and to Ann Lazier for helping to make the Conference such a pleasant social occasion and to Steven Vajda for “the perfect conference after dinner speech”. For their diligence, thanks are also due to the referees: Martin Beale, Steven Bradley, Dwight Crane, Mark Eisner, Bob Foote, Stanley Gartska, Richard Grinold, Roger Hartley, Peter Hutchinson, Morton Lane, Peter Lewy, Steven Littlechild, Geoffrey Lockett, Daniel Loucks, Kurt Marti, Leonard Mirman, Paul Olsen, András Prékopa, William Raike, Benjamin Schwartz, Richard Sutherland, John Tomlin, Paul Van Moeseke, Balder von Hohenbalken, Roger Wets, Douglass Wilde, Laurence Wolsey and William Ziemba. I am grateful to Masoud Derakshan-Now, András Prékopa and William Ziemba for detailed comments on a draft of the introduction. Kirsty Healey of Academic Press, Peter Lewy and Judith Wagstrom of Oxford Systems Associates Ltd supplied valuable editorial assistance. Secretarial services were cheerfully and painstakingly provided by Helena Smith, of the Center for Advanced Study in the Behavioral Sciences, and Judith Wagstrom.

May 1979

M. A. H. Dempster

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I

Introduction

1

Introduction to Stochastic Programming

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The papers in this volume, taken together, represent a survey of the current state of the art of stochastic programming. The term *stochastic programming* bears the same relation to the modern approach to deterministic optimization problems—*mathematical programming*—as the term *stochastic processes* bears to the classical theory of deterministic *dynamical processes*. Both terms refer to the non-negligible introduction of random variables into mathematical models in their respective fields; in the case of interest here, optimization theory, in its classical analogue, dynamical system theory. As with early progress in stochastic process theory, where discrete time Markov process results concerned with biological, medical and economic applications were developed relatively independently of continuous time results arising from problems in theoretical physics and engineering, theory for the optimization of stochastic systems has until recently been motivated by four separate fields of application—operations research, statistics, control engineering and economics. The term stochastic programming is usually taken to refer to models arising from the first of these fields, *operations research*.

Even within the operations research literature however there is a welter of terminology stemming from the various independent formulations of stochastic programming problems over two decades ago by Beale (1955), Dantzig (1955), Tintner (1955) and Charnes and Cooper (1959). Since then, several textbooks have appeared, Kolbin (1971), Sengupta (1972), Vajda (1972), Kall (1975), Kolbin (1977), see also Parikh (1968). Although extremely useful—indeed, this volume is intended as a supplementary reader for them—these texts display an unfortunate tendency to preserve terminological differences. While the first decade of research into stochastic programming problems largely concerned *theory*, Part IV of this volume demonstrates that *applications* of these models, in step with advances in

computational techniques for linear and nonlinear programming (see Part III) have become a practical reality. Research bibliographies attempting to classify both theory and applications have been produced by Fricks (1969a, 1969b) and Stancu-Minasian and Wets (1976). Various survey papers will be referred to in the appropriate context later in this Introduction.

The 30 papers comprising this volume represent the bulk of those presented at the first international conference to be devoted solely to stochastic programming—held in Oxford, England, under the sponsorship of the Institute of Mathematics and Its Applications—together with two papers solicited from authors in the Soviet Union. As well as providing an exchange of ideas at a critical point in the development of stochastic programming, it was a purpose of this conference to foster the correlation of related research results in the fields of management science, control engineering and economics using concepts central to the literature of statistical decision theory. Since then, at least two international conferences, see Wets (1976) and Davis *et al.* (1979), have been organized with this latter purpose specifically in mind under the *aegis* of the general term *optimization of stochastic systems*. It is to be hoped that this trend will continue and accelerate in the future. As is the case in the optimization of deterministic systems, where recently mathematical programming in abstract spaces has been introduced by a number of authors as a unifying theoretical structure for both classical and modern optimization theory, it is likely that the characterization and analysis of optimization problems for stochastic systems will benefit from a similar abstract mathematical programming approach. It is also possible that the computational and applications experience of stochastic (mathematical) programming may be relevant to general problems of stochastic system theory. As we shall see, computational and applications experience in stochastic programming has on the whole been with larger and more complex systems than has so far been (generally) the case in stochastic *optimization* problems in control engineering, economics or statistics.

The purpose of this Introduction is to tie the papers composing this volume both to each other and to related results, along the general lines indicated above. The structure of the sections of this Introduction mirrors the structure of the parts of the book. After a short section introducing the basic stochastic programming problems, brief comments on the individual papers are given under the headings—Theory, Computation and Applications—and the subheadings of the parts (and sub-parts) of the volume. Of course, a number of the theoretical papers are relevant to computation, while some of those on computation contain both practical applications and new theory, and the applied papers frequently present both new theoretical results and, more importantly, novel computational techniques.