OPTICS OF FLAMES

Including methods for the study of refractive index fields in combustion and aerodynamics

By F. J. Weinberg

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METHODS FOR THE STUDY OF REFRACTIVE INDEX FIELDS IN COMBUSTION AND AERODYNAMICS

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PREFACE

In recent years, the subject of combustion has grown and ramified enormously. Its boundaries with aerodynamics, chemistry, heat and mass transfer, and a variety of other subjects have become increasingly ill-defined, and the useful experimental and theoretical techniques overlap similarly. Such developments suggest the need for monographs which cover small fields exhaustively rather than for compendia which cover larger areas vaguely. In this monograph, an attempt has been made to cover all the aspects of what has proved to be perhaps the most informative single group of methods in making visible processes associated with flame phenomena, in their rapid recording and accurate measurement.

The optical methods discussed, which include the various 'schlieren' techniques, shadowgraphy, deflection mapping and interferometry, cannot be applied to combustion phenomena to best advantage without an understanding of the refractive index fields associated with various flame processes and, therefore, without some general understanding of combustion theory. Such considerations have determined the structure of the book. The necessary combustion background is provided in Chapter 3. Since the aim has been to provide the knowledge necessary for the design of optical apparatus to fulfil any particular function in combustion research—which is often carried out by chemists and engineers—it has been necessary to include some, in part elementary, revision of the minimum necessary optics, in Chapters 1 and 2.

In addition to helping to solve particular problems and attempting to inform the combustion worker of the potentialities of optical tools, it is hoped that the monograph will stimulate those working on the development of optical methods themselves by suggesting new and wider fields of application for such techniques. Whilst in the chapters on individual methods (4–6, 8), the particular requirements of combustion applications have always been borne in mind, the theory and designs presented are fairly independent of application, and these sections should be found of use in aerodynamics and other allied fields. This seems particularly appropriate in view of the ever closer association between combustion and wind-tunnel work, brought about by modern propulsion devices.

The spectroscopic study of flames by their own light has been the subject of a monograph[2] as early as 1948 and will not be considered here, except with regard to the effects on it of ray deflection caused by the refractive index fields of the flames (Chapter 7). High-speed photography and work on short-duration light sources, whilst not theoretically related to the present subject, have an important bearing on the 'freezing' of fast moving or transient combustion phenomena by the methods discussed. A selection of literature on available devices is presented.

Except for a brief preliminary review[3] which is confined to geometric-optical methods, no alternative source of the information here presented is available, at the time of writing. Relevant original publications are widely scattered over the journals of optics, combustion, physical chemistry, photography, aerodynamics, etc. and are accordingly couched in very diverse terminologies. Several combustion texts[4–6a] carry brief accounts of some applications of certain optical methods, usually under the heading of flame photography*. In aerodynamic applications, particularly to visualization of flow in high-speed wind tunnels[6b,7–8], rather more attention has been paid to underlying optical theory than in combustion texts, because of the particular needs in that subject for larger fields of view, optical path lengths and higher sensitivities. This has proved to be relevant in several instances, as will be indicated. Other fields of investigation, for instance the study of liquid diffusion and sedimentation in the ultracentrifuge, have given rise to rather different families of optical methods, and there seems to have been little interchange between the various disciplines.

The purposes of this monograph thus are: to establish the form of refractive index fields occurring in combustion; to examine the usefulness of a wide variety of optical systems in this context and supplement their theory as necessary; to provide a compact text, comprehensive yet elementary enough to assist those workers of a variety of academic backgrounds who are using, or wish to use, such techniques; and to draw the attention of that wider circle who have no previous experience of such methods to their scope, versatility and simplicity for visualization and quantitative measurements in combustion. Much of the work is original, and the treatment is a development of a course of post-graduate lectures given by the writer at the Imperial College of Science and Technology, London.

It is a pleasure to thank the many individuals and organizations who have made this work possible. The writer particularly wishes to express his gratitude for the opportunity to complete the monograph to Prof. D. M. Newitt, F.R.S., and Prof. A. R. Ubbelohde, F.R.S., of this Department, and to Prof. E. S. Campbell, of New York University; for many valuable discussions to Drs. J. H. Burgoyne and Prof. A. G. Gaydon, F.R.S., and to many of the writer's former and present research students, notably Drs. A. Levy, F. Peacock, T. P. Pandya, Messrs. W. S. Affleck, M. D. Fox and N. B. Wood; for sponsorship of research projects on which certain sections of the book are based, among several organizations, particularly to the Aeronautical Research and Development Command of the U.S. Air Force who have contributed extensively under contracts AF 49(638)-976 and AF 61(514)-1013, and to Pergamon Press Ltd., Oxford and Ente Nazionale Idrocarburi, Rome, for permission to quote some passages from his contributions to their publications.

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* In the recent new edition of GAYDON and WOLFHARD's[6] book, the investigation of flame structure by deflection mapping is also discussed.

LIST OF SYMBOLS

A	area
a	amplitude; radius (of circle, lens, mirror, etc.)
$\acute{a}$	displacement of oscillator
B	brightness (energy/area/solid angle/time)
b	fraction (e.g., of light transmitted, of fully developed turbulence); as suffix 'burnt'
C	concentration
c	velocity of light in vacuum; specific heat
D	diffusion coefficient; optical path $= \int n \, dx$
d	diameter (e.g. of source, of burner); separation of slits
E	eddy diffusivity; activation energy
e	displacement (of slit); exponent (in $k/c \propto T^e$)
F	generalized flux vector; mass fraction; when followed by (x) 'a function of x'
f	fraction (e.g. by volume, mole, of reaction completed); focal length
G	mass burning rate
g	velocity gradient at boundary
H	enthalpy
h	distance; axial coordinate in cylindrical system
I	illumination (energy/area/time)
i	angle of incidence
j	general index of a series
K	dielectric constant; general constant
k	thermal conductivity; $2\pi/\lambda$
L	characteristic length; limit of inflammability (as volume fraction of fuel)
l	length (e.g. of slit); scale of turbulence
M	molecular weight
m	magnification; mass (e.g. of droplet); fractional mass; as suffix 'medium'
N	mole number
n	refractive index
P	pressure; product molecule; as suffix 'product'
p	order (of reaction); general exponent; perpendicular coordinate
Q	heat of reaction; intensity of point source (energy/time/solid angle); limit of q
q	generalized source term; separation (e.g. between adjacent fringes, from optic axis)
R	molar refractivity; gas constant; radius of curvature; reactant molecule; as suffix 'reactant'
Re	Reynold's number
r	radial coordinate; specific refractivity; angle of refraction; ratio

S	flow velocity orthogonal to flame (S_u burning velocity)
s	sensitivity; length of arc
T	absolute temperature; $(t\lambda/\mathrm{T} - x)$
t	time; light transmitted
U	internal energy
u	intensity of turbulence; as suffix 'unburnt'
V	volume
v	velocity (e.g. of light in medium; flame speed)
w	rate of reaction; width (e.g. of slit)
X,Y,Z	finite distances along coordinates x, y, z; Z flame length
x,y,z	Cartesian coordinates (x along ray, y parallel to grad n where possible)
α,β	general angles
γ	ratio of specific heats (c_p/c_v)
Δ	small increment; flame thickness
δ	$(n - 1)$
η	viscosity
ϑ	angle of deflection
λ	wavelength
μ	diffusive mass flow rate; magnetic permeability
ν	thermal diffusivity $(k/\rho c)$; frequency
ρ	density
T	period of oscillation; lifetime of droplet
τ	(T/T_u)
ϕ	angle between ray and boundary
ψ	angle between flow line and flame; $(\pi\beta w/\lambda)$
Ω	angle between direction of flow line and grad (t)

CONTENTS

INTRODUCTION

THE LARGE changes in temperature and composition occurring in flames give rise to rapid refractive index variations. The effect of such refractive index fields on transluminating light is to introduce distortions into the wavefront which can be displayed as changes either in ray direction or in the phase of the wave. Both are consequences of the same effect, but it is usually convenient to treat methods based on ray deflections (e.g. shadowgraphy) as 'geometric optics' and those based on phase changes (e.g. interferometry) as 'physical optics'.

In all their aspects, optical methods provide a very versatile and powerful family of tools in combustion and allied research. In many cases they give information which is not otherwise obtainable at all. In all their applications they are characterized by not interfering with the easily disturbed flame processes, by absence of inertia and by their ability of producing virtually instantaneous records of rapidly moving or transient phenomena.

It is not generally realized how easy these methods are to apply. This is particularly true of ray deflection methods, for, despite the small differences of gaseous refractive indices from unity, the massive refractive index gradients associated with flames are, optically, relatively gross phenomena which yield records with surprisingly inelaborate equipment.

At the risk of some oversimplification, it is convenient to subdivide the purposes for which optical methods are generally used in combustion research and the corresponding extent to which the records obtained are analysed, into three main categories.

The first and simplest is the visualization of flames and associated flow phenomena, which are not normally visible because of the transparency of gases within which the reactions occur. The luminous zone, often loosely referred to as 'the flame', its position and its importance within the region of reaction are not generally definable with precision. In some cases of interest this zone is absent entirely, and in many others the light emitted is too feeble for photographic recording, in particular where short exposures for 'freezing' transient phenomena are required. Optical surfaces, i.e. loci of some particular criterion in the refractive index field such as those obtainable by schlieren and shadow methods, then provide definable records whose intensity is determined by an extraneous light source. Very inelaborate systems furnish images adequate for visualization of flames.

The second group constitutes a refinement of such qualitative visualization, in that the optical systems are designed to allow for accurate quantitative measurements on the geometry of the optical surfaces. The significance of this type of application lies particularly in the definability of such surfaces, whose positions within the flame zones are frequently better suited to the purposes of the measurement than is that of the luminous region. As an example, the schlieren cones of premixed flames are particuarly suitable for burning velocity measurements[9–10] because they have been shown[11–12] to occur at temperatures low enough to render gas expansion and consequent flow-line distortion unimportant. The techniques employed here are generally elaborations of those used for visualization, only in that no image distortions are permitted.

The third group consists of methods developed to record and measure refractive index distributions in the vicinity of flame fronts, shock waves, hot surfaces, etc., for the purpose of analysing the structure of such phenomena. The newcomer to the field of combustion is usually surprised to find that the understanding of what was, after all, the first chemical reaction consciously put to use by man, still leaves so much to be desired. It will be seen later that perhaps the main reason for the slow development of the theory of combustion processes lies in the extreme conditions prevailing in flame zones. Here heat and mass transfer processes occur simultaneously with chemical reactions, under conditions of high temperatures and formidable gradients of concentration, temperature and velocity over minute distances. This state of affairs forbids extrapolation of knowledge gained under more leisurely conditions and demands studies *in situ* on actual flames. Yet, at the same time, these conditions conspire to make flame zones particularly sensitive to any form of external interference. In this field the instantaneous, non-interfering methods of refractive index measurement offer unique advantages over methods involving material probes. In such work, employing various forms of deflection mapping or interferometry, interpretation of records is rather more elaborate, direct simultaneous visualization usually being sacrificed and particular (and rather academic) combustion systems, simple both in mechanism and in geometry, being chosen.

Unfortunately, the various techniques available do not lend themselves to classification within the framework of these three main groups of applications. While some of them are normally tied to only one kind of application (such as interferometry to refractive index measurement), others, such as the group of schlieren methods discussed in Chapter 4, have been used in all three capacities. It has, therefore, been necessary to divide the subject as follows.

Chapter 1 contains the minimum necessary optics background, including some elementary revision, its modification for the case of media whose refractive index is very close to unity, and the somewhat less familiar laws of refraction in continuous refractive index fields. Chapter 2 is concerned with the dependence of refractive index on the gas parameters. Optical records of combustion processes usually are a consequence of simultaneous variation of temperature and composition, and in quantitative work the separation of these two main interrelated variables must be considered. These two chapters provide the basis for an assessment of refractive index fields of flames, in terms of a summary of the mechanisms of various combustion phenomena, in Chapter 3.

The second half of the book then deals with individual methods, their theory and their modifications for, and applications to, combustion research. An exception is the short Chapter 7, which deals with the optical 'illusions' induced by refractive index fields of the flames when these are observed by their own light. It seemed logical to place this after 'deflection mapping', since the main difference in the theory is that the distributed light source is now the luminous zone of the flame itself.

1

ELEMENTS OF OPTICS

THE CONTENTS of this chapter, a mixture of some elementary revision with perhaps rather less familiar extrapolations, have been designed entirely around the requirements of the monograph. Nothing that is not required and used subsequently is included. Moreover, the presentation is unorthodox and rather weighted with emphasis on the approximate nature of 'ray optics' and its elementary relation to 'wave optics'. The fact that under certain conditions the former breaks down (and also the nature of this breakdown) happens to be of importance in this field, and this feature is often overlooked. This chapter, therefore, is not to be regarded as a general revision of optics outside the present context.

RAYS AND WAVES

In the teaching of school optics, we meet the properties of light rays as fundamental laws of nature, long before the concept of a wave is introduced. Thus we learn of refraction at a boundary first in the form

$$(\sin i)/(\sin r) = n \tag{1.1}$$

where i and r are angles between the perpendicular to the surface and the incident and refracted ray, respectively, and n = refractive index, and we come to regard as a property of the substance the ratio of the sines of those angles before the difference between velocities of a light wave is introduced.

The fact that we are physically so much larger than wavelengths of light makes it possible to use this concept to derive all the conventional geometric optics. We may briefly retrace the sequence and make use of the opportunity to draw some conclusions to be used later. By applying eqn. (1.1) to the surfaces of spherical lenses, we find that the latter deflect parallel rays close to the optic axis—the line through the lens centre—in such a way that they pass through a point focus at a distance f beyond the lens. An angular change of θ in the direction of the incident beam results in a displacement of $(f\theta)$ at the focus. Conversely, if the lens is used as a collimator, i.e. to produce a parallel beam of light, a source of extent d at the focus induces a maximum deviation from parallelity of (d/f).

Applying the equality of angle of incidence and reflection in the case of a mirror to reflection at a spherical surface leads to an analogous result. If the surface is parabolic, the focusing action is no longer confined to rays close to the optic axis.

The focusing action of optical elements is the only property necessary for image formation. Anything with a focal length f gives

$$1/X_1 + 1/X_2 = 1/f \tag{1.2}$$

where X_1 and X_2 are, numerically, the distances from the optical element of the conjugate points, i.e. points such that each coincides with the image of the other. Since the angles subtended at the centre of the lens or mirror by a finite object and image must be the same, the magnification at 2 with respect to 1 is X_2/X_1.

The next stage in the usual development of geometric optics is the theory of thick lenses and does not concern us. The thin lens approximation applies par excellence to the main optical elements involved here, because of their exceedingly long focal lengths. These are indeed often an embarrassment with regard to laboratory space, and it is worth noting at this stage that the object–image distance is given by eqn. (1.2), as

$$X_1 + X_2 = X_2^2/(X_2 - f) \tag{1.3}$$

the differentiation of which gives $X_2 = 2f$ as the condition for minimum separation. The conjugate positions are then symmetrical about the lens and the minimum length required is $4f$. In the case of a mirror, X_1 and X_2 are on the same side, so that only half the distance is necessary.

The one thick lens which is used in virtually every optical system to be discussed is a condenser. This is usually a short-focus large-diameter lens or assembly of lenses, often of poor optical quality, used to image the real source on a pinhole or other small aperture which then becomes the effective source. It is not usual, however, to calculate the best disposition of the condenser beyond ensuring that the angle it subtends at the aperture is at least equal to that subtended there by the next optical element, so that the latter is filled by the cone of light.

After this digression to mention a few relevant practical consequences of simple theory, we return to consider the circumstance that, once we accept the relationships between angles of incidence, reflection and refraction as fundamental laws of nature, we are perfectly capable of designing complex optical systems without having to realize at all that light propagates as a wave. Indeed, we are used to ignoring this fact, unless we are dealing with interference, diffraction or other explicitly physico-optical topics.

This state of affairs has its advantages and its dangers. The outstanding advantage is that, without the simplicity of geometric optics, the understanding and design of elaborate optical systems would be immensely complicated and in many cases wellnigh impossible. When dealing with a point source and a single train of waves, a lens or spherical mirror may be regarded simply as a device for altering the curvature of the wave front. As soon as a number of such point sources (i.e. an extended object) are concerned, however, the phase relationships between the various wave trains must be taken into account, in order that their contributions to any part of the image can be added. The complexity of extending such an approach to even the simplest schlieren system, even in quite an approximate manner, will become obvious in Chapter 4. To present the treatment of optical systems in terms of geometric optics as a matter of convenience is, therefore, rather a euphemism; for all but the simplest cases it is a matter of necessity.

The danger is that, this being so, we are tempted to forget that wave propagation is involved at all. A very elementary but rather striking example of this is the difficulty often encountered by workers who have forgotten all but their school optics and who tend to think of single-ray refraction as defined by eqn. (1.1), when trying to visualize refraction by a continuous refractive index gradient perpendicular to the ray. Rather more serious is the neglect of the effects of phase variation when predicting limits of the performance of optical systems on the basis of geometric optics. In the systems to be discussed, such effects usually become serious when limits, e.g. of sensitivity, are approached.

The nature of the failure of geometric optics under such circumstances is best illustrated by an example. *Plate 19* is a shadow photograph of a flat flame taken by parallel light and also shows the distorted shadow of a slit in the same beam. The slit was placed in front of, and oriented diagonally to, the plane of the flame. The geometric theory of the shadow (set out in Chapter 5) shows that the distribution of light intensity in the shadowgram corresponds to the projection of the slit record on a plane perpendicular to the flame. The photograph was taken[13a] in a beam of light accurately monochromatic and parallel, deviation from parallelity being kept down to less than 30 sec of arc. As close inspection of the record shows, the light distribution in the shadowgram is broken up into a series of light and dark fringes caused by diffraction. While it is true that the use of white light and/or a less parallel beam would give rise to an intensity distribution leading to little suspicion of the conclusions of geometric optics, the photograph reveals that, under such stringent conditions, these conclusions are valid only over distances larger than the fringe widths, i.e. that geometric optics informs us only of the shape of the outer envelope of the illumination. If now the method were applied to a refractive index field giving rise to a fringe spacing about equal to the width of the shadow, the predictions of geometric optics would break down completely.

It is of the utmost importance, therefore, to establish the limitations of the geometric approach by recourse to wave theory whenever a new field is broached. Fortunately it does not matter greatly if the intractability of wave theory calculation precludes exact treatment and geometric optics must be used for all the subsequent detailed routine work. Even an oversimplified physical treatment will generally establish the approximate limits to the validity of geometric optics and the parameters on which these limits depend. Particular cases will be dealt with in subsequent chapters; some more general references are given[14–19].

With these pitfalls in mind, it will be prudent to start with a simple wave and define all properties of rays in terms of those of the wave. The quantity oscillating in the plane perpendicular to the direction of propagation, is the magnitude of an electric field coupled with that of a magnetic field at right angles to it. Although we shall from time to time refer to conclusions of electromagnetic theory, in the main part of this text we shall not find it necessary to concern ourselves with the nature of the oscillator and can think in terms of e.g. ripples on a pond, if this appears simpler. Thus each point reached by the wave front may be regarded as a new source, because of its own induced oscillation. If we draw short circular arcs with equal small radii in the direction of propagation from centres lying on the instantaneous wave front, the common tangent to these elemental wavelets becomes the new wave front. *We can now define rays as continuous lines everywhere perpendicular to the field of such wave fronts.*

It is worth noting that one consequence of the definition is that the concept of a single ray becomes meaningless. For this reason, thinking of the refraction of a single ray leads to conceptual difficulties in the case of continuous refractive index fields. Before considering this and the other effect by which we assess distortion of the wave front—change in direction and change in phase—we must define some other parameters of the wave. Let us consider propagation in one dimension, x, and take a section along a rectilinear ray at a time t. The wave can conveniently be represented by

$$a' = a \cos 2\pi \,(t/\mathrm{T} - x/\lambda) \tag{1.4}$$

since at any constant x, the displacement, a', varies in simple harmonic motion with t, whilst at any particular time the equation represents a sinuosidal wave in distance. In the former case, that of the instantaneous frozen shape, a' has the same magnitude at all points separated from each other by λ which is, therefore, the wavelength. Similarly the oscillation pattern is repeated after each period T, and this is the reciprocal of the frequency, ν:

$$\nu = 1/\mathrm{T} \tag{1.5}$$

The maximum value of a' is a, the amplitude. Intensity is proportional to the square of the amplitude. (In terms of the analogy with a mechanical oscillation, this follows directly from the kinetic energy of the oscillator at point $a' = 0$, where its potential energy is zero and its velocity $(\mathrm{d}a'/\mathrm{d}t)_{a'=0}$).

The velocity of propagation of the wave, v, must be equal to the number of, for instance, crests generated in unit time, multiplied by the distance between them, i.e.

$$v = \nu\lambda = \lambda/\mathrm{T} \tag{1.6}$$

This velocity differs from medium to medium. Its maximum value, c, occurs in vacuum. It follows from eqn. (1.6) that either wavelength or frequency, or both, must also be a function of the medium. However, on considering the movements of two adjacent oscillators, one on each side of an interface traversed by the wave, it becomes obvious that frequency must be the fundamental constant. Accordingly,

$$v_1/v_2 = \lambda_1/\lambda_2 \tag{1.7}$$

wavelength is proportional to velocity and, in particular,

$$c/v_m = \lambda/\lambda_m \tag{1.7a}$$

where the suffix m denotes any medium and its absence, vacuum.

BEAM DEFLECTION

Discontinuous Boundaries

We now have the necessary minimum of wave properties to characterize what happens to rays in traversing refractive index fields. The simplest case is that of a straight interface between two media. The meaning of the word 'straight' here is that the radius of any curvature must be large in comparison with wavelength—which is almost, but not quite, the same as forgetting the qualification altogether and thinking in terms of a single ray.

It is worth giving an elementary proof of eqn. (1.1), just to emphasize the role of the ratio of velocities and of wavelengths and to underline the analogy to the case of the continuous field.

In drawing the elemental wavelet arcs, when locating the new wave front above, the radii were all equal because wavelength was constant. When the same procedure is applied across an interface between media 1 and 2 (*Figure 1.1*), each arc radius must be proportional to the appropriate wavelength (successive positions of an element of the wave front are represented by the heavy lines). It can be seen from the diagram that

$$(\sin i)/(\sin r) = \lambda_1/\lambda_2 = v_1/v_2 = {}_1n_2 \qquad (1.1a)$$

The relevant property of the medium is the velocity of the light wave within it. The ratio of the velocities is the refractive index of one medium with

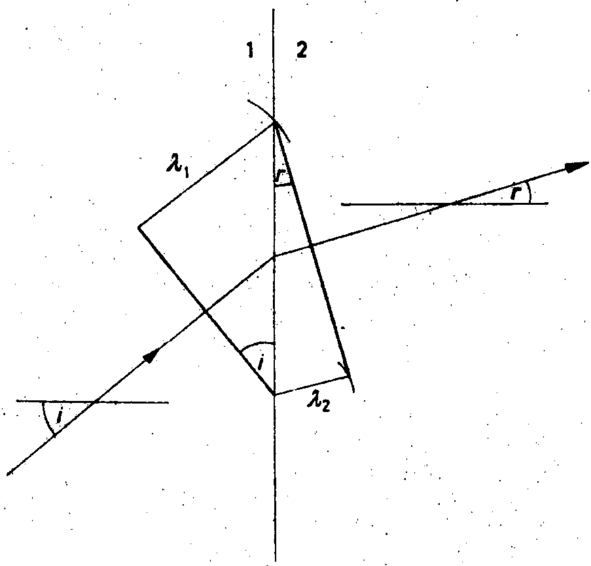

Figure 1.1

respect to the other, and this must always determine the refraction of a light wave, even in systems where it is no longer expressible by the ratio of (sin i) to (sin r).

The refractive index of medium m with respect to vacuum is termed the absolute refractive index of m and, unless otherwise stated, 'refractive index' shall hereafter mean *absolute* refractive index, i.e. $n_m = c/v_m$. The distinction is particularly important in the present context because the velocity of light in gases differs only little from that in vacuum. Thus in

$$n = 1 + \delta \qquad (1.8)$$

for gases at N.T.P., δ is of the order 0·0003 (0·000293 for air). This circumstance considerably simplifies almost all the subsequent calculations, and the quantity δ will be used throughout, more often indeed than the refractive index, n.

Since

$$n = c/v = (v + \Delta v)/v = 1 + (\Delta v/v) = 1 + \delta = 1 + \Delta\lambda/\lambda \quad (1.9)$$

δ is, in fact, the fractional change both in light velocity and in wavelength on going from the gas to vacuum, and it is this that at N.T.P. generally amounts to a few parts in 10^4. At elevated temperatures, which are the common feature of combustion applications, δ is normally even less, and its neglect by comparison with unity introduces an inappreciable error. We shall therefore write

$$\delta \ll 1 \quad (1.10)$$

a condition which will simplify most of the following equations.

Refractive indices are sometimes quoted with respect to air and must be converted to absolute values. Their relationship to each other in a series of media, say $A, B, C, \ldots, Z$, follows from eqn. (1.1a) as

$${}_An_B . {}_Bn_C \cdots {}_Yn_Z = (v_A/v_B)(v_B/v_C) \ldots (v_Y/v_Z) = v_A/v_Z = {}_An_Z \quad (1.11)$$

The corresponding relationship under condition (1.10) is

$${}_A\delta_B + {}_B\delta_C + \ldots + {}_Y\delta_Z \simeq {}_A\delta_Z \quad (1.11a)$$

If $A \equiv Z$, i.e. if the beam ultimately emerges into the first medium,

$${}_An_B + {}_Bn_C + \ldots {}_Yn_A = 1 \quad (1.11b)$$

A geometric consequence which follows on substitution of the respective ratios of $(\sin i)/(\sin r)$ for each of the refractive indices above is that, if the various media are assembled as parallel-faced slabs, the emergent ray must be parallel to its direction of incidence, irrespective of its directions within the slabs. The corresponding approximation under condition (1.10) is

$${}_A\delta_B + {}_B\delta_C + \ldots + {}_Y\delta_A \simeq 0 \quad (1.11c)$$

If the number of media is confined to two, P and Q,

$${}_Pn_Q = 1/{}_Qn_P \quad (1.11d)$$

or, for gases,

$${}_P\delta_Q \simeq {}_Q\delta_P \quad (1.11e)$$

Conversion of ${}_an_m$ (suffixes: air and medium) to its absolute value n_m follows from eqn. (1.11) and (1.11a) as

$$n_m = 1/{}_mn_a \cdot {}_an = n_a \cdot {}_an_m = 1{\cdot}000293\,{}_an_m \quad (1.12)$$

(The numerical value is for CO_2-free air at N.T.P. at the wavelength of sodium emission, $5{\cdot}893 \times 10^{-5}$ cm). Using the approximation for gases, this becomes

$$\delta_m = {}_a\delta_m + 0{\cdot}000293 \quad (1.12a)$$

Unless a solid or liquid boundary is involved, strictly speaking no permanent refractive index discontinuity can persist in a gas. Any discontinuous step in composition, temperature or pressure must be dissipated in a loose assembly of rapidly and randomly moving molecules whose separation is much greater than molecular dimensions. Nonetheless, the concept of refractive index boundaries in gases is a very useful one, for two reasons.