PREDICTIVE CONTROL

A Unified Approach

PRENTICE HALL INTERNATIONAL SERIES IN SYSTEMS AND CONTROL ENGINEERING

RONALD SOETERBOEK

Series Editor
M. J. GRIMBLE

Predictive Control

A Unified Approach

Ronald Soeterboek

Control Laboratory,
Department of Electrical Engineering,
Delft University of Technology,
The Netherlands

Prentice Hall

New York London Toronto Sydney Tokyo Singapore

First published 1992 by
Prentice Hall International (UK) Limited
Campus 400, Maylands Avenue
Hemel Hempstead, Herts HP2 7EZ
A division of
Simon & Schuster International Group

Printed and bound in Great Britain
at the University Press, Cambridge

Library of Congress Cataloging-in-Publication Data

Soeterboek, Ronald.
Predictive control : a unified approach / Ronald Soeterboek.
p. cm. – (System and control engineering)
Includes bibliographical references and index.
ISBN 0-13-678350-3
1. Predictive control. I. Title. II. Series: Prentice Hall International series in system and control engineering.
TJ217.6.S64 1992
629.8—dc20 92-2541
CIP

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library

ISBN 0-13-678350-3

1 2 3 4 5 96 95 94 93 92

Predictive Control

Prentice Hall International
Series in Systems and Control Engineering

M. J. Grimble, Series Editor

BANKS, S. P., *Control Systems Engineering: modelling and simulation control theory and microprocessor implementation*
BANKS, S. P., *Mathematical Theories of Nonlinear Systems*
BENNETT, S., *Real-time Computer Control: An introduction*
BITMEAD, R. R., GEVERS, M. and WERTZ, V., *Adaptive Optimal Control*
BUTLER, H., *Model Reference Adaptive Control*
CEGRELL, T., *Power Systems Control*
COOK, P.A., *Nonlinear Dynamical Systems*
ISERMANN, R., LACHMANN, K. H. and MATKO, D., *Adaptive Control Systems*
KUCERA, V., *Analysis and Design of Discrete Linear Control Systems*
LUNZE, J., *Feedback Control of Large-scale Systems*
LUNZE, J., *Robust Multivariable Feedback Control*
McLEAN, D., *Automatic Flight Control Systems*
OLSSON, G. and PIANI, G., *Computer Systems for Automation and Control*
PARKS, P. C., and HAHN, V., *Stability Theory*
PATTON, R., CLARK, R. N. and FRANK, P. M. (editors), *Fault Diagnosis in Dynamic Systems*
PETKOV, P. H., CHRISTOV, N. D. and KONSTANTINOV, M. M., *Computational Methods for Linear Control Systems*
SÖDERSTROM, T. and STOICA, P., *System Identification*
SOETERBOEK, A. R. M., *Predictive Control: A unified approach*
STOORVOGEL, A., *The $H\infty$ Control Problem*
WATANABE, K., *Adaptive Estimation and Control*
WILLIAMSON, D., *Digital Control and Instrumentation*

To Anita and Nick

Preface

The concept of predictive control originates from the late seventies and nowadays has evolved to a mature level. Predictive controllers have remarkable features. For example, they can be used to control a wide variety of processes, among which are nonminimum phase and unstable processes, without the designer having to take special precautions. Further, they are easy to tune and process constraints can be handled systematically. These features promote the practical applicability of predictive controllers as is illustrated by the many successful applications that have been reported in the literature. However, the reason why predictive controllers have these features is often not very well understood. Further, in some predictive controllers heuristics play an important role. From an academic point of view this makes predictive controllers less attractive.

The main objective of this book is to enhance insight into predictive controller design and to contribute to an improved understanding of how predictive controllers operate and why they can be used successfully in practical applications. For these reasons the theory of predictive controller design is presented in a unified fashion rather than focusing on one of the many predictive controllers proposed in the literature. Further, the book not only covers most existing predictive control theory but also considers a number of extensions which make the practical use of predictive controllers very attractive. Much attention is paid to providing insight and theoretical backgrounds to the various aspects of predictive controller design. To avoid losing the practical applicability of predictive controllers, many practical aspects, such as tuning and implementation issues, are discussed extensively. Much attention is given to handling process constraints efficiently. Also, a MATLAB implementation of a unified predictive controller is provided. The result of all this is a unified predictive controller that not only unifies many well-known predictive controllers but also more traditional design methods such as pole-placement and some time-optimal controllers. Moreover, the proposed concept is not only attractive from an academic point of view but, as is shown by simulation studies and some industrial applications, is also attractive from a practical point of view. This is a rare combination. A detailed overview of what this book contains can be found at page 13, and, just as important, an overview of what the book does *not* contain can be found at page 15.

Although only basic knowledge in sampled data systems and classical control theory is assumed, the book is primarily written for graduate students and research workers. It is intended for those who wish to become acquainted with predictive controller design, and for those already familiar with predictive control who wish to gain a deeper insight in and understanding of predictive controller design and all its remarkable features. Therefore, the book is suitable as background reading and as a reference.

In writing the text I have been helped by many people whom I would sincerely like to thank. My first thanks go to Paul van den Bosch and Henk Verbruggen who motivated my interest in this topic, encouraged me to write this book and provided many useful suggestions for improvements. I would also like to thank Jacques Richalet, Salwa Abu el Ata-Doss and Robain de Keyser with whom I had many pleasant discussions and who contributed to the book in various ways. Parts of the text have also been read by Rajamani Doraiswami whose comments were very much appreciated.

Further, I want to thank Addy Koster who helped me in the experiments on the distillation column and Arthur Pels for his help with the wind tunnel experiments. I also wish to thank Hans Butler for trying to convince me over and over that model-reference adaptive control is 'better' than predictive control, and who thus strengthened my belief in predictive control.

I am also indebted to the staff and students of the Control Laboratory for providing a pleasant working atmosphere. Finally, I would like to thank my wife, Anita, for her generous support and patience during the writing of this book.

Ronald Soeterboek

Glossary

This list contains the most important symbols and abbreviations used in this book.

Notation

- Italic upper case characters denote polynomials in q^{-1}. For example, A and B are polynomials in q^{-1}.
- Italic lower case characters denote elements of polynomials, matrices, vectors or signals as a function of k (= discrete time).
- Italic and bold upper case characters denote matrices. For example, $\boldsymbol{\Psi}$ and $\boldsymbol{M}$ are matrices.
- Italic and bold lower case characters denote column vectors. For example, $\boldsymbol{\lambda}$ and $\boldsymbol{s}$ are vectors. A row vector is denoted by using the transpose operator. For example, $\boldsymbol{\lambda}^T$ and $\boldsymbol{s}^T$ are row vectors.

Symbols:

q^{-1}	backward shift operator: $q^{-1}x(k) = x(k-1)$
q	forward shift operator: $q\,x(k) = x(k+1)$
T_S	sampling period in seconds
z	complex variable used in the z-transform: $z = e^{T_S s}$ in which s is the complex variable used in the Laplace transform
${}_r(\boldsymbol{A})_j$	jth row of matrix $\boldsymbol{A}$. If convenient, the symbol $\boldsymbol{a}_j^T$ is also used for this purpose. If matrix $\boldsymbol{A}$ is of dimension $n \times m$, then $[{}_r(\boldsymbol{A})_j] = 1 \times m$
${}_c(\boldsymbol{A})_j$	jth column of matrix $\boldsymbol{A}$. If matrix $\boldsymbol{A}$ is of dimension $n \times m$, then $[{}_c(\boldsymbol{A})_j] = n \times 1$
$(\boldsymbol{A})_{ij}$	element ij of matrix $\boldsymbol{A}$. If convenient, the symbol a_{ij} is also used for this purpose
x_i	ith element of a polynomial X
n_X	degree of a polynomial X
X_i	a polynomial which is a function of i. Its degree is denoted by n_{X_i} and its jth element is given by $x_{i,j}$
$X(1)$	'gain' of a polynomial: $X(1) = \sum_{j=0}^{n_X} x_j$
$(\hat{\cdot})$	estimated variables
x_{ss}	steady-state value of x(k): $$x_{ss} = \lim_{k\to\infty} x(k) = \lim_{z\to 1}(1-z^{-1})X(z^{-1})$$
$(.)^T$	transpose operator
[.]	matrix dimension
Δ	differencing operator: $\Delta = 1 - q^{-1}$
J	criterion function

$\mathcal{E}(.)$	expectation operator
$\epsilon(k)$	prediction error at $t = k$
$\partial(.)$	partial derivative operator
H_L	loop transfer function (= the loop transfer function based on the process)
$\hat{H}_L$	nominal loop transfer function (= the loop transfer function based on the model)
$\mathcal{P}(.)$	operator that transforms a row vector into a polynomial in q^{-1}:

$$\mathcal{P}(x^T) = Y$$

where:

$$\begin{aligned} x^T &= [x_1, \ldots, x_n] & [x] = 1 \times n \\ Y &= x_1 + x_2 q^{-1} + \cdots + x_{n-1} q^{-n+1} & n_Y = n - 1 \end{aligned}$$

ω	frequency in rad/s
$l(k)$	unit step: $L(z^{-1}) = \dfrac{1}{1 - z^{-1}}$

Model parameters

A, B, C, D	polynomials describing the process
$y(k)$	process output at $t = k$
$u(k)$	controller output and process input at $t = k$
$e(k)$	discrete white noise with zero mean
K_{dc}	DC gain of the process: $K_{dc} = \dfrac{B(1)}{A(1)}$
$\hat{K}_{dc}$	DC gain of the process model: $\hat{K}_{dc} = \dfrac{\hat{B}(1)}{\hat{A}(1)}$

d	time delay of the process in samples
$\hat{d}$	time delay of the model in samples
T_d	time delay of the process in seconds
T	estimate of C: $T = \hat{C}$
$\xi(k)$	disturbance acting on the output of the process

Controller parameters

H_p	prediction horizon
H_m	minimum-cost horizon
H_c	control horizon
ϕ_ξ	polynomial describing the disturbance acting on the output of the process
ϕ_w	polynomial describing the reference trajectory
ϕ	minimal polynomial of ϕ_ξ and ϕ_w
ρ	weighting factor
P, Q_n, Q_d	polynomials in the unified criterion function
$\underline{u}$	lower bound level constraint
$\overline{u}$	upper bound level constraint
$\underline{\Delta u}$	lower bound rate constraint
$\overline{\Delta u}$	upper bound rate constraint
$\mathcal{R}, \mathcal{S}, \mathcal{T}$	controller polynomials
$u(k)$	controller output and process input at time $t = k$
$w(k)$	reference trajectory at time $t = k$
Sp	set point
ϵ	accuracy of solution found by DGP
η	accuracy of solution found by ACH

Servo performance criteria

t_r	rise time in seconds
t_p	peak time in seconds
t_s	settling time in seconds
$\mathcal{O}$	overshoot in %
ϵ_{ss}	steady-state error in %

Robustness criteria

gm	gain margin
$\underline{gm}$	lower gain margin
$\overline{gm}$	upper gain margin
dm	delay margin in seconds
$\underline{dm}$	lower delay margin in seconds
$\overline{dm}$	upper delay margin in seconds

Abbreviations

ACH	Active Constraint Handling
ARIX	Auto-Regressive Integrated eXogenous
ARIMAX	Auto-Regressive Integrated Moving-Average eXogenous
ARMAX	Auto-Regressive Moving-Average eXogenous
ARX	Auto-Regressive eXogenous
CLCE	Closed-Loop Characteristic Equation
CLTF	Closed-Loop Transfer Function

DGP	Dedicated Gradient Projection method
DMC	Dynamic Matrix Control
EHAC	Extended Horizon Adaptive Control
EPSAC	Extended Prediction Self-Adaptive Control
ECM	Explicit Criterion Minimization
FIR	Finite Impulse Response
FLOP	Floating Point OPeration
FSR	Finite Step Response
GMV	Generalized Minimum Variance
GPC	Generalized Predictive Control
IMAC	Interlaced Multipredictor Adaptive Controller
LQ	Least Quadratic
MAC	Model Algorithmic Control
MIMO	Multi-Input Multi-Output
MSTC	Minimum Settling Time Control
MV	Minimum Variance
PCA	Predictive Control Algorithm
PFC	Predictive Functional Control
RFRC	Ripple-Free Response Control
SISO	Single-Input Single-Output
UPC	Unified Predictive Control

Contents