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To my wife Masako
and my son Hideaki

PREFACE

The main characteristics of the real-world decision-making problems facing humans today are multidimensional and have multiple objectives including economic, environmental, social, and technical ones. Hence, it seems natural that the consideration of many objectives in the actual decision-making process requires multiobjective approaches rather than single-objective. One of the major systems-analytic multiobjective approaches to decision-making under constraints is multiobjective optimization as a generalization of traditional single-objective optimization. Although multiobjective optimization problems differ from single-objective optimization problems only in the plurality of objective functions, it is significant to realize that multiple objectives are often noncommensurable and conflict with each other in multiobjective optimization problems. With this observation, in multiobjective optimization, the notion of Pareto optimality or efficiency has been introduced instead of the optimality concept for single-objective optimization. However, decisions with Pareto optimality or efficiency are not uniquely determined; the final decision must be selected from among the set of Pareto optimal or efficient solutions. Therefore, the question is, how does one find the preferred point as a compromise or satisficing solution with rational procedure? This is the starting point of multiobjective optimization. To be more specific, the aim is to determine how one derives a compromise or satisficing solution of a decision maker (DM), which well represents the subjective judgments, from a Pareto optimal or an efficient solution set. An enormous number of articles together with several significant monographs and books have been published since the First International Conference on Multiple Criteria Decision Making was held at the University of South Carolina in 1972, and nowadays, multiobjective optimization is considered to make a major contribution to decision-making under certainty or some probabilistic settings.

However, recalling the imprecision or fuzziness inherent in human judgments, two types of inaccuracies of human judgments should be incorporated in multiobjective optimization problems. One is the experts' ambiguous understanding of the nature of the parameters in the problem-formulation process, and the other is the fuzzy goals of the DM for each of the objective functions. For handling and tackling such kinds of imprecision or vagueness in human beings, it is not hard to imagine that the conventional multiobjective optimization approaches, such as a deterministic or even a probabilistic approach, cannot be applied. The motivation for multiobjective optimization under imprecision or fuzziness comes from this observation. For this reason, multiobjective optimization under imprecision or fuzziness seems to be particularly promising and applicable for dealing with human-centered decision-making problems in most practical situations. Fortunately, due to the theory of fuzzy sets initiated by Zadeh, if we look at recent developments in multiobjective optimization under uncertainty and imprecision, we can see remarkable advances in the field of so-called fuzzy multiobjective optimization.

Although a number of books in the field of multiobjective optimization have already been published in recent years, they focus mainly on multiobjective optimization under certainty or some probabilistic settings. In spite of its urgent necessity, there seems to be no book which is designed to incorporate both types of fuzziness of human judgments into multiobjective optimization in a unified way.

In this book, the author is concerned with not only presenting a unified presentation of some of the most important methods for both multiobjective optimization and fuzzy multiobjective optimization, but also introducing the latest advances in the new field of interactive multiobjective optimization under fuzziness together with a wide range of actual applications on the basis of the author's continuing research works. Special stress is placed on interactive decision-making aspects of fuzzy multiobjective optimization for human-centered systems in most realistic situations under fuzziness.

In addition to presenting the several interactive methods as well as basic notions, this book contains the interactive computer programs for almost all of the linear programming-based interactive methods introduced in the book with numerical examples. These programs are written in C language and compiled by MS-C (Microsoft C compiler) Ver. 6.0 for IBM PCs. The programs are provided on floppy diskettes for implementation on IBM PCs.

The intended readers of this book are senior undergraduate students, graduate students, and specialists in systems analysis. This includes scientists and executives who are mainly interested in decision-making problems with several conflicting objectives under fuzziness, such as public decision-making, administrative planning, and managerial decision-making, together with practitioners for social engineering in any field. In order to master all the material discussed in this book, the readers would probably be required to have some background in linear algebra and mathematical programming. However, by skipping the mathematical details together with using the interactive computer programs, much can be learned about interactive decision-making methods for human-centered systems in most realistic settings without prior mathematical sophistication.

This book was written while the author was an honorary visiting professor at the University of Manchester Institute of Science and Technology (UMIST), Computation Department, sponsored by the Japan Society for Promotion of Science (JSPS) from March to December 1991 on leave from Hiroshima University. The author would like to express his sincere appreciation to Professor Madan G. Singh of UMIST, whose arrangements and warm encouragement during his stay in Manchester made it possible for him to write this book. Special thanks should be extended to Professor Yoshikazu Sawaragi, chairman of the Japan Institute of Systems Research and emeritus professor of Kyoto University, Department of Applied Mathematics and Physics, for his invariant stimulus and encouragement ever since the author's student days at Kyoto University. The author also wishes to thank all of his undergraduate and graduate students at Kobe University, Iwate University, and Hiroshima University. Special thanks go to his former graduate students Hitoshi Yano, Ichiro Nishizaki, Toru Yumine, Yuji Nango of Kobe University, and Junya Takahashi of Iwate University for their invaluable assistance through discussions and computer works. The author would further like to thank Mr. Colin Lloyd, a teacher at Ducie High School, Manchester, who kindly read the manuscript and made corrections. Finally, the author would like to thank Dr. Ken Derham and John Matzka, Plenum Publishing Corporation, London and New York, for their assistance in the publication of this book.

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Manchester

CONTENTS

CHAPTER 1

INTRODUCTION

1.1 Introduction and historical remarks

Since the First International Conference on Multiple Criteria Decision Making, held at the University of South Carolina in 1972 (Cochrane and Zeleny (eds.) 1973), it has been increasingly recognized that most of the real-world decision-making problems usually involve multiple, noncommensurable, and conflicting objectives which should be considered simultaneously. One of the major systems-analytic multiobjective approaches to decision-making under constraints is multiobjective optimization as a generalization of traditional single-objective optimization. For such multiobjective optimization problems, it is significant to realize that multiple objectives are often noncommensurable and cannot be combined into a single objective. Moreover, the objectives usually conflict with each other and any improvement of one objective can be achieved only at the expense of another. With this observation, in multiobjective optimization, the notion of Pareto optimality or efficiency has been introduced instead of the optimality concept for single-objective optimization. However, decisions with Pareto optimality or efficiency are not uniquely determined; the final decision must be selected from among the set of Pareto optimal or efficient solutions. Consequently, the aim in solving multiobjective optimization problems is to derive a compromise or satisficing[†] solution of a decision maker (DM) which is also Pareto optimal based on subjective value judgments (see, for example, Chankong and Haimes 1983a; Cohon 1978; Goicoechea, Hansen and Duckstein 1982; Grauer, Lewandowski, and Wierzbicki (eds.) 1982; Grauer and Wierzbicki (eds.) 1984; Haimes, Hall, and Freedman 1975; Hwang and Masud 1979; Nijkamp 1979; Seo and Sakawa 1988; Steuer 1986; Zeleny 1982).

Two approaches for the determination of a compromise or satisficing solution for a DM in multiobjective optimization problems have been developed. They are

(1) goal programming approaches,

(2) interactive programming approaches.

The goal programming approaches, which assume that the DM can specify the goals of the objective functions, first appeared in a 1961 text by Charnes and

[†] "Satisficing" is a term or concept defined by J.G. March and H.A. Simon (1958). An alternative is satisficing if: (1) there exists a set of criteria that describes minimally satisfactory alternatives, and (2) the alternative in question meets or exceeds all these criteria.

Cooper (1961) to deal with multiobjective linear programming (MOLP) problems. Subsequent works on goal programming approaches have been numerous including Charnes and Cooper (1977), Ignizio (1976, 1982, 1983), and Lee (1972).

The interactive programming approaches, which assume that the DM is able to give some preference information on a local level to a particular solution, were first initiated by Geoffrion et al. (1972) and further developed by many researchers such as Chankong and Haimes (1983a), Choo and Atkins (1980), Korhonen and Laakso (1985), Musselman and Talavage (1980), Oppenheimer (1978), Sakawa (1981, 1982), Sakawa and Mori (1983a, 1983b), Sakawa and Seo (1980, 1982, 1983), Sakawa and Yano (1984), Steuer (1977), Steuer and Choo (1983),Wierzbicki (1979, 1980), and Zionts and Wallenius (1976, 1983).

The interactive goal programming method proposed by Dyer (1972) is a first attempt to provide a link between goal programming and interactive approaches. Since then, several goal programming-based interactive methods which combine the attractive features from both goal programming and interactive approaches have been proposed (e.g. Masud and Hwang 1981; Monarchi, Kisiel, and Duckstein 1973; Weistroffer 1982, 1983, 1984).

However, considering the imprecise nature of the DM's judgments in multiobjective optimization problems, fuzzy programming approaches (e.g. Carlsson 1982; Dubois 1987; Inuiguchi, Ichihashi, and Tanaka 1990; Kacprzyk and Orlovski (eds.) 1987; Kickert 1978; Luhandlula 1989; Slowinski 1986; Slowinski and Teghem (eds.) 1990; Zimmermann 1978, 1983, 1987, 1991; Zimmermann, Gaines, and Zadeh (eds.) 1984; Verdegay and Delgado (eds.) 1990) seem to be very applicable and promising for solving multiobjective optimization problems.

The concept of fuzzy mathematical programming on a general level was first proposed by Tanaka et al. (1974) in the framework of the fuzzy decision of Bellman and Zadeh (1970). Since then, fuzzy mathematical programming has been developed not only on a general level but also on a more practical level. Specifically, a relatively practical introduction of fuzzy set theory (Zadeh 1965) into conventional multiobjective linear programming problems was first presented by Zimmermann (1978) and further studied by Leberling (1981) and Hannan (1981). Following the fuzzy decision or the minimum operator proposed by Bellman and Zadeh (1970) together with linear, hyperbolic, or piecewise linear membership functions respectively, they proved that there exist equivalent linear programming problems. However, suppose that the interaction with the DM establishes that the first membership function should be linear, the second hyperbolic, the third piecewise linear and so forth. In such a situation, the resulting problem becomes a nonlinear programming problem and cannot be solved by a linear programming technique.

In order to overcome such difficulties, Sakawa (1983) has proposed a new method using a combination of the bisection method and the linear programming method together with five types of membership functions: linear, exponential, hyperbolic, hyperbolic inverse, and piecewise linear functions. This method was further extended for solving multiobjective linear fractional (Sakawa and Yumine

1983) and nonlinear programming problems (Sakawa 1984b). A brief survey of major approaches to so-called fuzzy mathematical programming proposed before 1984 can be found in the paper of Slowinski (1986) together with the proposed method for solving a water supply planning problem under fuzziness. More comprehensive surveys of the major fuzzy programming approaches proposed through the mid-1980s can also be found in Dubois (1987) and Kacprzyk and Orlovski (1987).

In these fuzzy approaches, however, it has been implicitly assumed that the fuzzy decision or the minimum operator of Bellman and Zadeh (1970) is the proper representation of the DM's fuzzy preferences. Therefore, these approaches are preferable only when the DM feels that the fuzzy decision or the minimum operator is appropriate when combining the fuzzy goals and/or constraints. However, such situations seem to occur rarely, and consequently, it becomes evident that an interaction with the DM is necessary.

Under these circumstances, assuming that the DM has a fuzzy goal for each of the objective functions in multiobjective linear, linear fractional, and nonlinear programming problems, several interactive fuzzy decision-making methods have been proposed by incorporating the desirable features of both the goal programming methods and the interactive approaches into the fuzzy approaches (Sakawa 1984c, 1986; Sakawa and Yano 1984a, 1984b, 1985b, 1986a, 1986b, 1986f, 1988b; Sakawa, Yano, and Yumine, 1987; Sakawa and Yumine 1983; Sakawa, Yumine, and Yano 1984, 1987).

However, when formulating a multiobjective programming problem which closely describes and represents the real decision situation, various factors of the real system should be reflected in the description of the objective functions and constraints. Naturally, these objective functions and constraints involve many parameters whose possible values may be assigned by the experts. In the traditional approaches, such parameters are fixed at some values in an experimental and/or subjective manner through the experts' understanding of the nature of the parameters.

In most practical situations, however, it is natural to consider that the possible values of these parameters are often only ambiguously known to the experts. In this case, it may be more appropriate to interpret the experts' understanding of the parameters as fuzzy numerical data which can be represented by means of fuzzy subsets of the real line known as fuzzy numbers (Dubois and Prade 1978, 1980b). The resulting multiobjective programming problem involving fuzzy parameters would be viewed as the more realistic version of the conventional one.

From this point of view, Orlovski (1983, 1984) first formulated general multiobjective nonlinear programming problems with fuzzy parameters. He presented two approaches to the formulated problems by making systematic use of the extension principle of Zadeh (1965, 1975) and demonstrated that there exist, in some sense, equivalent nonfuzzy formulations.

Along a similar line of fuzzy linear programming proposed thus far, without the explicit interpretation of fuzzy parameters as was pointed out by Orlovski

(1983, 1984), Tanaka and Asai (1984a, 1984b) formulated the multiobjective linear programming problems with fuzzy parameters. Following the fuzzy decision or the minimum operator proposed by Bellman and Zadeh (1970) together with triangular membership functions for fuzzy parameters, they considered two types of fuzzy multiobjective linear programming problems; one was to decide the nonfuzzy solution and the other was to decide the fuzzy solution.

Recently, in order to deal with the multiobjective linear, linear fractional, and nonlinear programming problems with fuzzy parameters characterized by fuzzy numbers, Sakawa and Yano (1985c, 1986c, 1986d, 1986e, 1986g, 1986h, 1987a, 1987b, 1987c, 1987d, 1988a, 1989a, 1989b, 1990b, 1990g) introduced the concept of α-multiobjective programming and (M-) α-Pareto optimality based on the α-level sets of the fuzzy numbers. They then presented several interactive decision-making methods, both without and with the fuzzy goals of the DM, to derive the satisficing solution of the DM efficiently from among an (M-) α-Pareto optimal solution set for multiobjective linear, linear fractional, and nonlinear programming problems as a generalization of their previous results. Recent excellent survey papers of Luhandlula (1989) and Inuiguchi et al. (1990) are devoted to reviewing and classifying the numerous major papers in the area of so-called fuzzy mathematical programming.

Finally, it is appropriate to mention some areas of application for multiobjective and/or fuzzy multiobjective programming approaches. Although most of the early practical applications have been accomplished in the areas of water resources planning (e.g. Haimes 1977, Cohon 1978, Slowinski 1986), regional planning (e.g. Rietveld 1980; Czyzak 1990), and environmental planning (e.g. Sommer and Pollatschek 1978; Nijkamp 1979; Sakawa and Seo 1980, 1982, 1983; Sakawa and Yano 1985d), many other real-world problems are inherently multiobjective. As we look at recent engineering, industrial, and management applications of the multiobjective and/or fuzzy multiobjective approaches, we can see continuing advances. They can be found, for example, in the areas of optimal design of shallow arches (e.g. Stadler 1983a, 1983b), electronic circuit design (e.g. Lightner 1979), media selection in advertising (Wiedey and Zimmermann 1978), operation of a packaging system in automated warehouses (e.g. Nakayama et al. 1980; Sakawa 1983; Sakawa, Yano, and Yumine 1987), management of the erection of a cablestayed bridge (Ishido, Nakayama, Furukawa, Inoue, and Tanikawa 1986) and, pass scheduling for hot tandem mills (Sakawa, Narazaki, Konishi, Nose, and Morita 1986; Sakawa, Narazaki, Nose, and Konishi 1987).

1.2 Organization of the book

In multiobjective optimization problems, multiple objectives are often noncommensurable and conflict with each other. Consequently, the aim is to find a compromise or satisficing solution for a decision maker (DM) from a Pareto optimal solution set on the basis of subjective value judgments.

However, considering the imprecise or fuzzy nature of human judgments, a fuzzy set approach seems to be very applicable and promising for multiobjective optimization problems under fuzziness. Two types of fuzziness of human judgments should be incorporated in multiobjective optimization problems. One is the experts' ambiguous understanding of the nature of the parameters in the problem-formulation process; the other is the fuzzy goals of the DM for each of the objective functions.

Organization of each chapter is briefly summarized as follows.

Chapter 2 reviews the fundamentals of the basic fuzzy set theory initiated by Zadeh, which will be used throughout the remainder of this book. Starting with several basic definitions involving fuzzy sets, Zadeh's extension principle is presented. It provides a general method for extending nonfuzzy mathematical concepts to the fuzzy framework. With the extension principle, operations on fuzzy sets, especially fuzzy numbers, are systematically developed. Bellman and Zadeh's approach to decision-making in a fuzzy environment, called fuzzy decision, is then examined in detail.

In Chapter 3, linear programming is briefly reviewed and the fuzzy linear programming approach proposed by Zimmermann is presented. Fundamental notions and methods of multiobjective linear programming are reviewed and fuzzy multiobjective linear programming is also introduced. Finally, interactive fuzzy multiobjective linear programming is explained in detail.

Chapter 4 can be viewed as the nonlinear version of Chapter 3 and is mainly concerned with interactive fuzzy multiobjective nonlinear programming as well as fuzzy nonlinear programming and fuzzy multiobjective nonlinear programming.

In Chapter 5, in contrast to the multiobjective linear programming problems discussed thus far, by considering the experts' imprecise or fuzzy understanding of the nature of the parameters in the problem-formulation process, the multiobjective linear programming problems involving fuzzy parameters are formulated. Through the introduction of extended Pareto optimality concepts, linear programming-based interactive decision-making methods, both without and with the fuzzy goals of the DM, for deriving a satisficing solution of the DM from among the extended Pareto optimal solution set are presented together with detailed numerical examples.

Chapter 6 is devoted to a nonlinear generalization along the same lines as Chapter 5. Through the use of nonlinear programming, considerable effort is devoted to the development of some refined interactive decision-making methods for multiobjective nonlinear programming problems with fuzzy parameters.

Chapter 7 presents interactive computer programs, which have been developed by the author's group based on the methods introduced in Chapters 3, 4, 5, and 6, in order to facilitate the interaction processes. Moreover, it demonstrates the feasibility and efficiency of both the methods and the corresponding computer programs. Interaction processes for several numerical examples for multiobjective linear and nonlinear programming problems, both without and with fuzzy parameters, are shown under the hypothetical decision maker together with the corresponding computer outputs.

Chapter 8 is concerned with some application aspects of interactive fuzzy optimization. As examples of Japanese case studies, interactive fuzzy optimization methods presented in this book are applied to the operation of a packaging system in automated warehouses, pass scheduling for hot tandem mills, and environmental planning.

Finally, Chapter 9 outlines related topics including multiobjective possibilistic programming, multiobjective programming with fuzzy decision variables and fuzzy parameters, fuzzy multiobjective 0-1 programming, and fuzzy regression analysis.

The Appendix presents generalized scalarizing methods for multiobjective optimization problems, called the hyperplane methods, by putting the special emphasis not only on generating Pareto optimal solutions but also on obtaining trade-off information. The results presented in the Appendix are the theoretical basis for the trade-off information used in Chapters 3, 4, 5, and 6.