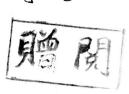
Basic Behavioral Statistics

Gehring

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# Basic Behavioral Statistics



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The Foundation for Pools and hins



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To the memory of my mother

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**Basic Behavioral Statistics** 

# **Preface**

# A New Twofold Approach to Introductory Statistics

Here is perhaps the first serious attempt to make introductory behavioral statistics as easy as possible to understand, yet at the same time challenging to more capable students. Compared with traditional approaches, which aim at either a relatively lower or a relatively higher level of student, this text deals with different motivational and ability levels by offering two books in one.

Every main topic gets double coverage. The initial treatment of each topic is an easy-to-understand presentation suitable for all students. Additional coverage is presented in the supplementary material at the end of each chapter. The supplementary material includes more detailed and specialized applications, plus mathematical relationships and derivations of formulas. This additional coverage offers extra precision, rigor, comprehensiveness, and thought-provoking stimulation for more ambitious students.

# A Study Guide Is Available

A companion Study Guide for students gives complete step-by-step instructions for performing the most important computations. The Study Guide also contains many basic exercises in behavioral applications. It is strongly recommended that students test and perfect their learning by working homework problems in the text, and if they need additional help and practice, in the Study Guide also.

The Study Guide incorporates an additional feature. A number of tables and figures from the text are reproduced in the Study Guide. This enables students to keep each table or figure in front of them for repeated reference while they are reading on in the text and turning to new pages. They do not have to keep their fingers constantly in the text to mark the location of important tables and graphs.

# Overcoming Mental Blocks and Substandard Math Backgrounds

I am particularly sensitive to students' problems in introductory statistics because I teach several courses based largely upon the introductory course. These include advanced parametric statistics, nonparametric sta-

tistics, and the type of experimental psychology laboratory course in which some half-dozen research projects must be statistically analyzed.

With this perspective, I am particularly concerned that introductory statistics students develop at least a certain practical mastery and general understanding of techniques. They should learn enough from the introductory course to function effectively at higher levels, whether or not they are "whizzes" in math.

In order to make the learning process as interesting and as easy as possible, I have employed the following principles:

Most examples are from data that students in the behavioral sciences have found most interesting or most relevant to their studies. This brings a bit of the excitement of current research into the discussion of less exciting methods.

In the first, or basic, portion of each chapter, there is only information that I believe every student should learn. This starts each student off with only essential and understandable information, devoid of unnecessary details, complications, and digressions. Optional or advanced information appears only in the supplementary material at the end of each chapter, in approximate order of usefulness. The following ordering of supplementary material is typical: (1) useful alternate formulas, (2) details and advanced remarks omitted from the basic section, (3) related topics, and (4) mathematical derivations.

Verbal and pictorial or graphic explanations of principles precede mathematical treatment and the introduction of statistical symbols. Thus, principles are initially explained in much the same understandable manner as in "content" courses in behavioral science.

Necessary mathematical passages are presented in a conversational, rather than in a forbiddingly formal, manner.

Distributions of data in examples and exercises often contain just a few numbers in order to make the repetitive element in calculations as simple as possible.

Tables and figures are carefully integrated with text explanations.

Various types of questions often asked by students—some relatively enlightened and some otherwise—are answered in the text (sometimes in parentheses). This adds an element of comprehensiveness and should minimize wasting valuable class time in answering questions that can be answered conveniently in the text.

In certain sections of all chapters, students are periodically asked by rhetorical questions to answer queries concerning the material being covered. Students are best advised to try to answer these questions to the best of their ability before proceeding with their reading. The questions provide spot-checks of progress and understanding. They are designed to remedy problems in understanding as they arise. The questions solidify understanding and help prevent more serious difficulties from occurring later, as students work on the extensive exercises at the end of both the basic and supplementary material in each chapter. While students should think out the answers to the questions themselves, correct answers are also given soon afterward, in the text material that follows.

At various places in the text, brief comments appear in parentheses. These parenthetical comments give extra explanation or clarification, which some students will find helpful to their understanding. When students find the extra parenthetical clarification unnecessary, they may simply skip over it with a quick glance.

Appendices include: (1) a review of basic math, (2) the algebra and geometry of straight lines, (3) statistical tables, (4) review material for a final exam, (5) a glossary of symbols, (6) references, and (7) answers to all text exercises.

Boxes are occasionally used in the basic sections to add useful, but not essential, information.

Coverage allows for the ever-increasing use of preprogrammed calculators owned by behavioral science departments or by students. These calculators automatically compute such statistics as correlation and t at the touch of a key. Their use is the reason for the emphasis on conceptual understanding of formulas rather than on routine mechanics of computation.

The first goal of each student should be to ensure a uniformly high level of mastery of all the basic material. The ambitious student is advised to understand and apply the basic material to the solution of the basic exercises and the Study Guide problems. Whenever it is practical, two or more different ways of solving each type of problem are taught. Thus, solutions arrived at by one process can be checked by a different process to ensure accuracy. Confident students should also test their competence over all the basic material in such a way as to prevent gaps and inconsistencies in their knowledge. Once mastery of the basic material has been indicated by means of successful solutions to the basic text exercises and Study Guide problems, students may in some cases proceed to the supplementary material.

# Challenging the More Ambitious and Capable Students

A substantial amount of useful information appears at the end of each

chapter, in the supplementary material. This material can be used in various ways, or not used at the discretion of the instructor.

Instructors who wish to concentrate on only the most basic and most useful material may assign only readings in the first, or basic, portion of each chapter. Other instructors may assign to all students the basic material plus certain selections from the supplementary material that they believe to be especially important. Or, if they wish, instructors may choose to assign readings in some supplementary sections only to students seeking the best grades or to students planning to study or use statistics beyond the introductory level. Supplementary sections that are not assigned by the instructor may be interesting and informative to students and studied at the students' option. Most supplementary sections are written to be comprehensible independent of the other supplementary material, in order to maximize the flexibility of instructor and student choices.

# **Summary of Main Objectives**

This text's approach should, above all, help to give interest, confidence, and mastery to students who are capable of verbal and graphic understanding, even though their mathematical skills may be in some respects deficient. The approach should also help prevent students from getting in over their heads on topics that they imperfectly understand. Finally, the approach should help minimize embarrassing gaps in basic knowledge and applications. Therefore, students capable of higher levels of mastery should be able to do uniformly superior work on all the main types of statistical applications.

Though the first statistical methodology course is plagued with certain mathematical obstacles not encountered in most so-called "content" courses, the obstacles need not be overwhelming to a student who is motivated and able to do well in the other types of courses. Indeed, it is quite satisfying to see statistics students with bad former experiences in mathematics, high anxiety, initial disinterest, and low self-confidence persevere to master introductory statistics as capably—perhaps even more capably—than they master their other courses. And it is equally satisfying to see the better students achieve uniformly high, insightful, and comprehensive levels of mastery of all types of statistical tests covered.

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R. E. G.

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