DIGITAL FILTERS AND THEIR APPLICATIONS

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Techniques of physics find wide application in biology, medicine, engineering and technology generally. This series is devoted to techniques which have found and are finding application. The aim is to clarify the principles of each technique, to emphasize and illustrate the applications and to draw attention to new fields of possible employment.

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4. V. Cappellini, A. G. Constantinides and P. Emiliani: Digital Filters and their Applications.

Preface

Signal processing methods and techniques now form the basis of very important developments in physics, electrical and electronic engineering, particularly in communications and radar–sonar systems, instrumentation and industrial process control. During the last decade digital methods for signal processing have become more significant in that now they not only replace the more classical analogue techniques in many relevant areas, but they are also being applied in many new areas.

There are several reasons for this development: the consistently high efficiency of digital techniques permits better signal processing and analysis; there is greater flexibility within applications; and there is an increasing availability of general purpose computers and minicomputers, or indeed special type digital processors, at a decreasing cost. Digital techniques have also become more important in two-dimensional (2-D) signal or image processing.

Of all the methods and techniques used for digital signal processing, *digital filtering* is the most important. In the past it has been limited to theoretical research, but recently has been used in many important practical applications for processing 1-D and 2-D signals. This fact may be attributed to:

(i) the availability of efficient and relatively simple design methods;

(ii) extremely fast and impressive technological advances in large and very large scale integration circuits for multipliers, accumulators, memories, with an increase in maximum working frequency and new devices, such as *charge coupled devices* (CCD) and *surface acoustic wave* (SAW) devices;

(iii) advances in computer *hardware* and *software*, particularly with the introduction of *microprocessors* and *microcomputers*, and implementation of fast *array processors*, which are useful as peripheral parts of a computing system or as the main processing system.

As a result of the above developments, digital filtering techniques have been introduced during the last few years in different and extremely important areas such as communications, radar–sonar, the processing of results of physical experiments, aerospace systems, biomedicine, earth resource satellites, etc.

In this book the theory and design of digital filters (1-D and 2-D) are presented together with a description of their practical utility and application in many areas of signal and image processing.

The book is divided into three main parts. The first part (Chapters 1 to 4) is essentially tutorial in character and summarizes the basic relationships for the different types of digital filters, presenting design methods and analysing the error and stability problems. From this mathematical basis, in the second part (Chapters 5 to 7, Appendices 1 to 4), criteria for the practical design of 1-D and 2-D digital filters are derived, and coefficient values, frequency responses and efficiency comparisons are presented. The implementation problems are also considered: useful computer programs are listed and hardware realizations are described. In Appendices 2 to 4, some relatively simple but efficient computer programs (FORTRAN IV) are presented, both for performing the design and the actual filtering. The third part (Chapter 8, Appendices 5 and 6) describes the impact of digital filters in research, operative and industrial areas, presenting interesting applications to signal and image processing in communications, radar, biomedicine, power systems protection and remote sensing.

We have included not only 1-D but also 2-D digital filtering methods and techniques. Thus we are able to compare 2-D problems with 1-D problems and we can clarify which of the methods and techniques used in 1-D can also be applied in 2-D.

It is hoped that the approach adopted in this book, with the inclusion of the theoretical aspects of the subject as well as the practical implementation procedures, will be of interest to researchers and also to practising engineers.

The co-operation of the research groups involved in signal processing at Imperial College, London, at Florence University and at IROE-CNR Institute has made the writing of this book possible. We have drawn both from our own individual experiences and from joint research efforts in bringing together the theoretical developments and the practical applications included in the book. We should also like to thank the British Council for their encouragement and support during the early stages of the above research and also the Consiglio Nazionale delle Ricerche for their support.

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To Grazia, Pamela and Silvana

Contents

Chapter 1

Digital Filters

IA Introduction

In this chapter the fundamental properties of linear digital filters are presented as linear transformations on discrete time functions. Firstly discrete functions are examined in general and subsequently the particular case of discrete functions derived by sampling continuous functions is considered in some detail for its direct relevance to practical applications. The fundamental properties of z-transform, the transform for discrete functions, are summarized. Then the definition of discrete linear systems is given, including descriptions in the time domain, frequency domain and z-transform domain.

Causality, recursivity and stability are discussed, and algorithms for stability tests and stabilization of unstable filters are included; these algorithms are of particular interest, as it will be observed, in the two-dimensional (2-D) case.

Realization structures from given transfer functions are discussed.

Finally the main properties of Discrete Fourier Transform (DFT) and of Fast Fourier Transform (FFT) are summarized, pointing out the aspects more useful for the design and implementation of digital filters.

IB Discrete functions

In the following, the properties of linear digital filters, defined as transformations of discrete functions, are described.[13,15,16]

Discrete functions are defined only for *discrete values* of their variables, that is they are *sequences of numbers*. These numbers can be obtained as quantized samples of a continuous function (representing an analogue signal or image) or they can be the values of a discrete variable such as the readings indicated by, or the output data of, a digital counter. The sequences thus obtained are then processed to obtain other sequences.

The notation which is used for one-dimensional (1-D) sequences is the following

$$\{x(n)\} \qquad N_1 \leq n \leq N_2 \tag{1.1}$$

where $N_1 = -\infty$ and $N_2 = \infty$ for infinite sequences.

For two-dimensional (2-D) sequences, the notation is

$$\{x(n_1, n_2)\} \qquad N_1 \leq n_1 \leq N_2, \qquad M_1 \leq n_2 \leq M_2 \tag{1.2}$$

where $N_1 = M_1 = -\infty$ and $N_2 = M_2 = \infty$ for double infinite sequences.

We can observe that within the terms *sequences of numbers* and *numerical transformations of sequences* the quantization aspect is conceptually included due to the implicit finite precision needed for any numerical representation. However in the development of the theory of discrete linear systems we consider the more general case of sequences of numbers with infinite precision. Thus we use functions whose variable is defined on a discrete set of values, but whose amplitude can assume any value in a continuous way within a specified range.

In Chapter 4 we discuss in some detail the effects and consequences of the amplitude quantization of the input and output sequences of a system, of the numbers which define a system and the arithmetic operations involved in a system.

In many applications, sequences of the type (1.1) and (1.2) are obtained by taking samples of continuous or analogue signals and images. Therefore we consider more precisely the way an analogue signal is related to its samples leading to the sampling theorem.

IC The 1-D sampling theorem

One of the most important applications of digital filtering techniques is in the processing of sequences of samples derived from continuous or analogue signals. This is made possible due to the results and implications of the *sampling theorem*.[8]

This theorem can be stated as follows: "A continuous analogue function $x(t)$ which has a limited Fourier spectrum, that is a spectrum $X(j\omega)$ such that $X(j\omega) = 0$ for $\omega > \omega_m$, is uniquely described from a knowledge of its values at uniformly spaced time instants, T units apart, where $T = 2\pi/\omega_s$ and $\omega_s \geq 2\omega_m$".

To prove this theorem, consider a function $x(t)$ with a Fourier transform $X(j\omega)$ given by

$$X(j\omega)=0 \qquad \text{for} \qquad \omega>\omega_m \tag{1.3}$$

as shown in Fig. 1.1(a).

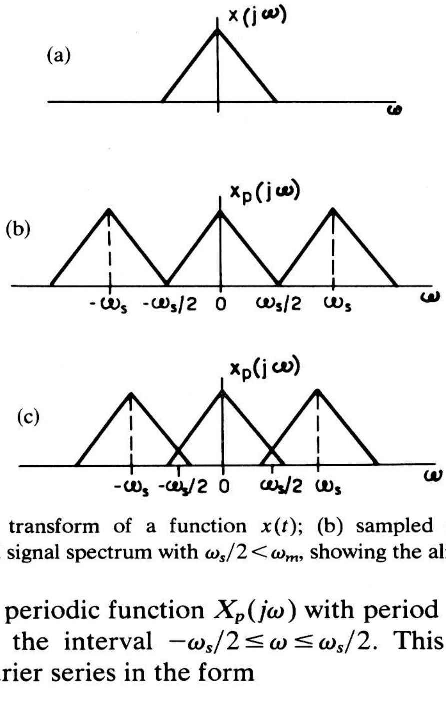

FIG. 1.1. (a) Fourier transform of a function $x(t)$; (b) sampled signal spectrum with $\omega_s/2=\omega_m$; (c) sampled signal spectrum with $\omega_s/2<\omega_m$, showing the aliasing phenomenon.

Consider now a periodic function $X_p(j\omega)$ with period ω_s, which is identical to $X(j\omega)$ in the interval $-\omega_s/2\leq\omega\leq\omega_s/2$. This function can be expanded in a Fourier series in the form

$$X_p(j\omega)=\sum_{k=-\infty}^{\infty} x_k\,\mathrm{e}^{-j\omega kT} \tag{1.4}$$

with coefficients

$$x_k=\frac{1}{\omega_s}\int_{-\omega_s/2}^{\omega_s/2} X(j\omega)\,\mathrm{e}^{+j\omega kT}\,\mathrm{d}\omega \tag{1.5}$$

Now the expression of $x(t)$ as a Fourier transform of $X(j\omega)$ is given by

$$x(t)=\frac{1}{2\pi}\int_{-\omega_s/2}^{\omega_s/2} X(j\omega)\,\mathrm{e}^{j\omega t}\,\mathrm{d}\omega \tag{1.6}$$

Setting $t=kT$, we obtain for the samples of $x(t)$, T units apart,

$$x(kT)=\frac{1}{2\pi}\int_{-\omega_s/2}^{\omega_s/2} X(j\omega)\,\mathrm{e}^{j\omega kT}\,\mathrm{d}\omega \tag{1.7}$$

and by comparing the two expressions (1.5) (1.7) it is easy to obtain

$$x_k = Tx(+kT) \tag{1.8}$$

Therefore, if we know the values of $x(kT)$ for $k=-\infty, \infty$, the Fourier series of the periodically repeated spectrum is uniquely determined by these samples. Further we can show that when $\omega_s \geq 2\omega_m$, an expression can be found for reconstructing the continuous analogue signal from its samples. Over the interval $-\omega_s/2$, $\omega_s/2$, the signal $x(t)$ can be expressed in the form

$$x(t)=\frac{1}{2\pi}\int_{-\omega_s/2}^{\omega_s/2}\left(\sum_{k=-\infty}^{\infty} x_k\, \mathrm{e}^{-j\omega kT}\right)\mathrm{e}^{j\omega t}\,\mathrm{d}\omega=\frac{1}{2\pi}\sum_{k=-\infty}^{\infty} x_k\int_{-\omega_s/2}^{\omega_s/2}\mathrm{e}^{-j\omega kT}\,\mathrm{e}^{j\omega t}\,\mathrm{d}\omega$$

$$=\frac{1}{2\pi}\sum_{k=-\infty}^{\infty} x_k\int_{-\omega_s/2}^{\omega_s/2}\mathrm{e}^{j(t-kT)\omega}\,\mathrm{d}\omega=\frac{1}{T}\sum_{k=-\infty}^{\infty} x_k\,\frac{\sin\left[\omega_s(t-kT)/2\right]}{\omega_s(t-kT)/2} \tag{1.9}$$

and by using (1.8) we obtain

$$x(t)=\sum_{k=-\infty}^{\infty} x(kT)\left(\frac{\sin\left[\omega_s(t-kT)/2\right]}{\omega_s(t-kT)/2}\right) \tag{1.10}$$

The relation (1.10) can be interpreted as the *convolution summation* of a sequence of pulses, with amplitude $x(kT)$ with the analogue filter impulse response

$$h(t)=\frac{\sin\omega_s t/2}{\omega_s t/2} \tag{1.11}$$

This is the impulse response of an ideal and therefore not physically realizable low-pass filter, having a constant amplitude transfer function equal to T in the interval $-\omega_s/2$, $\omega_s/2$ and zero elsewhere. It can be observed that, whilst this filter is indeed not physically realizable (its impulse response is anticipatory), it can be approximated nevertheless by using a sufficiently long time delay.

It is now important to point out that the samples of the signal determine only the periodic spectrum $X_p(j\omega)$, which is called the *sampled signal spectrum*. This spectrum has been obtained through the periodic repetition of the original band-limited spectrum, using a period equal to ω_s. The sampled signal spectrum can also be obtained in general by shifting the spectrum of the continuous signal to all the multiples of ω_s and by summing all these shifted spectra, that is

$$X_p(j\omega)=\frac{1}{T}\sum_{n=-\infty}^{\infty} X(j\omega-jn\omega_s) \tag{1.12}$$