

DEDUCTION SYSTEMS IN ARTIFICIAL INTELLIGENCE

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Contents

Preface	9
I The History of Deduction Systems and Some Applications (J.H. Siekmann)	11
1. Introduction	11
2. The Early History	13
3. The First Deduction Systems	21
4. The Development Since 1965	24
5. The Subdisciplines	29
6. Some Applications	32
A Note in Conclusion	33
References	34
II The Foundations (N. Eisinger & H.J. Ohlbach)	37
1. Introduction	37
2. First-Order Predicate Logic (PL1)	40
2.1 PL1 – The Syntax	40
2.2 PL1 – The Semantics	42
2.3 PL1 – Normal Forms	44
2.4 PL1 – Limitations and Modifications	47
3. Calculi for First-Order Predicate Logic	51
3.1 Gentzen Calculi (Natural Deduction Calculi)	53
3.2 Resolution	56

3.3 Theory Resolution	62
4. Representation	67
4.1 Tableaux	68
4.2 Clause Graphs	72
4.2.1 Clause Graphs as a Data Structure and Indexing Device	72
4.2.2 Clause Graphs with General Reduction Rules	77
4.2.3 Clause Graphs with Specific Reduction Rules	80
4.2.4 Graphs as Representations for Proofs	84
4.2.5 Extracting Refutation Graphs from Clause Graphs	89
4.3 Matrices	92
4.3.1 The Relationship between Matrix Method and Tableau Method	95
4.3.2 The Relationship between Matrix Method and Resolution	96
4.4 Abstract View of the Representation Level	98
5. Control	104
5.1 Restriction Strategies	104
5.2 Ordering Strategies	106
6. An Actual Deduction System	111
References	113
III The Equality Relation	117
1. The Equality Problem (K.-H. Bläsius & H.J. Ohlbach)	117
1.1 Formalizing Equality Within Predicate Logic	118
1.2 Equality as a Subproblem	120
1.3 Subareas of Equality Handling	122
2. General Equality Procedures (K.-H. Bläsius)	123
2.1 Subterm Replacement: Paramodulation	123

2.2	Controlling Resolution with Equality: E-Resolution	126
2.3	Distance Reduction: RUE-Resolution	129
2.4	Equality Graphs	132
2.5	Conclusion	138
	References	139
3.	Unification Theory (Hans-Jürgen Bärkert)	141
3.1	Robinson Unification	141
3.2	Theory Unification	142
3.3	Properties of Solution Sets	145
3.4	Unification Hierarchy	146
3.5	Some Results for <u>Special Theories</u>	148
3.6	Combining Theories and Universal Unification	149
3.7	Example: Unification in Boolean Rings	151
	References	153
4.	Term Rewriting Systems (N. Eisinger & A. Nonnengart)	154
4.1	Introduction	154
4.2	Rewrite Rules	156
4.3	Properties of Term Rewriting Systems	157
4.4	Critical Expressions and Critical Pairs	160
4.5	The Knuth-Bendix Procedure	163
4.6	Extensions of the Knuth-Bendix Procedure	165
4.7	The Knuth-Bendix Procedure as a Proof Procedure	169
	References	174
IV	Computational Logic (H.-J. Bärkert)	177
1.	Introduction: Logic Programs	177
2.	Resolution for Horn Formulas	181
3.	Compilation of Logic Programs	186
4.	Theory Unification in Logic Programs	189
5.	Sorts and Types	194

6. Feature Types	197
References	200
V Complete Induction (D. Hutter)	203
1. Introduction	203
2. The Structure of a Data Base	206
2.1 Definition of Data Structures	206
2.2 Definition of Functions	209
2.2.1 Termination	210
2.2.2 Uniqueness and Completeness	214
3. Proving Function Properties (Lemmas)	217
3.1 Using the Induction Axioms	217
3.2 Special Strategies to Be Used in Induction Proofs	223
3.3 Generalization	224
4. Current State of Research and Open Questions	227
References	228
Suggested Readings	229
Index.....	231

PREFACE

The ability to draw logical conclusions is of fundamental importance to intelligent behavior. For this reason, deduction components are an integral part of numerous artificial intelligence systems. Even though the original motivation for developing these systems was to automatically prove mathematical theorems, their applications now go far beyond. Logic programming languages such as PROLOG have been developed from deduction systems, and these systems are used within natural language systems and expert systems as well as in intelligent robot control. In addition, this field's logic-oriented methods have influenced the basic research of almost all areas of artificial intelligence.

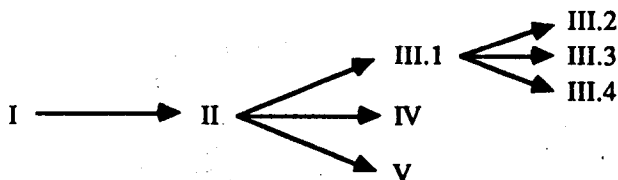
In the past, scientists as well as users from different areas of AI have repeatedly noted the need for introductory material on deduction systems. Such suggestions, voiced most frequently at AI conferences, led us to work out a tutorial on deduction systems for the 1987 GWAI (German Workshop on Artificial Intelligence) and to publish it as a book containing introductory papers on this subject.

This book was written for students, scientists, and users interested in artificial intelligence. It was intended to give the reader an easily understandable yet comprehensive and up-to-date view of deduction systems. Fundamental terms and methods used in these systems will be introduced and explained using many examples and diagrams. The reader will find a certain familiarity with predicate logic useful. In spite of its introductory nature, in many cases this book does represent the current state of research and thus points out new and interesting aspects even to the expert in this field.

The book has been divided into five chapters. The first chapter gives a review of the history of automated deduction and points out several applications of deduction systems. Chapter two makes up the core of this book: here the fundamental methods of this field are explained with the aid of numerous examples. The authors base this chapter on a four level model of an automated deduction system and discuss relevant logics, important

calculi, forms of representation, and control mechanisms. Chapter three deals with equality reasoning. Contributions on general equality reasoning, unification theory, and rewriting systems point out the importance of equality and resulting problems. The fourth chapter deals with foundations of logic programming, and chapter five provides an introduction to automated theorem proving using complete induction.

Because of the relative independence of the contents of chapter III, IV, and V, these can be read in any order. The same is true for sections 2, 3, and 4 of chapter III. The following diagram can be used for orientation:



In Germany, automated deduction has had its own tradition. Even before the publication of the resolution principle in 1965, a team based in Bonn, led by G. Veenker, worked on automated theorem proving methods. W. Bibel has been leading an internationally known team in Munich since the early 1970's. Roughly ten years ago, the "Markgraf Karl Group" was founded in Karlsruhe and later, along with J. H. Siekmann, moved to Kaiserslautern. The authors of the papers collected in this book are or were members of this research team and have worked on developing the Markgraf Karl System, which is known as one of the most powerful deduction systems existing today.

We would like to thank the authors for making this book possible. With their joint contribution, Norbert Eisinger and Hans Jürgen Ohlbach established the basis for the remaining papers and thus for the book as a whole. In addition, their numerous suggestions made the comparatively uniform representation of the different papers possible. Jörg H. Siekmann was willing to provide the necessary historical framework and a review of different applications of deduction systems in the introductory chapter; for this and his helpful criticisms of the individual papers we sincerely thank him. The authors and editors would also like to thank Susanne Biundo, Peter Borst, Richard Göbel, Birgit Hummel, Manfred Kerber, Achim Posegga, Robert Rehbold, Wolfgang Reif, Michael M. Richter, Ingrid Walter, Christoph Walther, and Martin Weigele, who all read parts of the manuscripts and supplied important suggestions.

Konstanz and Kaiserslautern, July 1987

Karl-Hans Bläsius and Hans-Jürgen Bürckert

CHAPTER ONE

THE HISTORY OF DEDUCTION SYSTEMS AND SOME APPLICATIONS

(Jörg H. Siekmann)

1. INTRODUCTION

The development of the first mechanical calculating machines by W. Schickard (1623), B. Pascal (1642), and G. W. Leibniz (1671) in the 17th century is generally seen as a milestone of technology. A reconstruction of the very first machine built by W. Schickard in 1623 in Tübingen is exhibited at our computer science department and draws new admirers from professional as well as students every year. What makes these machines so attractive and what is their historical significance? Neither the mathematical principles they are based on, nor the craftsmanship necessary to build such a machine are exceptional compared to other accomplishments of that century. Yet we know more about the history and development of these machines than about other, academically possibly more impressive contributions of the time. What is it, then, that makes them so outstanding?

These early pieces of evidence from the long history of mechanizing human thought [McC 79] had two important prerequisites for their realization: For one thing, the craftsmanship to manufacture mechanical instruments and clocks were already advanced enough in Tübingen, Paris, Nürnberg and other places such that the technical realization of such a

computing machine did not present a problem too large to overcome. Secondly, computation with numbers, which among the Romans for example had been an art mastered only by a few select specialists, had been systemized to a degree that it had become a comparatively easy and mechanical task even for a layman.

The development of today's deduction systems had two similar prerequisites: the mechanical realization of fast electronic computers on the one hand, and the invention of a logical calculus, i.e. the systemization of the important human ability to draw deductive conclusions from given premises, on the other. The idea to join these two seemingly unrelated developments was at least as historically significant as the early calculating machines. Its consequences can be seen in the field of artificial intelligence (AI), for which automated deduction became a foundational discipline, as well as within traditional computer science: programming and computing not only rely on logic as their foundational science – as anticipated by J. McCarthy in the 1960's – they **are** logical deduction. Using R. Kowalski's famous equation, this can be expressed as:

COMPUTATION = DEDUCTION + CONTROL

This insight has radically changed our view of the computer and of the nature of computation [Kow 79, HS 85, FGCS 84], and is considered by many – because of its scientific and technical consequences – as one of the great discoveries of our century.

While the development of electronic computers from Zuse's Z1 in 1936 to today's LISP machines (lambda calculus) or the fifth generation machines (Horn logic) is probably familiar to most of the readers of this book, the second historical prerequisite, namely the development of a mechanizable logical calculus, has not received the amount of attention it deserves.

2. THE EARLY HISTORY

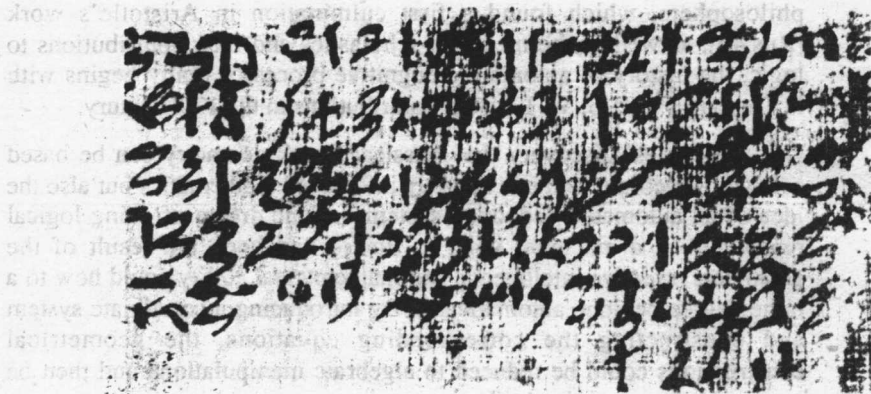
If we ignore for the moment the early logic-oriented studies of Greek philosophers, which found a first culmination in Aristotle's work [Rus 46], as well as the medieval scholastics and their contributions to logic, the history of automating cognitive processes really begins with R. Descartes' and G. W. Leibniz' contributions in the 17th century.

Descartes' discovery that classical Greek geometry can be based solely on algebraic methods not only influenced mathematics but also the idea of an automated deduction system, i.e. the dream of doing logical reasoning on a machine: what to Euclid had been the result of the geometers' creative intelligence and mathematical ability could now to a large degree be done automatically. By introducing a coordinate system and constructing the corresponding equations, the geometrical constructions could be reduced to algebraic manipulations and then be processed almost mechanically.

Descartes was aware of this aspect of his work and put much emphasis on it: "... it is possible to construct all the problems of ordinary geometry by doing no more than the little [namely the four operations suggested by him] covered in the four figures that I have explained. This is one thing which I believe the ancients did not notice, for otherwise they would not have put so much labor into writing so many books in which the very sequence of the propositions showed that they did not have a sure method of finding all..." [Des 37]. His belief, strongly rejected by many mathematicians of his time, was that thus geometry had lost its appeal to the creative mathematician, since it had become mechanizable.

What R. Descartes had done for Euclidian geometry, G. W. Leibniz hoped to extend to the entire area of human thought. Influenced by ideas of the "Ars Magna" of Ramonius Lullus in the 14th century, he suggested a long-range research project (as a note to our funding agency: it took more than 300 years till its first completion): to develop a universal formal language, the "lingua characteristica", in which any proposition could be formulated, as well as a calculus to go with it, the "calculus ratiocinator".

Just as it is possible to translate the following Hau-problem (1800 BC) from one natural language into another, e.g. from demotic characters:



into possibly easier to read hieroglyphs:



or into medieval German:

Form der Berechnung eines Haufens, gerechnet anderthalb mal zusammen mit vier. Er ist gekommen bis zehn. Der Haufe nun nennt sich?

or, for the convenience of our anglo-saxon readers into:

Computation of a heap, taken half as much again together with four. It came unto ten. Knowest thou ye heap?

one can translate it into a formal language more common today:

$$1.5 \cdot x + 4 = 10$$

In this formal language we can compute using only the syntactical operations of a calculus, in this case e.g.

$$y + a = b \Rightarrow y = b - a \quad \text{and} \quad c \cdot z = d \Rightarrow z = \frac{d}{c}$$

In the same manner, Leibniz wanted to translate natural language descriptions also of nonmathematical facts into a formal language ("lingua characteristica") and an appropriate calculus ("calculus ratiocinator").

Even though Leibniz' technical contributions to this research project are considered minor from today's point of view, his conception of the long-range objective and the significance of the project turned out to be most influential. He argued that, in comparison to the Greek mathematicians, who obtained immortality by studying mathematical laws inherent in geometrical bodies even though these do not occur in nature and are of little use in practical applications, how much more important would a mathematical genius be, who was able to study the logical laws inherent in human thought? Leibniz' hope was that such a calculus, once established, would be mechanizable just as the calculating machines for arithmetic developed at his time, and that thus humans could be spared all boring intellectual tasks. "The intellect is freed of all conception of the objects involved, and yet the computation yields the correct result." Leibniz' idea of such a universal language and a corresponding calculus climaxes with the touching description of two people of good will engaged in a philosophical dispute who, in search for the truth, translate their arguments into the "lingua characteristica" and, instead of arguing like two philosophers would, act like two computer scientists and say: "Calcalemus – let's just compute it" [Ger 90].¹

A true fragment of the calculus in the sense of Leibniz was not developed until 200 years later by A. de Morgan and G. Boole and is familiar to most of us as **propositional logic** since our school days. This Boolean algebra, originally interpreted as referring to sets or properties, has, as Boole well recognized, the propositional logic interpretation common today. Boole viewed his algebra as a continuation of Leibniz' programme. The extent to which Boole's works tried to mechanize logical calculations is well demonstrated by a machine developed by the economist and logician Stanley Javins in 1869 to evaluate

¹ "Quo facto, quando orientur controversiae, non magis disputatione opus erit inter duos philosophos, quam inter duos computistas. Sufficiet enim calamos in manus sumere sedereque ad abacos, et sibi mutuo (accito si placet amico) dicere: calcalemus."

Boolean expressions – a machine not unlike the cash registers found in offices during the last century.

Perhaps the most important contribution to the development of a logical calculus, upon which deduction systems of today are based, was made by Gottlob Frege in his book “Begriffsschrift”, which he saw as the fulfillment and elaboration of Leibniz’ research program (restricted to the area of mathematics). To E. Schröder’s [Sch 82] accusation of not sufficiently taking Boole’s work into consideration, Frege answered: “Ich wollte nicht (wie Boole) eine abstracte Logik in Formeln darstellen, sondern einen Inhalt durch geschriebene Zeichen in genauerer und übersichtlicherer Weise zum Ausdruck bringen, als es durch Worte möglich ist. Ich wollte in der That nicht einen blossen “calculus ratiocinator”, sondern eine “lingua characterica” [sic] im Leibnizschen Sinne schaffen, wobei ich jene schlussfolgernde Rechnung immerhin als einen nothwendigen Bestandteil einer Begriffsschrift anerkenne.” [Fre 82].²

Frege’s “Begriffsschrift” contains the first complete development of that part of mathematical logic we today call **first-order predicate calculus**. Its enormous significance becomes apparent when we compare it with the degree of confusion about the different styles and interpretations given to a mathematical logic by Frege’s fellow and predecessor mathematicians – in particular, the discussion of whether the set theoretical interpretation or a propositional interpretation of the Boolean algebra was to be seen as primary. In Frege’s “Begriffsschrift”, the propositional logic interpretation is assumed to be fundamental and is the basis for all further constructions, including the usage of quantifiers. In particular, it established the essential **functional** structure of logic, as it is now common to all of us and developed the basic principles by which today’s logic textbooks introduce the logical connectives, quantifiers, and relations. Thus, the original programme of Leibniz, to reduce logical human thought to a purely syntactical operation had, to an important extent, been realized and completed.

The development and formal construction of a logical calculus is not the only reason for the relevance of the “Begriffsschrift”: it is moreover the methodological contribution by which the syntax and semantics of a formal

² “Unlike Boole, I did not want to represent an abstract logic using formulas; instead I wanted to express a content in formal symbols in a more clearly and precise manner than it is possible using words. In fact, I did not want to create just a “calculus ratiocinator” but a “lingua characterica” [sic] in the sense Leibniz anticipated, indeed recognizing that deductive computation is a necessary component of a “Begriffsschrift”.