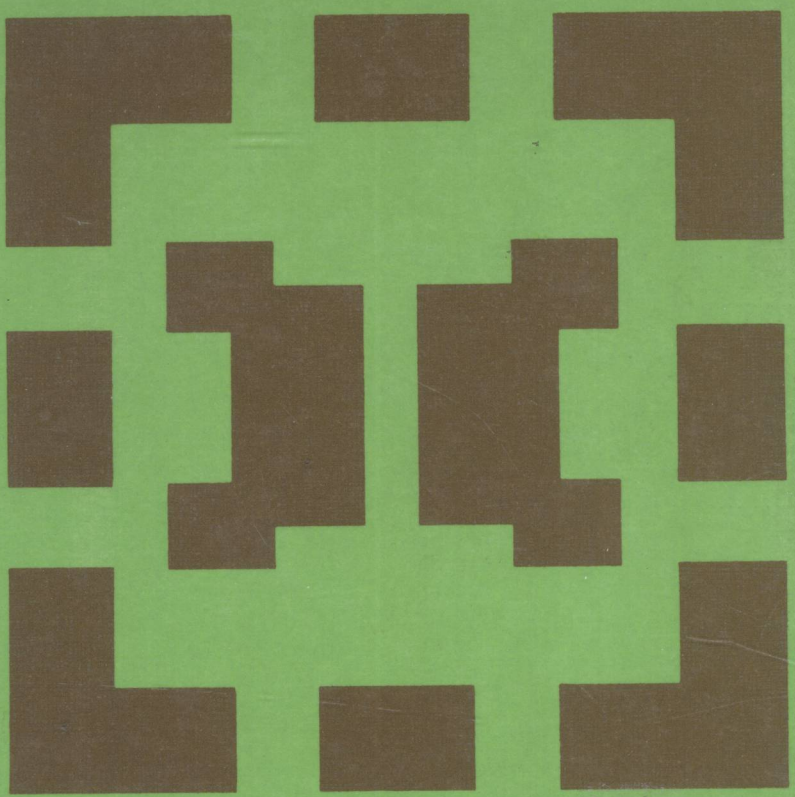
Mathematics and Its Applications

Annie Cuyt (ed.)

Nonlinear Numerical Methods and Rational Approximation



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Nonlinear Numerical Methods and Rational Approximation

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Nonlinear Numerical Methods and Rational Approximation

Mathematics and Its Applications

SERIES EDITOR'S PREFACE

Approach your problems from the right end and begin with the answers. Then one day, perhaps you will find the final question.

'The Hermit Clad in Crane Feathers' in R. van Gulik's *The Chinese Maze Murders.*

It isn't that they can't see the solution. It is that they can't see the problem.

G.K. Chesterton. *The Scandal of Father Brown* 'The point of a Pin'.

Growing specialization and diversification have brought a host of monographs and textbooks on increasingly specialized topics. However, the "tree" of knowledge of mathematics and related fields does not grow only by putting forth new branches. It also happens, quite often in fact, that branches which were thought to be completely disparate are suddenly seen to be related.

Further, the kind and level of sophistication of mathematics applied in various sciences has changed drastically in recent years: measure theory is used (non-trivially) in regional and theoretical economics; algebraic geometry interacts with physics; the Minkowsky lemma, coding theory and the structure of water meet one another in packing and covering theory; quantum fields, crystal defects and mathematical programming profit from homotopy theory; Lie algebras are relevant to filtering; and prediction and electrical engineering can use Stein spaces. And in addition to this there are such new emerging subdisciplines as "experimental mathematics", "CFD", "completely integrable systems", "chaos, synergetics and large-scale order", which are almost impossible to fit into the existing classification schemes. They draw upon widely different sections of mathematics. This programme, Mathematics and Its Applications, is devoted to new emerging (sub)disciplines and to such (new) interrelations as exempla gratia:

- a central concept which plays an important role in several different mathematical and/or scientific specialized areas;
- new applications of the results and ideas from one area of scientific endeavour into another;
- influences which the results, problems and concepts of one field of enquiry have and have had on the development of another.

The Mathematics and Its Applications programme tries to make available a careful selection of books which fit the philosophy outlined above. With such books, which are stimulating rather than definitive, intriguing rather than encyclopaedic, we hope to contribute something towards better communication among the practitioners in diversified fields.

Rational or Padé approximation, the subject of this volume, is still something of a mystery to this editor. Not the basic idea itself, which is lucid enough. But why is the technique so enormously efficient, and numerically useful, in so many fields ranging from physics to electrical engineering with continued fractions, orthogonal polynomials, and completely integrable systems tossed in for good measure.

Anyway, that it is, as a topic, slowly, beginning to be appreciated, is shown, as the editor notes, by the (exponentially) increasing number of papers and conferences on the topic. Reason enough to

take stock and present something of a (partial) survey of the current state-of-the-art. That is what the current volume does.

The unreasonable effectiveness of mathematics in science ...

Eugene Wigner

Well, if you know of a better 'ole, go to it.

Bruce Bairnsfather

What is now proved was once only imagined.

William Blake

As long as algebra and geometry proceeded along separate paths, their advance was slow and their applications limited.

But when these sciences joined company they drew from each other fresh vitality and thenceforward marched on at a rapid pace towards perfection.

Joseph Louis Lagrange.

Bussum, December 1987 Michiel Hazewinkel

EDITOR'S PREFACE

These are the proceedings of a conference on "Nonlinear numerical methods and Rational approximation" organised by Annie Cuyt and Luc Wuytack at the University of Antwerp (Belgium), 20–24 April 1987. The conference focused on the use of rational functions in different fields of Numerical Analysis with sections on Padé approximation and rational interpolation, rational approximation, multidimensional and multivariate problems, orthogonal polynomials and the moment problem, continued fractions, convergence acceleration and their applications. The conference took place at the new campus of UIA, one of the three schools of the University of Antwerp. The organisation of such a conference has almost become a tradition. In the past 15 years the area of rational approximation has developed very rapidly and widely as one can tell from the existing literature. What's more, the number of participants in conferences on this subject keeps growing. This time a total of 58 participants from 16 countries took part. Previous international conferences on the subject were held at Boulder (1972) [11], Canterbury (1972) [8,9], Toulon (1974), Toulon (1975) [4], Tampa (1976) [12], Lille (1977), Lille (1978), Antwerp (1979) [15], Amsterdam (1980) [5], Leuven (1981) [3], Warsaw (1981) [6], Köja (1982) [13], Bad Honnef (1983) [14], Tampa (1983) [10], Bar-le-Duc (1984) [2], Lańcut (1985) [7], Marseille (1985) [1], Segovia (1986) and Havana (1987).

Several one-hour lectures were held by specialists in the field and also a number of short communications were presented. All these lectures were grouped in main sections. Each chapter in the proceedings deals with such a section from the conference. For each section the speakers in that section and the papers contained in the chapter are listed. Thus the proceedings very much reflect the structure and organization of the conference. The included papers are both original research papers and survey papers. I hereby want to thank all the referees whose comments and advices were greatly appreciated. Their work contributed enormously to the speedy publication of this volume.

The organizers would also like to thank the National Fund for Scientific Research (NFWO), the Department of Education (Ministerie van Onderwijs) and the University of Antwerp (UIA) for the financial support as well as IBM, ASLK, SABENA and MIVA for the logistic support. Thanks are also due to "Antwerpen Congresstad" for their very kind assistance and the many interesting brochures, to the "Congrescentrum UIA" for their hospitality and technical know-how, to L. Janssens and F. Schoeters for their administrative help. Without all these people the conference wouldn't have been possible. Last but not least I want to thank the participants: I very much enjoyed meeting all of you !

Annie Cuyt.
Antwerp, 20 October 1987.

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WELCOME

It is my pleasure to welcome you, the participants of the international congress on "Nonlinear numerical methods and Rational approximation". I am glad that the organising committee has chosen this university to host the congress and particularly I want to stress the involvement of the local organisers Dr. Annie Cuyt and Prof. Lucas Wuytack. This meeting gives me the opportunity to say a few words about the University of Antwerp and then to express some views — views of an outsider — on mathematics within the research enterprise in general.

The University of Antwerp is the youngest university in Flanders and has at present about 8000 students. The university is situated on three locations in and around Antwerp. Undergraduate humanities and economics is situated in the downtown area, in the shadow of the cathedral. Undergraduate science and medicine is located in the park area just North of here, close to the open air sculpture museum Middelheim. In the campus area where you are now, most of the upper undergraduate education and also the graduate schools are located with research in mathematics, computer science, physics, chemistry, biology and biochemistry, medicine, pharmacy, law, political and social sciences, literature and linguistics. A university hospital with 610 beds is connected with the medicine department and is, of course, the biggest structure in this campus.

In the program of this congress, I could see that the general topics of interest belong to the field of applied mathematics. I have the feeling that many of the talks deal with fundamental aspects of mathematics. This is certainly not surprising and illustrates the basic relation between fundamental research and science as a means for solving the real problems in life and in society. The relation between fundamental and applied research is controversial, especially now in a period of crisis. Though everybody believes that basic fundamental science is a long term necessity for the development and the economic competitivity of modern states, emphasis is very often laid on strategic and applied research work, in this country but also in the others. In his famous book "The Mathematician's Apology" G.H. Hardy wrote in 1940 (on his research work):

> "I have never done anything useful. Not one discovery of mine has ever contributed to the livability of the world, not for the better, nor for the worse. I did my part in educating mathematicians of my type; their work, as far as I did help them, is as useless as mine."

This citation illustrates the traditional way of thinking of the university scientists, who consider scientific work as an art rather than as a practical tool within society. In short Hardy bragged that his work was of absolutely no practical use to anyone. But now 47 years later it is not sure at all that Hardy's work was as useless as he made believe back in 1940. Science and its results are totally unpredictable and mathematics — even the most fundamental mathematics — are of utmost

importance for present day technology and for present day science in other disciplines. That is, for instance, why the largest multinational firms such as AT&T, Exxon and IBM, just to name a few at random, have large research centers where mathematicians play a very important role.

Think for instance of the computerized axial tomography scanner, the CAT scanner which revolutionised radiology in which classical mathematical techniques play a key-role. Think of the methodology of X-ray crystallography, which was intractable until Herbert Hauptman accomplished the major breakthrough using mathematical techniques which provided him his Nobel prize last year. Think also of the field of elementary particle physics. The string theory for instance was at one time a highly abstract mathematical theory, developed by mathematicians who had no thoughts of applications in mind. Now string theory is a hot topic in elementary particle physics and a key instrument in the interpretation of the most fundamental processes controlling the universe, especially the interactions between elementary particles.

Today there is a growing pressure to make university research more productive on a short term basis. The need of financial resources makes it necessary to find mechanisms to incorporate fundamental research in techniques for solving real problems. It is your task as mathematicians to convince the policy makers of the importance of your discipline in modern society. If mathematics and the fundamental part of it is so important within the scientific complex, let me tell you that more than the practitioners of the other sciences, you mathematicians have an image problem. Your work is more than that of the other sciences inaccessible and your works are often written in a lapidary style. Nobel prize winner Steven Weinberg termed it up this way:

> "When physicists write an article, they generally start with a paragraph saying in more or less understandable terms what the research is all about, but in mathematics I have seen not only articles but entire books in which the first sentence in the preface was "Let A be a nilpotent subgroup"."

The idea is that there should be no word that is not absolutely necessary inserted to help the reader understand what is going on. This also implies that mathematicians are inexperienced in translating their work, and its importance for the general public and hence for the policy makers. I realize however how difficult this must be: major and critical aspects of mathematics often involve the essence of the technical language, and hence by nature are difficult to explain, except to a mathematically literate audience.

We are glad that our university can give you the opportunity to interchange ideas on occasion of this congress. We hope that you will enjoy your stay on our campus. I wish you a pleasant and successful meeting !

F. Adams, rector UIA.
Antwerp, 21 April 1987.

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