Mathematical discourse
language, symbolism and visual images

Kay O'Halloran.

Mathematical Discourse

Language, Symbolism and Visual Images

Kay L. O'Halloran

continuum

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In memory of my father, Jim O'Halloran.
For my brother Greg and his family.

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1   Mathematics as a Multisemiotic Discourse

1.1 The Creation of Order

> Success is right. What does not succeed is wrong. It was, for example, wrong to persecute the Jews before the war since that set the Anglo-Americans against Germany. It would have been right to postpone the anti-Jewish campaign and begin it after Germany had won the war. It was wrong to bomb England in 1940. If they had refrained, Great Britain, so they believe, would have joined Hitler in the war against Russia. It was wrong to treat Russian and Polish [prisoners of war] like cattle since now they will treat Germans in the same way. It was wrong to declare war against the USA and Russia because they were together stronger than Germany.

In this extract from *Berlin: The Downfall 1945*, Beevor (2002: 429) summarizes the views of over three hundred pro-Nazi generals after Germany's defeat in the Second World War, based on a report of interviews by the Supreme Headquarters Allied Expeditionary Force in Europe (SHAEF). The German generals are seen to possess a view of events; one they envisaged would have worked towards victory rather than defeat. Their guiding principle, as expressed by Beevor (2002: 429), is 'Success is right. What does not succeed is wrong.' Many millions participated in the enactment of those views, and the familiar question arises as to how this could be possible. How could so many people be persuaded to take part in the events which unfolded during the course of the Second World War? There have been a variety of responses to this question. Goldhagen (1996), for instance, suggests that most of the ordinary Germans involved in the holocaust were 'willing executioners' who actually believed in the events that took place. No doubt a variety of means were used incrementally over a long period of time in order to mobilize the population in the war effort. In the past century, such massive mobilizations have not been confined to Germany. Weitz (2003), for example, documents the unprecedented programmes of genocide which have taken place in the twentieth century, including Stalin's Soviet Union, Cambodia under the Khmer Rouge and the former Yugoslavia. In these cases and many others, significant portions of the population take part in the war effort. But how can so many people be convinced of the necessity of such programmes, the impact of which lasts for generations?

In attempting to answer such a question, it is worthwhile to consider a simple reformulation of the German generals' guiding principles 'Success is right. What does not succeed is wrong' (Beevor, 2002: 429). That is, if the phrase *for the Nazi party* is inserted, the statement becomes 'Success [for the Nazi party] is right. What does not succeed [for the Nazi party] is wrong.' Such a reformulation introduces in unequivocal terms the basis upon which the guiding principles are constructed. The simple inclusion of the beneficiary 'the Nazi party' makes clear the premise underlying the linguistic statement, and the specific interests which are being served. Such an inclusion also provides room for argumentation and negation, whereas the finality accompanying the original clichéd statement 'Success is right' is much more difficult to counteract. In a similar manner, the import of linguistic choices may be seen in George W. Bush's statement to the world after 11 September 2001 attacks on the United States: 'Either you are with us, or you are with the terrorists' (CNN.com/US 20 September 2001). Expressed in simple terms of a relational set of circumstances, the dichotomy is based on pro-American interests ('with us') versus anti-American interests ('against us'). Such a simple division of the world into two opposing sets of relations leaves few options for a negotiated peace settlement along other possible lines of interest. Language functions in this way to structure the world largely in terms of categories, the nature of which depends upon the choices which are made.

The value of using language and other systems of meaning to create a world view conducive to the war effort was well recognized in Nazi Germany. These strategies included the use of the media for news reports and documentaries (involving language, visual images and music), political speeches and rallies (for example, language, visual images, embodied action, music, and architectural features of the platform and seating arrangements), and particular styles of dress and the distinctive salute of the Nazi party (for example, the uniforms, insignias, actions and gestures). These strategies have direct parallels in existence today, where choices from the different resources combine to create particular meanings to the exclusion of others. However, the contexts which give rise to the ordering of reality are not confined to those which are specifically designed for mass consumption in the form of 'propaganda' programmes. Order is maintained, negotiated and challenged in *every* situation which involves choices from language, visual images, gesture, styles of dress and so forth.

Page (2001: 10) comments: 'There is a privilege in being raised in a time of peace. A luxury that your life is not under immediate threat. War becomes something labelled as heroic, often patriotic, nationalistic. There is a cause, it is just and right, and it somehow excuses all the pain and all the loss.' The use of language and other sign systems for the structuring of thought and reality in the ways described by Page is the subject of this study. This approach is not intended to downplay strategies of physical and mental coercion and abuse. However, violence commences somewhere, and in many cases, for ordinary citizens at least, the starting point is the ordering

of reality along certain lines through semiosis; that is, acts of meaning through choices from language and other sign systems. The major aim of this study is to introduce a theory and approach for examining the nature and impact of semiosis in contexts which span the supposedly inane to the discourses of immense influence, which include the subject matter of this investigation; namely, mathematics and science. War is chosen as the topic to introduce this approach.

The role of language for structuring thought and reality is well recognized today within a wide range of disciplines which include sociolinguistics, critical discourse theory, communication studies, psychology and sociology (for example, Berger and Luckmann, 1991; Bourdieu, 1991; Fairclough, 1989; Gumperz, 1982; Halliday, 1978; Herman and Chomsky, 1988; Vygotsky, 1986). In addition, the functions of visual images are increasingly taken into account (for example, Barthes, 1972; Lynch and Woolgar, 1990; Mirzoeff, 1998; van Leeuwen and Jewitt, 2001). This is especially important in the electronic age where the ease with which pictorial representations may be reproduced is expanding. Beevor (2002), for example, includes visual images in the form of black and white photographs and maps to depict the advance of the Red Army and the final collapse of the Third Reich. *Berlin: The Downfall 1945* is a text or discourse constructed through choices from the English language, photographs and maps. These choices work together to create Beevor's account of the horror of the final months of the Second World War in Germany. In what follows, the types of meanings afforded by Beevor's (2002) photographs are investigated and compared to meanings which are made using language.

Photographs 43–45 displayed in Plate 1.1(1) appear in Chapter 24 in Beevor (2002: 354–369). These photographs appear among a group of inserted photographs which are numbered 30–49. As seen in Plate 1.1(1), Photograph 43 is a picture of a German teenage conscript at the end of the war, Photograph 44 shows a Russian female medical assistant attending to a wounded Russian soldier, and the official signing of the final surrender by General Stumpff, Field Marshal Keitel and Admiral von Friedburg in May 1945 is shown in Photograph 45. In Beevor (2002), these pictures are preceded by photographs of Russians engaged in street fighting in Berlin, scenes outside the Reich Chancellery, convoys of Russian-controlled armed forces, German soldiers surrendering in Berlin, Russian soldiers washing and civilians cooking in the streets of Berlin, victory celebrations between delegates from the Red Army and the US Army, and German civilians escaping across the Elbe River to American territory. Immediately following Plate 1.1(1), there are further photographs of soldiers in the streets of Berlin, the Russian victory parade, and a full-page photograph of Red Army officials visiting the battleground inside the Reichstag. The photographs displayed in Plate 1.1(1) have contextual meaning within this sequence of photographs. Beevor's (2002) linguistic account of the fall of Berlin similarly unfolds as a staged text consisting of sentences, paragraphs, pages

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Plate 1.1(1) Photographs from Beevor (2002: Chapter 24)

and chapters which have contextual meaning within the sequence of the narrative. However, there are differences in the types of meaning afforded by Beevor's linguistic and photographic account of the fall of Berlin. These

differences relate to the meaning potential of language and visual images. This point is developed below.

From existing photographs of the fighting and aftermath in Berlin in 1945, a selection of photographs has been chosen to be included in Beevor (2002). In turn, each photograph in the sequence represents a set of choices made by the photographer, which, in the case of war, most likely happen more by chance rather than design. The photographer captures an instance of time according to the camera angle, the camera distance, the perspective and light conditions, for example. Certain scenes are frozen within the frame, and within those frames human figures are engaged in some form of action in a setting. Further to this, the photographs are developed and reproduced under certain conditions which include choices in terms of paper quality, darkroom techniques, and the possibility for various forms of editing, including cropping and erasure. Putting aside the materiality of the medium and the production process, following O'Toole's (1994) framework for the analysis of paintings, each photograph represents choices at the rank of the whole frame or the Work (in terms of the setting, actions and circumstance), the Episodes in each frame (the activities which are captured), the Figures (the individual people and other objects) and their Members (in terms of body parts and parts of the objects). The impact of these choices in the photographs displayed in Plate 1.1(1) merit close attention.

The settings, physical actions, gestures, facial expressions and the nature of the averted gazes of the human figures in the photographs are juxta-posed in what is a grotesque opposition between the devastation faced by those involved in the fighting (Photographs 43–44) and the well-fed and well-attired defiance of those taking part in the official surrender (Photo-graph 45). This opposition is marked at each rank of the Work, Episode, Figure and Member. For example, the contrast between the physical and emotional state of the soldiers, the medical attendant and the German generals becomes evident in a glance. The quality, style and condition of their respective uniforms at the rank of Figure and Member are similarly diametrically opposed. Compositionally, even the grainy quality of the street scene where the Russian medical assistant attends to the injuries suffered by a soldier (Photograph 44) is placed in stark opposition to the smooth textual quality of the photograph of the official German surrender (Photograph 45).

The situational contexts, actions, experiences and the emotional and physical states of the participants in the fall of Berlin according to circum-stance, nationality, age, gender and position are thus constructed by the photographs. Even if Beevor had the space to describe these dimensions, the meanings of these black and white photographs are impossible to exactly reproduce in narrative form. A linguistic description cannot make the same meanings as Photographs 43–45. The scenes, the interplay of Episodes, the actions and events, the mood of the Figures realized through their embodied actions and appearance cannot be captured using words.

In the same manner, 'Success [for the Nazi party] is right. What does not succeed [for the Nazi party] is wrong' cannot be captured pictorially. Different resources such as language and visual images have different potentials to create meaning. In simplest terms, language tends to order the world in terms of categorical-type distinctions, while visual images such as photographs create order in a manner which to varying degrees accords with our dynamic perceptual experience of the world. The two types of meanings afforded by language and visual images combine in Beevor's account of the fall of Berlin and the collapse of the Third Reich.

The semantic realm explored in this study is not war, rather it is the world offered by mathematics, the discourse which underlies the scientific view of the world. This world came into being largely through the development and refinement of a new sign system, namely mathematical symbolism, which was designed to function in co-operation with language and specialized forms of visual images. The mathematical and the scientific ways of ordering the world permeate our everyday existence, and thus the aim of this study is to understand the nature and the implications of such a view. Before moving to the field of mathematics, Michael Halliday's social-semiotic approach which informs this study is introduced.

1.2 Halliday's Social Semiotic Approach

We impose order on the world, and that order is expressed semiotically through choices from a variety of sign systems. These semiotic resources, or sign systems, include language, paintings and other forms of visual images, music, embodied systems of meaning such as gesture, action and stance, and three-dimensional man-made items and objects such as clothes, sculptures and buildings. A culture may be understood as typical configurations of choices from a variety of semiotic resources. The lecture, the pop song, the political speech, the news report and the textbook are to a large extent predictable configurations of semiotic choices. In a general sense, this understanding of semiotics pertains to 'the specificity of human semiosis' (Cobley, 2001: 260) where 'Semiosis is the name given to the action of signs. Semiotics might therefore be understood as the study of semiosis or even as a "metasemiosis", producing "signs about signs" '(Cobley, 2001: 259). As Cobley (2001: 259) claims, 'Behind this simple definition [of semiotics] lies a universe of complexity.'

Nöth (1990) describes the diversity in theoretical and applied approaches to study of semiotics and Chandler (2002: 207) sees semiotics as 'a relatively loosely defined critical practice rather than a unified, fully-fledged analytical method or theory'. There are many schools and branches of theoretical and applied semiotics, with various definitions and meanings. Nöth (1990), for instance, categorizes semiotics as being concerned with the study of language and language-based codes, text (for example, rhetoric and stylistics, poetry, theatre and drama, narrative, myth, ideology and theology), non-verbal communication, aesthetics and visual

communication. Nöth (1990: 5–6) provides alternative subdivisions which include the semiotics of culture, multimedia communication, popular culture, anthropology, ethnosemiotics, and other topics such as psychosemiotics, socio-semiotics and semiotic sociology, together with the semiotics of disciplines such as mathematics, psychiatry, history and so forth.

Michael Halliday's (1978, 1994, 2004) social-semiotic theory of language known as Systemic Functional Linguistics (SFL) is located within the theoretical realm of what Nöth (1990: 6) terms 'socio-semiotics'. Halliday is concerned with the social interpretation of the meaning of language, and this view is extended to include other semiotic resources such as the maps and photographs found in Beevor (2002) and the mathematical symbolism and diagrams found in the discourse of mathematics. While the basic tenets of the Hallidayan approach to language are introduced below, more comprehensive accounts may be found elsewhere (for example, Bloor and Bloor, 1995; Eggins, 1994; Martin, 1992; Martin and Rose, 2003; Thompson, 1996).

Halliday (1978, 1994) sees language as a tool, where the means through which language is used to achieve the desired results are located within the grammar. The grammar is theorized according to the functions language is required to serve. Halliday (1994) identifies the 'metafunctions of language' as (i) the experiential – the construction of our experience of the world, (ii) the logical – the construction of logical relations in that world, (iii) the interpersonal – the enactment of social relations, and (iv) the textual – the means for organizing the message. The grammatical systems through which these four metafunctions of language are realized are described in Chapter 3.

From the Hallidayan perspective, meaning is thus made through choices from the metafunctionally based grammatical systems. The meaning of a choice (the sign or the syntagm) is understood in relation to the other possible choices within the system networks (the paradigmatic options). Halliday uses the term 'social semiotic' to explain that the meanings of the signs (the semiotic choices) depend on the context of use (the social). The meanings arising from choices from the system networks are negotiated within the social and cultural context in which those choices are made. For example, a linguistic statement such as 'Success is right' does not exist as an abstract independent entity. Rather, the statement means within a context of use, in this case in Beevor's (2002) account of the fall of Berlin. In the same fashion, contexts are established semiotically. For example, the fall of Berlin is constructed by Beevor (2002) and other historians through choices from the semiotic resources of language, maps and photographs. Similarly, the academic lecture is a typical configuration of semiotic choices from the resources of language, visual images, dress, gesture, objects, architecture, seating, lighting and so forth. The configuration of the academic lecture is recognizable by members of a culture, even though the form varies according to discipline and institution.

In order to communicate, members of a culture, or groups within that culture, must possess some sense of shared contextual meaning. Being part of a culture means learning, using and experimenting with the meaning potential of the semiotic systems to create, maintain and negotiate the reality which is socially constructed. Semiotic activity is also used for acts of resistance, which may materialize, for example, in the form of email messages or websites where 'standard' linguistic practices are subverted from the point of view of grammar, lexical choice, text colour and graphics. The dynamic nature of the electronic medium is such that the distinction between the spoken and written modes becomes increasingly blurred with the variations in genre configurations, language choices and graphical representations. However, these new practices eventually become in themselves standardized in much the same way that video texts in the music industry become predictable. The resistance which some discourses initially appear to offer (for example, in the music and film industry, sport and the internet) typically become absorbed into mainstream culture, often in the form of re-packaged commercial products.

The contextual values attached to different choices or combinations of choices from semiotic resources are socially and culturally determined. Members of a culture recognize and maintain or resist those values. Companies such as McDonalds, Nike and Coca Cola, for example, invest large amounts of money in advertising to ensure that their brands and accompanying icons maintain 'the right' social value among the other products on offer. In this way they seek to create and maintain a market for groups of consumers. In one study, Cheong (2004) found that apart from the interpersonally salient component of an advertisement designed to attract the attention of the reader (in many cases a visual image), the only obligatory item in a print advertisement is the company logo. Presumably if the logo was missing, the intertextual relations with other texts in the advertising campaign would ensure that the brand is easily identifiable. Advertising as such means creating an image so that the product or service is viewed as desirable by groups of members of a community. Buying the product thus means acquiring the social and cultural connotative value of that product (Barthes, 1972, 1974).

Human life is negotiated through semiotic exchange within the realms of situational and cultural contexts. Certain combinations of selections function more prominently to structure reality to the exclusion of others. Studies in Systemic Functional Linguistics (SFL) attempt to document and account for the typical linguistic patterns in different types of social interaction or genres; for example, casual conversation (Eggins and Slade, 1997), service encounters (Ventola, 1987), pedagogical discourse (Christie, 1999; Christie and Martin, 1997; O'Halloran, 2000, forthcoming b; Unsworth, 2000) and scientific writing (Martin, 1993b; Martin and Veel, 1998). Other studies of language look at typical patterns along contextual parameters such as gender (for example, Tannen, 1995) and sexuality (for example, Cameron and Kulick, 2003). Forensic linguistics, on the other

hand, is concerned with identifying typical language patterns of the individual (for example, Coulthard, 1993).

Bourdieu's (1991) notion of symbolic and cultural capital of the 'habitus', which is the set of acquired dispositions of an individual or group of individuals, may be conceptualized as semiotic capital; that is, the ability to construct, interpret and reconstruct the world in contextually specific ways. However, the ability to make appropriate meanings in a range of contexts through the use of semiotic resources is unevenly distributed across sections of any community or culture. The reason for this unequal distribution of semiotic capital is related to the educational, economic, social and cultural background of individuals and groups within any community. For example, Bernstein (1977, 1990) identifies the disadvantages students from lower social class backgrounds face in participating in the linguistic practices rewarded in educational institutions. In a sense, being 'educated' means being able to participate in certain types of 'valued' semiotic exchange; for instance, the discourses of medicine, science, business, law, music and art. Certain groups within a society, typically those with wealth and connections, are relatively well placed within the semantic domains which are rewarded (usually by members of that same group). Other groups to varying degrees are marginalized. Increasingly the market-driven practices adopted in schools and universities, such as making entrance dependent on money rather than merit, function to reinforce these divisions of inclusion and exclusion.

Participation in everyday discourse includes semiotic exchange in terms of performative action; that is, selections in the form of gesture, stance, proxemics and dress. Whether delivering a conference paper or giving a political speech, the speaker needs to *talk the talk* (using appropriate linguistic and phonological choices), *walk the walk* (in terms of non-verbal behaviour and action), and increasingly *look the look* (in terms of clothing, hairstyle, make-up, body size, body shape, height, and skin and hair colour, for example) according to the parameters established as desirable in that culture. More generally men and women are urged to identify their 'unique selling point' (USP), be it the talk, the walk or the look. Increasingly acts of meaning inscribed on and through the human body (for example, physical appearance which is increasingly the product of medical procedures and other forms of practices involving drugs, chemicals and so forth) often outweigh the import of other acts of meaning (the talk). In the electronic medium, the performative action and physical creation of identity becomes a textual act. One is no longer constrained by semiosis emanating from the body and the immediate material context. Multiple identities can be established according to the limits of the electronic medium and platforms that are offered, and the user's ability to make use of different semiotic resources, including language, visual images, music and so forth. Semiotic capital comes into play in new ways through computer technology.

The social-semiotic construction of reality (Berger and Luckmann, 1991)

is determined as much by what is included as to what is excluded. As seen in the example of the German generals' 'guiding principles', (i) there are limits to what options *are* selected, and (ii) there are limits to what *can* be selected from the existing systems. In the first case, semiotic selections function as meaning through choice, and so some options (for example, 'success') are chosen to the exclusion of others (for example, 'justice' or 'freedom'), while other possible options are left out (for example, 'for the Nazi party'). In the second case, although systems are dynamic and constantly changing with each contextual instantiation, there are nonetheless at any one time a limited number of options available. We are contained within particular semantic domains according to the limitations of the systems which are available. These systems, however, constantly evolve so that meaning making is a dynamic practice in which change is possible.

Realms of meaning do not exist until they become semiotic choices; for example, the concepts of women's rights, gender and Freud's (for example, 1952, 1954) concept of psychoanalysis are comparatively new linguistic choices. Although perhaps pre-existing as disparate practices, the introduction of these options in language led to radically new ways of conceptualizing women, women's roles and what has become the inner psychosexual self. Similarly, the scientific revolution in the seventeenth century introduced radically new ways of conceptualizing the physical world. The basis for this scientific re-ordering of reality was the development of mathematics which offered new resources in the form of the symbolism and visual display. These semiotic resources combine in significant ways with language to create a new world order. The nature of that order is investigated in this study.

1.3 Mathematics as Multisemiotic

Mathematics and science are considered as 'multisemiotic' constructions; that is, discourses formed through choices from the functional sign systems of language, mathematical symbolism and visual display. These discourses are commonly constituted as written texts, although mathematical and scientific practices are not confined to these forms of semiotic activity. There are many different 'multimodal' genres constituting mathematical and scientific practices; for example, lectures, conference papers, software programs and laboratory investigations. In addition to the written mode, these types of semiotic activity involve spoken discourse, physical action and gesture in environments, which include digital media and day-to-day three-dimensional material reality. The major line of enquiry in this study, however, is directed towards multisemiosis in printed discourses of mathematics, largely because modern mathematical symbolism is a semiotic resource which developed in written format. In order to develop theoretical frameworks for mathematical symbolism and visual display, the print medium has been chosen for investigation. In addition, the effects of computer technology on the nature of mathematical discourse are also

considered in this study. With the exception of the systemic functional (SF) approach to mathematics (Lemke, 1998b; O'Halloran, 1996, 1999b, 2000, 2003a, 2003b, forthcoming a; Veel, 1999), few studies exist in the field of the semiotics of mathematics (for example, Anderson *et al.*, 2003; Rotman, 1987, 1988, 1993, 2000).

Mathematical discourse involves language, mathematical symbolism and visual images as displayed in Plate 1.3(1), a page reproduced from *Physica D*, a journal for research in dynamical systems theory. Plate 1.3(1) contains equations (11), (12) and (13), which are mathematical symbolic statements spatially separated from the main body of the linguistic text. Symbolic statements and elements are also embedded within the linguistic text. For example, symbolic elements function as elements within the linguistic statements in the text located between equations (11) and (13). In addition, there are visual images in the form of mathematical graphs in the three panels labelled Fig. 2 in Plate 1.3(1). Mathematical written discourse may also contain tables which are forms of textual organization where the reader may access information quickly and efficiently (Baldry, 2000a; Lemke, 1998b). As seen in Plate 1.3(1), mathematical printed texts are typically organized in very specific ways which simultaneously permit segregation and integration of the three semiotic resources.

An SF approach to mathematics as social-multisemiotic discourse means that each of the three semiotic resources – language, visual images and mathematical symbolism – is perceived to be organized according to unique discourse and grammatical systems through which meaning is realized. That is, each semiotic resource is considered to be a functional sign system which is organized grammatically. Mathematical texts such as those displayed in Plate 1.3(1) represent specific semiotic choices from the available grammatical systems in each of the three resources. As seen in the graphs and linguistic and symbolic components of the mathematics text in Plate 1.3(1), choices from the three semiotic resources function integratively. That is, the linguistic text and the graphs contain symbolic elements and the symbolic text contains linguistic elements. This feature of mathematical discourse means that the grammars of each resource must be considered in relation to each other.
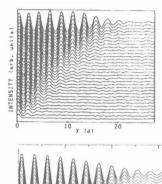
The similarities and differences in the organizing principles of the three semiotic resources are considered *intra-semiotically* in terms of the grammars and functions of each resource. In addition, mathematical discourse is considered *inter-semiotically*; that is, in terms of the meaning which arises from the relations and shifts between the three semiotic resources. Royce (1998a, 1998b, 1999) refers to intersemiotic semantic relations between linguistic and visual components of a text as 'intersemiotic complementarity', and Iedema (2003: 30) calls the process of semiotic shift as 'resemioticization', which he defines as 'the analytical means for . . . tracing how semiotics are translated from one into another as social processes unfold'. In mathematics, intersemiotic shifts take place on a macro-scale across stretches of text, and they also take place on a micro-scale within stretches
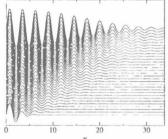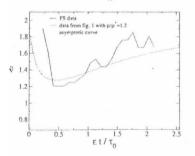
of the wavelength λ_c of the patterns is at criticality about 13% off from the theoretical value; however, we are not interested here in the absolute value, but in the relative variation of λ_c/λ.

The difficulty of comparing theory and experiment on the variation of the wavelength is that the only theoretically sharply defined quantity is the wavelength

sufficiently far behind the front, λ_{as}, and that one has to go beyond the lowest order Ginzburg–Landau treatment to be able to study the pattern wavelength left behind. For example, if we use a Swift–Hohenberg equation for a system with critical wavenumber k_c and bare correlation length ξ_0.

$$\partial_t u = -\frac{(\xi_0 k_c)^2}{4}\left(1+\frac{1}{k_c^2}\frac{\partial^2}{\partial x^2}\right)^2 u + \epsilon u - u^3, \quad (11)$$

then a node counting argument [4,6] yields for the *asymptotic* wavelength λ_{as} far behind the front [6]:

$$\frac{\lambda_c}{\lambda_{as}} = \frac{3\left(3+\sqrt{1+24\epsilon/(k_c^2\xi_0^2)}\right)^{3/2}}{8\left(2+\sqrt{1+24\epsilon/(k_c^2\xi_0^2)}\right)}$$

$$\approx 1 + \frac{\epsilon}{(2k_c^2\xi_0^2)} \quad (\epsilon \ll 1). \quad (12)$$

In the Rayleigh–Bénard experiments, $k_c \approx 2.75/d$, where d is the cell height; the theoretical value is $\xi_0 = 0.385d$, so if our conjecture that the value is some 15% larger is correct, we get $\xi_0 \approx 0.44d$. This then gives

$$\frac{\lambda_c}{\lambda_{as}} \approx 1 + 0.34\epsilon. \quad (13)$$

As we stressed already above λ_{as} is the wavelength far behind the front; for a propagating pulled front, there is another important quantity which one can calculate analytically, the local wavelength λ^* measured in the leading edge of the front. For the Swift–Hohenberg

Fig. 2. Top panel: shadowgraph trace of a propagating front in the experiments of FS for $\epsilon = 0.012$ [16]. The time difference between successive traces is $0.42t_v$, where t_v is the vertical diffusion time in the experiments, and the distances are measured in units d (the cell height) (from [9]). Middle panel: similar data obtained from numerical integration of the Swift–Hohenberg equation also at $\epsilon = 0.012$ starting with a localized initial condition. The time difference between successive traces corresponds to $0.42t_v$. Bottom panel: velocity versus time in the experiment, as obtained by interpolating the maxima of the traces in the top panel, as explained in the text. The dashed line shows the analytical result (8) and the dotted curve the result of the amplitude equation simulation of Fig. 1 with $\mu/\mu^* = 1.2$. Note that the curves are not fitted, only the absolute scale is affected by adjusting ξ_0.

Plate 1.3(1) Language, visual images and symbolism (Kockelkoren *et al.*, 2003: 173)

of text. The potential of intersemiotic processes to produce metaphorical construals is formulated through the notion of 'semiotic metaphor'.

Through close examination of the meaning realized within and across the three semiotic resources, the functions and the semantic realm of

mathematics as a discourse are tentatively formulated in this study. It must be stressed that this is not an account of the entire field of mathematics. Rather it is an account of the semiotic processes and the discourse and grammatical strategies through which mathematics operates to structure the world. From this position, the semantic realm with which mathematics is concerned may be appreciated. This is in part achieved through a comparison of the functions of mathematics with those of language. However, mathematics evolved as a discourse capable of creating a world view which extends beyond that possible using linguistic resources alone. The result of that re-ordering in what is viewed as the scientific revolution is also considered in this study. The implications of viewing mathematical discourse as a multisemiotic construction are considered below.

1.4 Implications of a Multisemiotic View

The multisemiotic approach, where language, visual images and mathematical symbolism are considered as semiotic resources (O'Halloran, 1996), originally stems from O'Toole's (1994, 1995, 1999) extensions of Halliday's (1978, 1994) SF approach to displayed art, and Lemke's (1998b, 2000, 2003) early work in mathematical and scientific discourse. The SF approach to mathematics is welcomed by Rotman (2000: 42) who explains that such an approach offers 'a linguistic/semiotic framework well grounded in natural language that . . . [is] abstract enough to include the making of meaning in mathematics'.

Halliday's (1994) Systemic Functional Grammar (SFG) includes documentation of the metafunctionally based systems which are the grammatical resources through which meaning is made. Halliday's account of the abstract language systems includes statements of how these choices are realized in text. SFG is essentially a 'natural' grammar as it explains how language is organized to fulfil the metafunctions of language: the experiential, logical, interpersonal and textual. Halliday's (1994) model of language described in Chapter 3 provides the basis for the Systemic Functional Grammars (SFG) presented for mathematical symbolism and visual images in Chapters 4 and 5 respectively. These grammars and a framework with systems for intersemiosis are used for discourse analyses of mathematical texts in Chapters 6 and 7. The discussion includes an account of the educational implications of a multisemiotic view of mathematics and the nature of pedagogical discourses in mathematics classrooms.

The SFGs for mathematical symbolism and visual images are inspired by O'Toole's (1994, 1995) systemic frameworks for the analysis of semiosis in paintings, architecture and sculpture. O'Toole (1994) demonstrates how the SF frameworks may be used so that the viewer can learn to engage directly with instantiations of displayed art rather than depending on the 'knowledge' handed down by art historians and other accredited experts. Bourdieu (1989) further explains that aesthetics and art appreciation are discourses which function covertly to maintain existing social class

distinctions. In this view, 'taste' is a social and cultural product through which group and individual identities are indexed and, as with all symbolic investments, different values are placed on those indices. Needless to say, the highest values are accorded to those who constitute the powerful in society.

> the main reason for this close [semiotic] engagement with the details before our eyes is that it enables everyone to sharpen their perceptions and join in the discussion as soon as they begin to recognize the systems at work in the painting. And everyone can say something new and insightful about the work in front of them. Art history, on the other hand, requires a long apprenticeship ... before they are expected to be able to contribute any new information to a discussion of the work in question. And what kind of information might this be? ... Don't they in fact 'mystify' the painting and make us feel we have nothing to contribute? ... the result is to build an insurmountable wall around this precious property.
>
> (O'Toole, 1994: 171)

Following O'Toole's (1994) example, rather than producing a discursive commentary about the nature of mathematics and its intellectual achievements, the intention behind the SF approach in this study is active engagement with mathematical text in order to understand the strategies through which the presented reality is structured, the content of that reality and the nature of the social relations which are subsequently established. The result is an appreciation and understanding of the functions of mathematical discourse and the strategies through which this is achieved. This is essentially a new approach to mathematics for practising mathematicians, and teachers and students of mathematics. This approach also offers insights for outsiders who typically possess a limited understanding and knowledge of mathematics. The implications of an SF approach to mathematics as a multisemiotic discourse are outlined below in relation to the key ideas and formulations developed in this study. These ideas are revisited in Chapter 7 after the theory and approach have been developed in Chapters 2–6.

Mathematical and Scientific Language

The view of mathematics as multisemiotic has implications about the ways mathematical and scientific language are understood. Traditionally, the nature of scientific language has been viewed in isolation rather than as a semiotic resource which has been shaped through the use of mathematical symbolism and visual display. Scientific language developed in certain ways as a response to the functions which were fulfilled symbolically and visually. On a more global scale, our entire linguistic repertoire has been shaped by the use of other semiotic resources, with the result that many of our contemporary linguistic constructions are metaphorical in nature. For example, certain views become common sense under the guise of metaphorical labels such as 'economic rationalism', 'entrepreneurship' and

'freedom and democracy'. Despite their grammatical instantiation as nouns, these are not concrete or material objects. On the contrary, rather complex and dynamic sets of practices are subsumed under such labels. An understanding of the functions of mathematical symbolism, visual images and other semiotic resources permits a re-evaluation of the role of language in constructing such a naturalized view of the world. As with the vested interest behind the guiding principle, 'Success is right', metaphorical terms need to be critically understood in a historical and contextual manner in order to appreciate the premises behind their construction.

The Grammar of Mathematical Symbolism

An SF framework for mathematical symbolism is presented so that the grammatical strategies through which meaning is encoded symbolically can be documented. This is significant because the grammatical strategies for organizing meaning in symbolic statements differ from those found in language. While members of a culture are capable of using language as a functional resource in various ways, typically the use of mathematical symbolism is restricted to certain groups. One reason for this limited access is that the grammar of mathematical symbolism is not generally well understood. It is important to demonstrate how mathematical meaning is organized, and how the unique grammatical strategies specifically developed in mathematical symbolism so that this semiotic could be used for the solution of mathematics problems. The underlying premise is that mathematical symbolism developed as a semiotic resource with a grammar which had the capacity to solve problems in a manner that is not possible with other semiotic resources. The SFG of mathematical symbolism presented in Chapter 4 explains how this functionality is achieved.

Grammar of Visual Images in Mathematics

Visual images in mathematics are specialized types of visual representation, most typically in the form of abstract graphs, statistical graphs and diagrams. The systemic functional framework for abstract graphs is used to explain how the systems are organized to make very specific meanings which provide a link between the linguistic description of a problem and the symbolic solution. Once again, the functions fulfilled by mathematical visual images are different to those achieved linguistically and symbolically. The systems through which the functions of abstract graphs are achieved are discussed in Chapter 5. This discussion includes insights into the changing roles of visual images in mathematics due to the impact of computer technology. Visualization is undergoing a rapid resurgence due to the increasing sophistication of computer graphics which display numerical solutions generated by the computer. The new ways of manipulating and viewing data through computers are discussed in Chapter 5.

Intrasemiosis and Intersemiosis

While the three semiotic resources in mathematics fulfil individual functions which are not replicable across the other resources (Lemke, 1998b, 2003; O'Halloran, 1996), the success of mathematics depends on utilizing and combining the unique meaning potentials of language, symbolism and visual display in such a way that the semantic expansion is greater than the sum of meanings derived from each of the three resources. Lemke (1998b) refers to this expansion of meaning as the multiplicative aspect of multi-semiosis. Mathematical discourse thus depends on *intrasemiotic* activity, or semiosis through choices from the grammatical systems within each resource, and *intersemiotic* activity, or semiosis through grammatical systems which function across the three resources. Intersemiosis involves reconstrual of particular elements in a second or third resource through intersemiotic shifts or 'code-switching'. Intrasemiosis, or meaning within one semiotic resource, is important because the types of meaning made by each semiotic are fundamentally different. Intersemiosis, however, is equally important because not only is the new meaning potential of another resource accessed, but also metaphorical expressions can arise with such shifts. This important process, which may arise in any multi-semiotic discourse, is developed in this study through the notion of semiotic metaphor. The functions of mathematics are therefore achieved through intrasemiosis and intersemiosis; that is, meaning through each semiotic resource, and meaning across the three semiotic resources where metaphor plays an important role in the expansion of meaning.

Intersemiotic Mechanisms, Systems and Semiotic Metaphor

Intersemiotic mechanisms provide a description of the ways in which intersemiosis takes place across language, visual images and mathematical symbolism. The intersemiotic mechanisms take place through metafunctionally based systems which are documented in Chapter 6. Semiotic metaphor refers to the phenomenon of metaphorical construals which arise from such shifts across semiotic resources. This process means that expansions in meaning can occur when a functional element is reconstrued in a different resource. For instance, an action realized through a verb in language (for example, 'measuring') may be reconstrued as an entity in a second semiotic resource (for example, a visual line segment or a symbolical distance). Such reconstruals permit expansions of meaning on a scale which is not possible within a single semiotic resource. As explained in Chapter 6, one of the key elements in the success of mathematics is the metaphorical reformulation of elements across the three semiotic resources.

Mathematics Education

The view of mathematics as a multisemiotic discourse is significant in a pedagogical context as often teachers and students do not seem to be aware

of the grammatical systems for mathematical symbolism and visual display, and the types of metaphorical construals which take place in mathematics texts and in the classroom. The ways in which a social-semiotic perspective can inform mathematics teaching and learning are described in Chapter 7. This discussion is based on the functions of language, visual images and the symbolism, their respective grammatical systems and the nature of the intersemiotic activity. Chapter 7 includes a discussion of the nature of pedagogical discourse in mathematics.

1.5 Tracing the Semiotics of Mathematics

In order to introduce the types of meaning found in modern mathematics, a historical perspective is adopted in Chapter 2 to examine the semiotic unfolding of mathematics from the period of the early Renaissance to modern contemporary mathematics. The nature of the projects of early modern mathematics, as exemplified by Descartes and Newton, is seen to lead to the creation of a mathematical and scientific reality which is located within a limited semantic domain. However, at the same time, the semantic expansions afforded by the visual images and mathematical symbolism permitted expansions in the form of scientific description, prediction and prescription. Contemporary thought in mathematics, for example, chaos and dynamical systems theory, also reveals the changes in mathematical theorizations of reality.

Significantly, the mathematical practices advocated by Descartes and Newton have been re-inscripted into new contexts in contemporary times. The beginnings of modern mathematics and science developed in what was originally conceived as a transcendental realm which necessitated the existence of God, as seen in the discussion of Cartesian and Newtonian philosophy in Chapter 2. The re-inscription of the supposedly 'value free' discourses of mathematics and science as universal truth into new realms of human endeavour such as the social sciences, education, business, economics and politics is questioned from the relatively fresh perspective of the socio-semiotics of mathematics in Chapter 7. This discussion also contributes to an appreciation of the metaphorical nature of our semiotic constructions and the limitations of the contexts in which mathematics may be usefully applied. Mathematics is thus first viewed in a historical context so the functions for which mathematics was originally designed and the context of that development may be appreciated. From this point, the new contexts in which mathematics is re-inscribed are critically examined. Although mathematics has expanded into new fields, the semiotic resources nonetheless essentially remain linguistic, visual and symbolic. Computation is considered a symbolic undertaking which is instantiated in an electronic medium.

An understanding of the scientific view of the world made possible through mathematics is an overriding theme because such a view is vital for an understanding of contemporary Western culture which

materializes as a technological project shaped by the discourse of mathematics and science. Looking back, the rationalist project of the eighteenth century and the consequent mathematical, scientific and technological achievements of the modern period appeared to hold much promise for the world. As Horkheimer and Ardorno (1972) claim, the much-touted aim of progress was the improvement of the human condition accompanied by freedom, equality and justice. In retrospect, however, such progress seems to have been made for the advantage of the relatively privileged few. In addition to providing the infrastructure for unequally distributed goods and services such as healthcare and education, advances in mathematical and scientific knowledge appear to have primarily provided the means for technological development which is directed and controlled by military, business and political interests. As Davis (2000: 291) claims: 'Through advanced science and technology, warfare utilizes many mathematical ideas and techniques. The creation of vast numbers of new mathematical theories over the past fifty years was due in considerable measure to the pressures and the financial support of the military.'

The self-evident deliverables of the scientific project were underscored in the aftermath of the Second World War and, in a more recent case, the US-led war in Iraq in 2003 where the destructive power of military technological innovation was widely televised. As Horkheimer (1972: 3) claims: 'In the most general sense of progressive thought, the Enlightenment has always aimed at liberating men from fear and establishing their sovereignty. Yet the fully enlightened earth radiates disaster triumphant.' Today the extent to which the military, business and the political institutions can be differentiated as separate functioning bodies becomes increasingly difficult to ascertain. One could include universities on the list of institutions which increasingly function pragmatically along the lines of business-orientated commercial interests.

The soundness of reason depends on the explicit or implicit premises upon which that reasoning is based. The view that mathematical and scientific reasoning is constructed to order the world along certain principles which change is not new (for example, Derrida, 1978; Foucault, 1970, 1972; Kuhn, 1970). However, the approach adopted in this study is to understand the systems and strategies through which that ordering takes place. In this way, the functions of these discourses may be understood, and through such awareness we can understand our own positions and explore possibilities other than those directly offered. This is an exploration of the world view offered by mathematics and science, a view which dominates our everyday thinking. It is also a critique of that world view which is so often misunderstood as universal truth.

The path is developed through an excursion through early printed mathematical texts to understand the context behind modern mathematics. SFGs are used to critically interpret the nature of meanings made in contemporary mathematics. Through an understanding of the discourse,

we may start to count the gains and costs of the mathematical and scientific view of the world. In the view of Davis (2000: 291):

> The mathematical spirit both solves problems and creates other problems. What is the mathematical spirit? It is the spirit of abstraction, of objectification, of generalization, of rational or 'logical' deduction, of universal quantization, of computational recipes. It claims universality and indubitability. I have the conviction . . . that this spirit is now . . . pushing us too hard, pushing us to the edge of dehumanization.

The ways in which 'mathematics is pushing us too hard' are investigated through an understanding of mathematics as a multisemiotic resource. Only then can we begin to appreciate the ways in which this discourse and scientific order function to shape our view of ourselves, and our relations to others and the world around us.

1.6 Systemic Functional Research in Multimodality

This study of mathematics represents part of a growing movement in SFL (see Iedema, 2003) where language is conceptualized as one resource which functions alongside other semiotic resources. This research field is commonly called 'multimodality', or the study of 'multimodal discourse' (for example, Baldry, 2000b; Baldry and Thibault, forthcoming a; Kress, 2000, 2003; Kress et al., 2001; Kress and van Leeuwen, 1996, 2001; Levine and Scollon, 2004; O'Halloran, 2004a; Unsworth, 2001; Ventola et al., forthcoming). Apart from the research in mathematics (Lemke, 2003; O'Halloran, 1996, 1999b, 2003b, forthcoming a), studies have been completed in a wide range of fields including science (Baldry, 2000a; Kress et al., 2001; Lemke, 1998b, 2000, 2002), biology (Guo, 2004b; Thibault, 2001), multiliteracy (Lemke, 1998a; Unsworth, 2001), film and television (Iedema, 2001; O'Halloran, 2004b; Thibault, 2000), music (Callaghan and McDonald, 2002), museum exhibitions (Pang, 2004), shopping displays (Ravelli, 2000), TESOL (Royce, 2002), hypertext and the electronic medium (Jewitt, 2002; Kok, 2004; Lemke, 2002) and advertising (for example, Cheong, 2004). Research in the field of multimodality also includes the development by Anthony Baldry et al. (Baldry, 2004, forthcoming; Baldry and Thibault, 2001, forthcoming a, forthcoming b) of an on-line multimodal concordancer, the Multimodal Corpus Authoring (MCA) system, which is web-based software for the analysis of phase and transitions in dynamic texts such as television advertisements, film and web pages.

There have been attempts to construct grammatical frameworks for different semiotic resources (see Kress and van Leeuwen, 1996, 2002; O'Halloran, 2004a). However, with the exception of Thibault's (2001) approach to the theory and practice of multimodal transcription and Baldry and Thibault's notion of phase for the analysis of dynamic texts (Baldry, 2004, forthcoming; Baldry and Thibault, 2001, forthcoming b), few comprehensive theoretical and practical approaches have been developed in the field of multimodality. Consequently, a meta-language for

an overarching model for the theory and practice of multimodal discourse analysis remains at a preliminary stage. Partly as a consequence of this lack of a meta-theory, there exist problems of terminology in studies of multi-modality, as noted by Iedema (2003: 50). For example, there is confusion over the use of the terms 'mode' versus 'semiotic', and, consequently, 'multi-modal' versus 'multisemiotic'. Given that this field represents a relatively new area of research, this is to be expected as the much needed frameworks undergo development.

As an example of mixed terminology, Kress and van Leeuwen (2001: 21–22) define 'mode' as the 'semiotic resources which allow the simul-taneous realization of discourses and types of (inter)action. Designs then use these resources, combining semiotic modes, and selecting from the options which they make available according to the interests of a particular communication situation.' From this position, Kress and van Leeuwen (2001: 22) see Narrative, for example, as a mode. In this study, however, the term 'semiotic' is used to refer to semiotic resources such as language, visual images and mathematical symbolism. These semiotic resources have unique grammatical systems through which they are organized. Any dis-course that involves more than one semiotic resource is therefore termed 'multisemiotic' rather than 'multimodal'. The use of the term 'multimodal' is explained below.

The term 'mode' in SFL, following Halliday and Hasan (1985), typically means the role language is playing (spoken or written) in an interaction. This sense, adopted in this study, is concerned with the nature of the action of semiosis; that is, whether it is auditory, visual or tactile, for example. It follows that different semiotic resources are constrained in terms of pos-sible modes through which the semiotic activity can take place. For example, language may be instantiated orally or visually, but visual images are instantiated through the visual mode in different media such as print, electronic media and three-dimensional space. On the other hand, Kress and van Leeuwen (2001: 22) use the term 'medium' to refer to the '[m]aterial resources used in the production of semiotic products and events, including both the tools and the materials used (for example, the musical instrument and air; the chisel and the block of wood. They are usually specially produced for this purpose, not only in culture (ink, paint, cameras, computers), but also in nature.'

In order to maintain existing systemic terminology, in this study the term *mode* is used to refer to the channel (auditory, visual or tactile, for example) through which semiotic activity takes place, *medium* for the material resources of the channel, and *genre* for text types such as the Narrative (which is realized through language in the spoken or written form). The term *multisemiotic* is used for texts which are constructed from more than one semiotic resource and *multimodality* is used for discourses which involve more than one mode of semiosis. A radio play featuring speech, music and diegetic sound is therefore multisemiotic rather than multimodal as it involves multiple semiotic resources realized through the auditory mode of

sound through the medium of the radio. However, a website which contains written linguistic text and a music video clip is multisemiotic (involving language, visual images, music) and multimodal (visual and auditory). The practices adopted here do not attempt to solve the problems of mixed usages of terminology, rather they seek to clarify the use of the terms adopted in this study. In this respect, mathematics is referred to as multi-semiotic as it consists of three semiotic resources, language, visual images and mathematical symbolism. Mathematics is considered to be primarily a written discourse produced in printed and electronic media. There are also multimodal genres in the field of mathematics, such as the academic lec-ture, which involves spoken discourse and other semiotic resources. The multimodal nature of mathematical pedagogical discourse is discussed in Chapter 7.