TELECOURSE GUIDE TO ACCOMPANY

INTRODUCTION TO

Contemporary Mathematics

TELECOURSE GUIDE TO ACCOMPANY

FOR ALL PRACTICAL PURPOSES

INTRODUCTION TO

Contemporary Mathematics

W. H. FREEMAN AND COMPANY
NEW YORK

Major funding for the FOR ALL PRACTICAL PURPOSES: INTRODUCTION TO CONTEMPORARY MATHEMATICS telecourse and for the television series, FOR ALL PRACTICAL PURPOSES, has been provided by the Annenberg/CBP Project. Additional funding has been provided by the Carnegie Corporation of New York. The Alfred P. Sloan Foundation funded the development of the textbook.

Printed in the United States of America

ISBN 0-7167-1898-7

2 3 4 5 6 7 8 9 0 HA 6 5 4 3 2 1 0 8 9 8

Preface

For All Practical Purposes: Introduction to Contemporary Mathematics is an introductory mathematics course for students in the liberal arts or other nontechnical curricula. The course consists of twenty-six half-hour television shows, the textbook, and this Telecourse Guide. This series shows mathematics at work in today's world. Part of the power of television is its ability to bring you to locations where you can see mathematics at work solving practical problems.

For All Practical Purposes aims to develop conceptual understanding of the tools and language of mathematics and the ability to reason using them. We expect most students will have completed elementary algebra and some geometry in high school. We do not assume students will be pursuing additional courses in mathematics, at least none beyond the introductory level.

The organization of this Telecourse Guide parallels the television shows. For each of the five subject-area clusters, the television series contains an overview show and four additional programs. For each of the programs in the series you will find a section in this guide.

How to Use This Guide

For each program in the series this Guide includes

- an outline and summary of the program.
- a list of learning objectives.
- sample examinations with answers. The examinations consist of multiple choice questions and long-answer questions.

Following each section in the textbook is a list of review vocabulary. This list of terms will help you understand the most important concepts. Before viewing a program you will find it valuable to read over the respective section in this Telecourse Guide, including the list of objectives. The objectives will provide you with an idea of the topics to be covered in each program.

The chart on page vii lists the appropriate textbook chapter to be read in conjunction with each show. Textbook page numbers are included for easy reference. You will find additional explanation of key concepts in the summary sections of the Telecourse Guide, and additional worked-out examples in those same sections.

The same mathematical concepts are explained and illustrated in slightly different ways in the television programs, Telecourse Guide, and textbook. Each part of the presentation of a subject area complements the others. For example, you will find that the worked-out examples and discussions contained in this Telecourse Guide may give different emphases for a particular topic than does the textbook discussion. Thus, to get the greatest benefit from the course, you should study all parts of the instructional package.

Contents

Reference Chart to Textbook

Television Program	Telecourse Guide Pages	Textbook Chapter	Textbook Pages
Overview Show *Management Science*	1–2	Part I Introduction	1–3
Show One Street Smarts	3–9	1 Street Networks	5–19
Show Two Trains, Planes, and Critical Paths	10–19	2 Visiting Vertices	24–39
Show Three Juggling Machines	20–32	3 Planning and Scheduling	49–64
Show Four Juicy Problems	33–41	4 Linear Programming	69–83
Overview Show Statistics	42	Part II Introduction	87–88
Show One Behind the Headlines	43–49	5 Collecting Data	89–108
Show Two Picture This	50–55	6 Describing Data	113–130
Show Three Place Your Bets	56–60	7 Probability: The Mathematics of Chance	137–150
Show Four Confident Conclusions	61–67	8 Statistical Inference	155–165
Overview Show Social Choice	69–70	Part III Introduction	171-173
Show One The Impossible Dream	71–75	9 Social Choice: The Impossible Dream	174–186
Show Two More Equal Than Others	76–78	10 Weighted Voting Systems: How to Measure Power	191–208
		12 Fair Division and Apportionment	230–245
Show Three Zero-Sum Games	79–82	11 Game Theory: The Mathematics of Competition	213–220
Show Four Prisoners' Dilemma	83–86	11 Game Theory: The Mathematics of Competition	220–224
Overview Show On Size and Shape	87–88	Part IV Introduction; 13 Patterns	251–267
Show One How Big Is Too Big	89–94	14 Growth and Form	270–283
Show Two It Grows and Grows	95–99	15 The Size of Populations	285–299
Show Three Stand Up Conic	100–105	17 Measuring the Universe with Telescopes	325–330
Show Four It Started in Greece	106–112	16 Measurement	304–317
		17 Measuring the Universe with Telescopes	331–339
Overview Show Computer Science	113	Part V Introduction	349–350
Show One Rules of the Game	114–117	18 Computer Algorithms	351–365
Show Two Counting by Two's	118–121	19 Codes	368–373
		20 Computer Data Storage	379–385
Show Three Creating a Code	122–124	19 Codes	367–368 ; 373–377
		20 Computer Data Storage	385–392
Show Four Moving Picture Show	125–127	21 Computer Graphics	395–413

Part I Management Science
Joseph Malkevitch
York College, CUNY

INTRODUCTION

The Management Science television programs and the accompanying textbook are intended to introduce elementary mathematical ideas that make it possible for businesses and governments to perform their functions better. You will learn not only that mathematics is at work in numerous situations of which you were unaware, but also that you can put some of these same ideas to work in situations you face yourselves. Furthermore, many of the ideas presented are based on easy to understand diagrams and do not require complicated algebra. Obviously, supporting details cannot be provided by the video. This is the job of the text. However, the shows make it possible for you to get an overview of the subject matter and if a still picture is worth 1,000 words, who is to say the value of moving and talking images and computer graphics?

We firmly believe that mathematics cannot be a spectator sport. You no doubt learned to ride a bicycle by getting on one, and you probably had an occasional spill. The analogue here will be taking the risk of trying the exercises and having the confidence you can learn to "ride." Also, it is important to understand that mathematics is a study of patterns. You may have had the experience of being introduced to a fellow student by a friend, and found subsequently you were seeing that person all over campus. Once you see the power that abstraction and looking for patterns brings, you too will see the themes presented here at work in many unexpected places.

OVERVIEW SHOW

You are probably so accustomed to seeing bread on the shelves of the supermarket, to watching skyscrapers and houses rise, to hearing fire engines and ambulances rushing to a fire, and to seeing the garbage disappear from the cans beside your house that you have probably given little thought to how these services are provided. The delivery of goods and services involves expensive equipment and personnel. The goal in such systems is to keep the costs down and the level of services high. Companies and governments have always hired people to work on problems of this sort. Businesses have always had to solve problems with an analytic component. Inventory analysis is something that every company that makes a product has to worry about. Years ago, this used to be part of the art of being a businessman. You could save yourself a lot of money if you were skilled. Similarly, routes for trucks that make deliveries to stores were figured out by hand. Good routes could save a company lots of money. These ad hoc methods, however, are now increasingly being replaced by formal methods and principles from the discipline of operations research. Mathematical ideas have replaced ad hoc methods so that now classes of problems can be attacked simultaneously, without attention to the details of a specific problem. For example, routing problems in general can be studied and algorithms to solve them generated. This is done rather than looking at a garbage collection problem or a snow removal problem for a specific section of a specific town. Management science sheds light on how to schedule "processors," whether these processors are doctors in an OR, checkout clerks in a supermarket, runways at an airport, or photocopy machines.

This show is designed to show the tremendous scope of applicability of management science concepts. Among the many ideas touched on are scheduling and organization of resources, queues (waiting lines), shortest paths and related routing problems, and various optimization problems.

SHOW ONE: STREET SMARTS: STREET NETWORKS

Many routing problems involving traversing streets in a city can be solved using a geometric tool called a *graph*. Given the portion of a map shown in Figure 1(a) below one can use dots called *vertices* to represent the corners, and line segments called *edges* to represent the streets joining two corners. This gives the graph in part (b) of Figure 1.

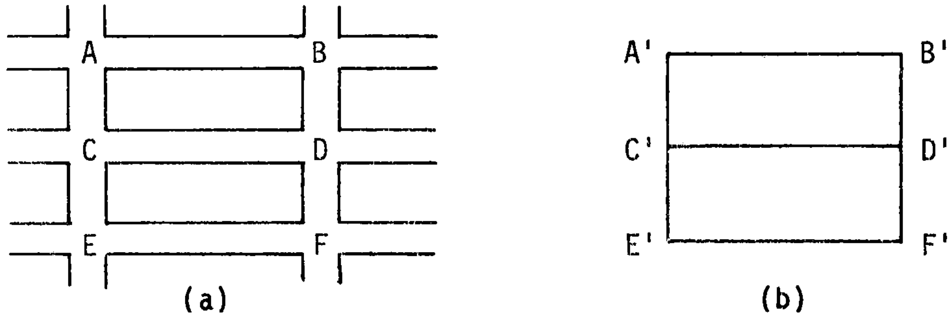


Figure 1.

If instead of streets sidewalks are to be represented (modeled) the resulting graph would be:

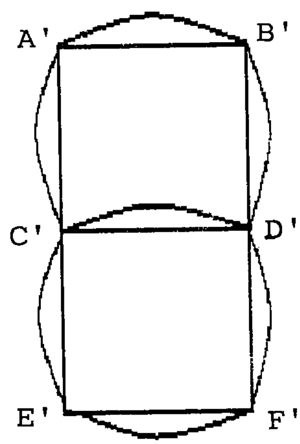


Figure 1(c).

For the purpose of delivering mail or collecting coins from parking meters, it is desirable to start at a corner, traverse each stretch of sidewalk once and only once and return to the start vertex The problem of finding such a route in a graph consisting of one piece is called finding an Euler circuit. It is not hard to show that if a graph is connected (one piece) it has an Euler circuit if all its vertices are even-valent, that is have an even number of edges which meet at the vertex. (The graph in Figure 1(c) has this property.) In practice, it will rarely happen that the graph representing a street network is connected and even-valent. Hence, one would like to traverse the edges of the network starting and ending at the same vertex, with a minimum number of repeated edges. This new problem is known as the "Chinese Postman Problem," and is associated with Meigu Guan (Peoples Republic of China) who first studied the problem in detail. This problem can be solved by duplicating a minimum number of edges in the original graph so that the new graph has only even-valent vertices, a process called "Eulerizing" the original graph. It is not hard to see that the number of repeated edges is at least the number of odd-valent vertices divided by 2. There is no guarantee, however, that this few edges can be repeated. In some graphs (Figure 2 below) every edge must be repeated to Eulerize the graph.

Figure 2.

The power of the ideas described here lies in the large number of settings where ideas of traversing the edges of a graph can arise. These include delivering mail, inspecting curbs, sweeping streets, sanding streets after a snowstorm, and many others.

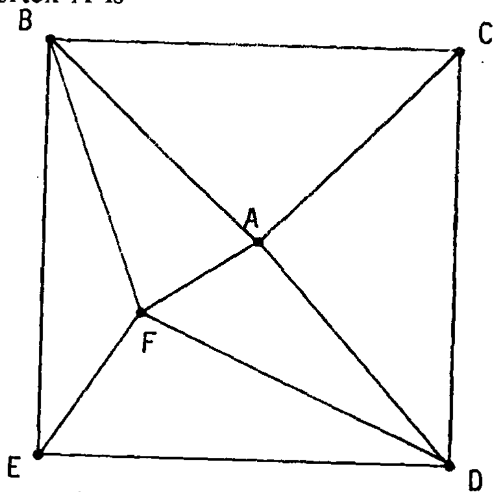
Skilled Objectives:

1. Learn the graph concept.
2. Learn to use graph models for simple problems in operations research.
3. Learn to find valances of vertices of a graph.
4. Be able to find Euler circuits in a graph.
5. Be able to solve chinese postman problems by Eulerizing a graph.
6. Be able to find situations modeled by edge traversal in a graph.

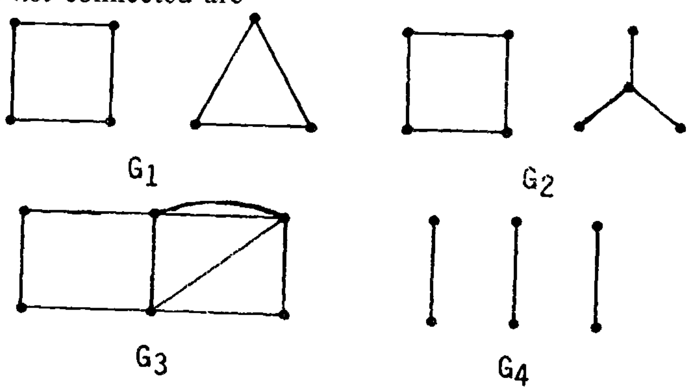
Sample Examination:

MULTIPLE CHOICE

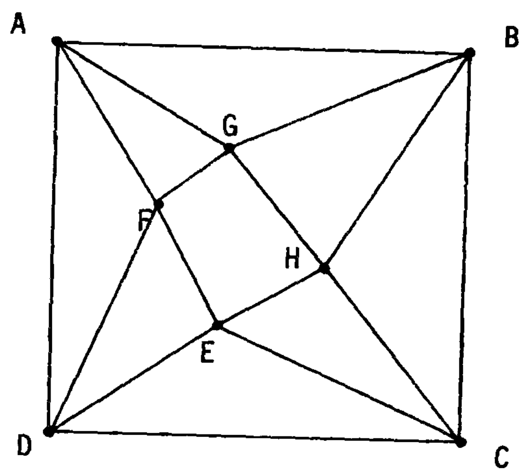
1. In the graph below the valance of vertex A is
   (a) 2
   (b) 4
   (c) 6
   (d) none of the above



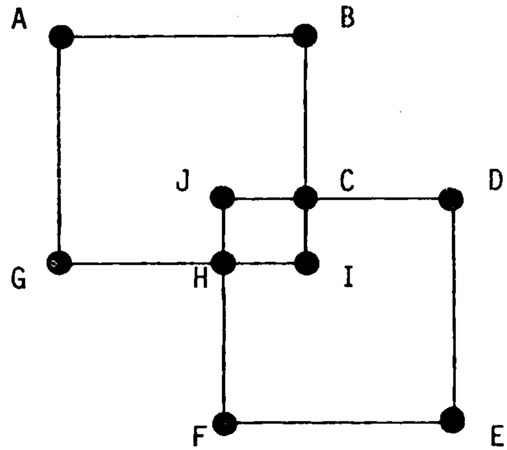
2. The graphs below which are *not* connected are
   (a) G_1, G_3
   (b) G_1, G_2, G_4
   (c) G_3
   (d) G_2, G_3



3. The graph below
   (a) has an Euler circuit
   (b) is not connected
   (c) has an odd-valent vertex
   (d) has only vertices of valence 6

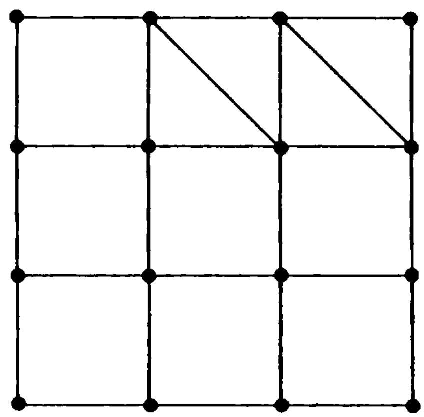


4. An Euler circuit for the graph below is
   (a) A B C J H I C D E F H G A
   (b) A B C I H G A
   (c) J C D E F H A
   (d) J C B A G H I C J



5. Determining if a graph has an Euler circuit might arise from a problem which models
   (a) assigning workers to the jobs they are best qualified for
   (b) scheduling operations
   (c) snow removal for a portion of a city's streets
   (d) inspecting traffic lights at the corners of a small village

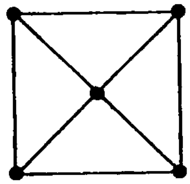
6. Since the graph below has 6 odd-valent vertices, to Eulerize the graph requires the duplication of at least
   (a) 1 edge
   (b) 2 edges
   (c) 0 edges
   (d) 3 edges



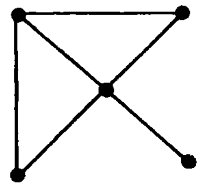
7. The minimum number of duplications to Eulerize the graph in problem 5 is
   (a) 1 duplication
   (b) 3 duplications
   (c) 4 duplications
   (d) 7 duplications

8. A graph which has vertices of valence 4, 3, 3, 2, 2 is

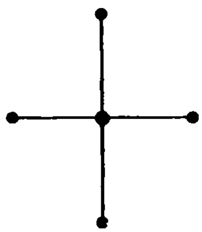
(a)



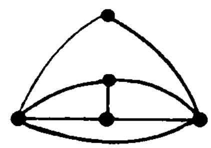
(b)



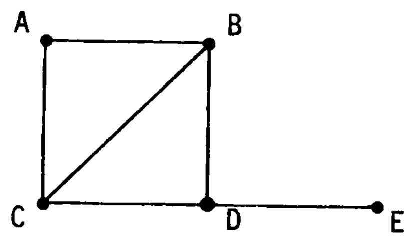
(c)



(d)



9. A route which traverses each edge at least once with a minimal number of repetitions is
   (a) A, B, D, E, D, C, A
   (b) B, C, A, B, D, E, D, B
   (c) A, C, E, D, C, B, A
   (d) A, B, C, D, E, D, B, C, A



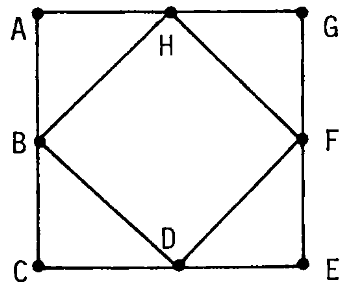
10. A graph G which is connected has a Euler circuit if
   (a) it has 20 edges.
   (b) it has 10 vertices.
   (c) it has no circuits.
   (d) it has vertices all of valance 4.

ANSWERS:
1. (b), 2. (b), 3. (a), 4. (a), 5. (c), 6. (d), 7. (c), 8. (b), 9. (d), 10. (d).

LONG ANSWER

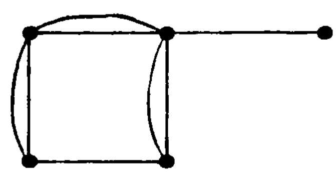
1. Write down an Euler circuit for the graph below or give a reason why it has none.

ANSWER: A, B, H, F, D, B, C, D, E, F, G, H, A



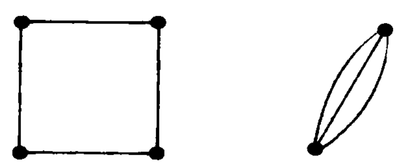
2. Draw a graph which is connected and has valences 5, 4, 3, 3, 1.

ANSWER: There are many possible answers, here is one:



3. Draw a graph which is not connected, and has vertices of valences 2, 2, 2, 2, 3, 3.

ANSWER: Here is one possible solution:



4. Solve the postman problem for the graph shown.

ANSWER: The graph with the duplicated edges shown below is a solution with four repeated edges: