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JAMES LIGHTHILL

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Preface

One of the major modern areas of successful practical application of mathematics is *fluid mechanics*. This is concerned with analysing the motion of either liquids or gases. Both of these, of course, are 'fluids'; that is, they are mobile substances lacking any large-scale order which are capable, consequently, of unlimited deformation and of yielding in time to any disturbing force however small.

Study of the mechanics of fluids is important in many contexts such as the following.

1 Locomotion through fluid media

All animals live immersed in fluid (air or water) and their capability of motion through it is of crucial importance for their life style. Man has greatly modified his life style by devising machines for improved locomotion. A most valuable flexibility is conferred when locomotion is achieved not by pushing the ground (as a walker or a train does) but by pushing the fluid (as in animal swimming or flying, or as in ships or aircraft). Study of all these matters (ranging from zoology to engineering) involves advanced fluid mechanics.

2 Circulation systems

Systems of circulating fluid offer important means for distributing things where they are needed. Once more, zoology gives a good illustration of this through the vital importance of the circulation of the blood. Modern chemical processing plant depends just as crucially upon the liquid or gaseous convection of dissolved chemicals, of suspended particles, or of energy in circulation systems. The atmosphere is a vast circulation system, driven by the heat of the sun, and engaged in the transport of heat, water-vapour, oxygen, carbon dioxide and various pollutants. The ocean is another great circulation system of practically equal importance to man; who needs to investigate the mechanics of the fluid motions involved in all of these.

3 Transfer of energy in engines

Energy stored as potential energy, chemical energy or heat energy becomes converted into kinetic energy in a water turbine, a gas turbine or a steam turbine, in each case by means of fluid flow acting on rotating blades. Such flow is studied in order to improve the efficiency of turbines; which may also, in many cases, depend upon effective fluid motions for transferring heat quickly from one part to another in such an engine.

4 Resistance of structures to wind and water

The design of structures intended to resist strong winds, river erosion, or violent sea motions requires an understanding of what determines the forces exerted by winds, currents or waves upon stationary structures. Although these are complex problems, we may note that two of the relatively less complex problems under headings 1 and 4 here are essentially the same: the fluid mechanics determining the resistance to a vehicle moving through still air; and the fluid mechanics determining the force of a steady wind on a stationary structure. It helps, indeed, to study the former problem *from the standpoint of the moving vehicle*: in a frame of reference in which the vehicle is at rest, the air is blowing past it with equal and opposite velocity, and the vehicle becomes effectively a stationary structure in a wind.

One of the reasons why the motions of fluids are so complex derives from the fundamental property that the fluid is capable of unlimited deformation (unlike solid structures which, in general, are capable of only a very limited degree of deformation without breaking). Some other reasons for complexity will be specially studied in this book under the headings of *boundary layers* and *turbulence*.

An essential characteristic of the application of mathematics to systems of great complexity (like fluids in motion) is that progress can be made only through an efficient cooperation between theory and experiment. There is a big contrast here with the study of much simpler systems for which the basic physical laws are known which allow computer programs to be developed that will predict reliably the behaviour of the system. Most fluid motions, as we shall see, are much too complex for that to be possible even if the largest and fastest of the nineteen-eighties generation of computers is being used.

Great progress with the effective study, and the effective computation, of fluid motions has been made, however, through the realization that such progress required creative inputs on a continuing basis both from theory and from experiment. Even though the basic physical laws underlying the mechanics of fluids are known with precision, typical problems encountered under headings 1 to 4 above involve motions of such complexity that the most powerful computers cannot infer those motions as a straightforward deductive exercise from those basic physical laws. At the same time, experiments on the intricate details of particular fluid motions are possible although they are in general very expensive. How, then, is it possible to make predictions about the vast majority of fluid motions: those which have not been subjected to such detailed experimental probing?

This book seeks to exemplify the answers to that question in an informal, introductory way. Briefly, those answers are based on the creative use of data from experimental studies and data from theoretical analyses to generate practically useful mathematical models (including manageable computer models) of a wide range of important fluid flows. Some of the analyses, as we shall see, involve mathematically exciting theories which, incidentally, are of a strikingly nonlinear character. The book's prime emphasis, however, is on the problem of how to use those as strong supports at one end of an effective bridge spanning the world of mathematics and the world of experiment and observation.

The illustrative examples given are concerned entirely with water and air, rather than with more complicated fluids such as blood or the fluids (including highly viscous fluids) used for various lubrication purposes. They do include something about circulation systems and about resistance, as well as about the fluid mechanics that both makes flight possible and underlies the energy transfer in certain types of turbine. Features special to flows at very high velocities (significantly greater than 100 m/s) or very low velocities (significantly less than 1 m/s) are left out, however, while for matters concerned with water waves, sound waves, shock waves and the mechanics of stratified fluids, readers are referred to the present author's *Waves in Fluids* (Cambridge University Press, 1978) which they may find a suitable text for study after they have read (say) Chapters 1 to 8 of the present work.

London J.L.
July 1985

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1
Principles of mechanics applied to lumps of fluid

All the analysis in this book is founded upon the basic principles of mechanics, such as Newton's laws of motion and the momentum and energy principles. These principles are assumed to be known, along with the most elementary physical properties of fluids. The present chapter, however, takes the study of fluid mechanics only as far as can be achieved by applying the principles of mechanics to 'lumps' of fluid on a large scale.[†]

1.1 Elementary mechanics of a fluid in equilibrium

The word **hydrostatics** is often used to describe the mechanics of a fluid in equilibrium; in other words, to describe the statics of a fluid. The elements of hydrostatics form a simple body of knowledge which is assumed to be known to the reader, and which is now briefly summarized.

The forces that act on a lump of fluid in equilibrium (Fig. 1) consist of (*i*) its weight and (*ii*) forces acting normal (that is, perpendicular) to its boundary.[‡]

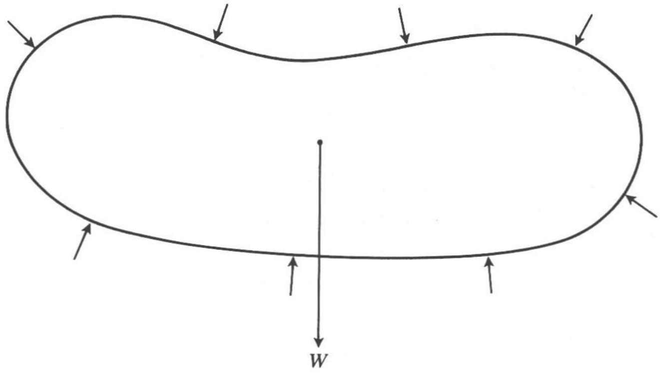

Fig. 1. The forces acting on a lump of fluid in equilibrium consist of its weight W and pressure forces acting normal to its boundary.

[†] Rather than by more systematically building up knowledge from their application to very small 'particles' of fluid as in later chapters.

[‡] Here is an important difference from a lump of solid material, which can be in equilibrium under a system of forces including tangential components.

The magnitude of these latter forces per unit area of boundary is called the pressure.

At each point in a fluid in equilibrium the pressure has a definite value which for *any* lump of fluid with that point on its boundary is the normal force per unit area acting on that lump. In other words, the pressures acting in every direction are equal.

Another very well known result, that the pressure is the same at all points of the fluid which are at the same height, has some quite remarkable consequences. It means that a lump of fluid can be used (Fig. 2) to permit a kilogram weight to balance a tonne if their areas of application are in the ratio 1 : 1000. This is the principle of the hydraulic press: a device which can have an almost indefinitely large 'mechanical advantage' (ratio of the output force, used here to support, or perhaps to raise, the tonne weight, to the input force, here supplied by a weight of one kilogram; or, for raising, slightly more).

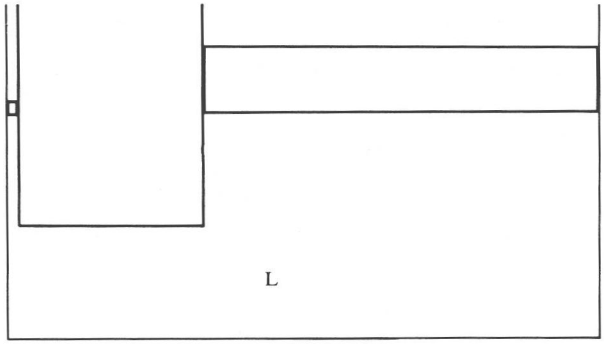

Fig. 2. Schematic illustration of the principle of the hydraulic press: the small weight acting on a small area generates the same fluid pressure as the much larger weight acting on a proportionately larger area.

The balance of forces upon a lump of fluid in equilibrium (Fig. 1) tells us that the *resultant* of the normal pressures must be a force equal and opposite to the weight of the lump. This is Archimedes' Principle, which for fluid of effectively uniform density ρ (that is, with negligible *stratification* of density), gives the upward resultant as

$$\rho V g, \tag{1}$$

where V is the volume of the lump and g is the acceleration due to gravity. This result applies also if the lump is replaced by a solid lump of the same size and shape: the distribution of pressures in the fluid is unchanged and they have the same upward resultant ρVg. In words, the *buoyancy force* on the body is equal to the weight of fluid which the body has displaced.

Finally, Archimedes' Principle applied to a lump in the form of a cylinder with vertical generators, which stretches between two levels at heights H_1 and H_2 above some 'reference level' (e.g. the ground), tells us that the difference in the pressures p_1 and p_2 at the two levels is

$$p_2 - p_1 = \rho g(H_1 - H_2); \tag{2}$$

an equation identifying buoyancy force with weight per unit cross-sectional area of that cylinder. The distribution of pressure in a fluid of uniform density in equilibrium is, in short, specified by the rule

$$p + \rho g H = \text{constant}. \tag{3}$$

1.2 Flow through a contraction in a horizontal pipe

From elementary statics we now move to how the basic principles of dynamics can be applied to fluids in motion. The impact of a horizontal jet of fluid, of density ρ, velocity v, and cross-sectional area S, on a wall at right angles to the jet is commonly used in dynamics texts to illustrate the application of the momentum principle. The jet delivers a mass of fluid $\rho S v$ per second, and so the jet force on the wall is estimated as

$$\rho S v^2, \tag{4}$$

this being the rate of delivery by the jet of horizontal momentum, all of which is assumed destroyed at the wall.

Even if we are satisfied with relatively crude estimates, however, there are severe limitations to the range of problems for which such simple considerations of mass and momentum can give useful information. In order to illustrate this we study the flow of fluid through a gradual contraction in a horizontal pipe (Fig. 3), and focus attention on a large lump L of fluid

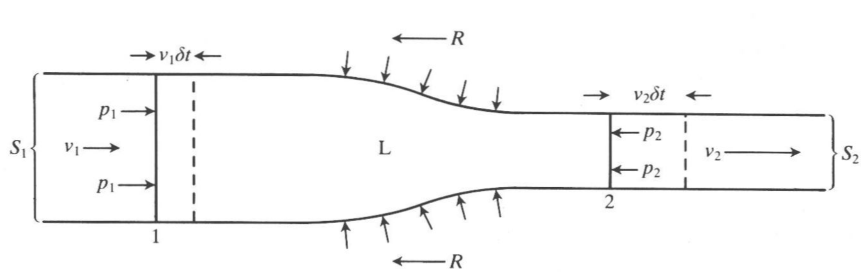

Fig. 3. Flow through a horizontal pipe with a gradual contraction in cross-sectional area from S_1 to S_2. In time δt the lump L of fluid moves from a position between the two plain lines at stations 1 and 2 to a position between the two broken lines. Pressure forces, shown as acting normally to the pipe wall in the region of the contraction, have the horizontal resultant R which opposes the net pressure force $p_1 S_1 - p_2 S_2$ acting on the lump L at stations 1 and 2.

stretching from station 1, of cross-sectional area S_1, upstream of the contraction, to station 2, of smaller cross-sectional area S_2, downstream of it.

In a small time δt, the front face of lump L is carried downstream a distance $v_2 \delta t$ where v_2 is the fluid velocity at station 2, and the back face is carried downstream a distance $v_1 \delta t$ where v_1 is the velocity at station 1. Consequently, the region occupied by lump L changes, by taking in an additional volume $S_2 v_2 \delta t$ at the front and vacating a region of volume $S_1 v_1 \delta t$ at the back. The *rate of change* of mass for lump L consists then of two terms: from the additional volume a positive contribution $\rho S_2 v_2$ and from the vacated volume a negative contribution $-\rho S_1 v_1$. As the mass is necessarily conserved we deduce that at each instant

$$S_1 v_1 = S_2 v_2, \tag{5}$$

where both sides of the equation represent the instantaneous rate of volume flow through the pipe.

We now give particular consideration to a 'steady-flow' case, i.e. one in which the rate of volume flow is not changing with time. Although the volume flow is constant, eqn (5) shows that $v_2 > v_1$ (as $S_2 < S_1$); the fluid *speeds up* as it enters the contraction.

What force produces this acceleration? In order to generate such a force it seems clear that the fluid pressure p_1 at station 1 must exceed the pressure p_2 at station 2, but the amount of that excess is not easily estimated. For example, consideration of the rate of change of momentum for lump L fails to determine the pressure drop $p_1 - p_2$ because the *horizontal resultant* R of the pressure forces between the pipe and the fluid is unknown (see Fig. 3 where these forces act at right angles to the boundary so that in the region of the contraction they must possess components along the axis of the pipe).

The net horizontal force accelerating lump L is $p_1 S_1 - p_2 S_2 - R$, consisting of a pressure force at station 1 opposed by a smaller pressure force at station 2 and by the reaction at the pipe. The corresponding rate of change of momentum consists of a positive contribution $\rho S_2 v_2^2$ at station 2 (the velocity v_2 times the rate of change of mass $\rho S_2 v_2$ due to lump L moving into an additional volume at the front) and a negative contribution $-\rho S_1 v_1^2$ at station 1 (similarly associated with the volume vacated by lump L at the back; note that in steady flow the momentum in any fixed region of space remains constant, so that any change in the momentum of a lump of fluid arises from changes in the region it occupies). Equating the force to the rate of change of momentum gives us an equation

$$p_1 S_1 - p_2 S_2 - R = \rho S_2 v_2^2 - \rho S_1 v_1^2, \tag{6}$$

but as this equation (to which we return in Section 1.4) includes the unknown reaction R it fails to give us information about the pressure drop $p_1 - p_2$.

The way to obtain a useful estimate of that pressure drop is, rather, by considerations of energy. The rate of change of kinetic energy of lump L consists of a positive contribution $(\frac{1}{2}v_2^2)(\rho S_2 v_2)$ at station 2 (half the velocity squared times the rate of change of mass) and a corresponding negative contribution $-(\frac{1}{2}v_1^2)(\rho S_1 v_1)$ at station 1. If we equate this rate of change to the rate at which external forces do work on the lump L we obtain a positive contribution $(p_1 S_1)v_1$ (force times velocity component in the direction of the force) at station 1, while the contribution $-(p_2 S_2)v_2$ from station 2 is negative (because the force is in the opposite direction to the velocity). On the other hand, the pressure forces between the pipe wall and the fluid act at right angles to the velocity (Fig. 3) *and therefore can do no work*, so that, on this analysis, the unknown reaction R makes no contribution.

Equating the rate at which work is done on lump L to its rate of change of energy, we obtain the equation

$$p_1 S_1 v_1 - p_2 S_2 v_2 = (\tfrac{1}{2}v_2^2)(\rho S_2 v_2) - (\tfrac{1}{2}v_1^2)(\rho S_1 v_1) \qquad (7)$$

which can be simplified by taking out a factor given by either side of eqn (5) to give

$$p_1 - p_2 = \tfrac{1}{2}\rho v_2^2 - \tfrac{1}{2}\rho v_1^2. \qquad (8)$$

This equation for estimating the pressure drop is discussed further, and critically examined, in the next section.

1.3 The total head of a steady stream

There are several reasons why eqn (8), although often very useful, can at most be only a rough approximation to the pressure drop which, indeed, it tends to underestimate. The first of these reasons is very familiar from elementary mechanics, where energy arguments may produce only crudely approximate results when they neglect the dissipation of kinetic energy due to *friction.*

Indeed, frictional forces (that is tangential forces) do occur between a solid boundary and a *moving* fluid, even though the well established and experimentally well supported laws of hydrostatics (Section 1.1) rule out such forces for a fluid at rest. Later, we shall see that the magnitude of any tangential force acting at the boundary of a lump of fluid that moves in a particular flow pattern depends on a well defined physical property of the fluid called the *viscosity*. We shall find, furthermore, that the fluids upon which this book concentrates (air and water) are fluids of *small* viscosity, in a certain well defined sense.

It might be tempting to infer in rather general terms from this that energy arguments such as were used to derive eqn (8) give results which are good approximations for air and water. The truth is more complicated, however. Equation (8) gives results that agree quite reasonably with experiment for the case actually illustrated in Fig. 3 (a gradual contraction) but gives results that are very inaccurate for some other pipe geometries, for example an *expansion* of cross-section (see Section 1.5), or an *abrupt* contraction.

Much of the present book is concerned with expounding the intricate reasons for these distinctions. They are important not merely because they set limits on the applicability of a method of calculation but also because (as in other branches of mechanics) the motions which keep energy dissipation to a minimum are most advantageous for many engineering applications. In this section, however, we just briefly initiate that discussion.

Frictional forces can be important not only between a lump of fluid and its *solid* boundary (as so far discussed) but also between neighbouring lumps of fluid. Such *internal* friction is able to dissipate kinetic energy into heat energy. Admittedly, the rate of frictional dissipation for a fluid moving in a particular flow pattern is another quantity proportional to the viscosity of the fluid. Nevertheless, there are certain pipe geometries which (for reasons that will emerge later) lead to flow patterns of a type especially prone to dissipate energy even for fluids of very small viscosities, and this ruins the accuracy of eqn (8).

One further obstacle to the accuracy of the equations of Section 1.2 exists. Friction tends to produce an uneven distribution of fluid velocities across the pipe, with the flow retarded more near a solid wall; yet in the arguments leading to eqn (5), for example, the velocity v_1 was assumed uniform across the cross-section (and similarly with v_2). Admittedly, a detailed study of those arguments shows that eqn (5) must remain correct if v_1 is the fluid velocity at station 1 *averaged* across the cross-sectional area (and similarly with v_2). However, this interpretation makes for difficulties in eqn (6) which, on a similar basis, would remain correct only if v_1^2 were the *fluid velocity squared* averaged over the cross-sectional area. Clearly, this is incompatible with the former determination of v_1 since the average of the square of a quantity always exceeds the square of its average. Similar difficulties arise in eqns (7) and (8).

For certain pipe geometries, however, including that of Fig. 3, typical flows of water (or of air) in the ranges of speed studied in this book involve velocity distributions that are almost uniform across the pipe except *very* near the wall. In such a case the above difficulties (as well as those others remarked on earlier) lead to only modest errors.

If the pipe in Fig. 3 is not horizontal, stations 1 and 2 may be at different heights H_1 and H_2 above the ground. In this case, the rate of change of potential energy due to gravity (gH per unit mass) has to be added on to the right-hand side of eqn (7), giving an extra term

$$(gH_2)(\rho S_2 v_2) - (gH_1)(\rho S_1 v_1). \tag{9}$$

When the factor $S_2 v_2$ (or $S_1 v_1$, which is the same) is taken out of expression (9), this expression provides an extra term $\rho g H_2 - \rho g H_1$ on the right-hand side of eqn (8), which can then be written

$$p_1 + \rho g H_1 + \tfrac{1}{2}\rho v_1^2 = p_2 + \rho g H_2 + \tfrac{1}{2}\rho v_2^2. \tag{10}$$

The quantity

$$p + \rho g H + \tfrac{1}{2}\rho v^2, \tag{11}$$

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